

Optimal Gear Design for Equal Strength Teeth Using Addendum Modification Coefficients

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ABSTRACT:

The addendum modification coefficient or shift factor is first defined and explained in detail in terms of gear design terminology. Two types of helical gear design equations are derived, one with the true generating shift factor, and the other with the conventional shift or addendum modification factor used in the United States. Nakada's equation and a cantilever beam equation used to design equal bending strength teeth are reviewed and discussed. Then a new design method to synthesize equal strength teeth between mating pinion and gear, which may have different material properties, is presented. This method uses the shift factors as design variables and an effective nonlinear equation solver to find the solution. Various numerical examples are given to illustrate this new method.

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Introduction :

There must be many advantages in the involute tooth form that has enabled it to practically eradicate all other tooth forms that have existed during the last several decades. One of the major advantages is as stated by Buckingham [1] that when the involute tooth form provides conjugate action it becomes the only tooth form that yields a constant pressure angle from point to point. It is well known, however, that the most important attribute is the manufacturability of involute gears: the involute gears can be cut by a straight-edge cutter. A "straight-line cutter" can not only be easily manufactured but also easily checked, and is thus easily maintained for the high precision production of gears. This paper is focused on yet another important advantage of the involute tooth form - the addendum modification coefficient or involute shift factor for gears with variable or non-standard center distance, which can be used to balance the teeth strength, increase the contact ratio and minimize the weight for mating gears.

It is generally known that Maag [2] was one of the first to introduce gears with addendum modification coefficients. Maag used the coefficient to balance the bending strengths of the teeth for mating pinion and gear, but the method as well as most criteria used in the design of his Maag gear were proprietary and remained undisclosed to gear design engineers for several decades. Nakada [3]

first published an equation containing x_1 and x_2 , the addendum modification coefficients of the pinion and gear, which provides for equal tooth thicknesses of the pinion and gear on the base circle, and stated this tooth form to approximate the Maag gear. Mabie et. al. [4] used cantilever beam stress theory as a model for predicting gear stresses and an interpolation process to balance the stresses in mating teeth. In this paper, three methods to balance the bending fatigue safety factors based on the AGMA bending stress formulae, and the solution of nonlinear equations, are described and illustrated with various numerical examples.

List of Symbols :

C_r	operating center distance
D_1	operating pitch diameter of pinion
D_2	operating pitch diameter of gear
D_{b1}	base circle diameter of pinion
D_{b2}	base circle diameter of gear
D_{o1}	outside diameter of pinion
D_{o2}	outside diameter of gear
ee	sum of hob offsets ($ee = e_1 + e_2$)
e_1	hob offset for pinion ($e_1 = x_1 m_n$)
e_2	hob offset for gear ($e_2 = x_2 m_n$)
F	effective face width
h	modified tooth whole depth
h_{ao}	hob addendum



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J	AGMA J geometry factor
k	top clearance factor
K_a	AGMA application factor
K_b	AGMA rim thickness factor
K_L	AGMA life factor
K_m	AGMA load-distribution factor
K_R	AGMA reliability factor
K_T	AGMA temperature factor
K_s	AGMA size factor
K_v	AGMA dynamic factor
m	module for spur gear
m_o	gear ratio
m_n	normal module
m_s	transverse module
n₁	pinion operating speed
n₂	gear operating speed
N₁	pinion tooth number
N₂	gear tooth number
P_N	normal base pitch
Q_v	AGMA Quality number
R_{b1}	base circle radius of pinion
R_{b2}	base circle radius of gear
SF_b	desired safety factor for bending
T	transmitted torque
xx	sum of addendum modification coefficients
x₁	pinion addendum modification coefficient or pinion shift factor
x₂	gear addendum modification coefficient or gear shift factor
x_{g1}	pinion generating (real) shift factor
x_{g2}	gear generating (real) shift factor
Y, Y_c	coefficient of increase in center distance
W_n	normal load
W_t	tangential load
β	angle between W_t and W_n
δ_o	tool protuberance
Δc_n	total backlash on base circle
Δc_{n1}	amount pinion tooth thinned for backlash on base circle
Δc_{n2}	amount gear tooth thinned for backlash on base circle
Δs_n	total backlash on pitch circle
Δs_{n1}	amount pinion tooth thinned for backlash on pitch circle
Δs_{n2}	amount gear tooth thinned for backlash on pitch circle
ρ_{ao}	tool tip radius
φ	standard transverse pressure angle
φ_n	standard normal pressure angle
φ_r	operating pressure angle
σ	bending stress
σ_{a11}	AGMA allowable bending stress
σ_{at}	bending strength
σ_t	AGMA bending stress
ψ	standard helix angle
ψ_b	base helix angle

Definitions of Addendum Modification Coefficient :

In many European countries and Japan the term addendum modification coefficient or rack shift factor, **x**, is meant to include both shifting of the rack for addendum modification and shifting of the rack for thinning of the teeth to obtain backlash. While American gear designers, following the AGMA standards [5], refer to two separate shift coefficients - the generating rack shift coefficient, **x_g**, applies to the completely finished tooth, and includes shifting of the rack for addendum modification plus shifting of the rack for backlash. **x**, the rack shift coefficient for addendum modification, only includes the shifting of the rack necessary for the specified addendum modification.

The relationship between the shift coefficients is summarized as follows:

x_g (real shift factor)

= **x** (in Europe)

= **x** (in USA) - $\frac{\Delta s_n}{2 m_n \tan \phi_n}$

General Equations of Profile Shifted Helical Gears :

The following equations are the general equations for profile shifted helical gears, with derivations given in Appendix A. There are two forms of these equations because of the above mentioned alternative definitions for the shift coefficient of the rack.

Design Equations of Helical Gears with **x_g**, (the generating shift factor)

$$\text{inv} \phi_r = 2 \tan \phi_n \left(\frac{x_{g1} + x_{g2} + Y_c}{N_1 + N_2} \right) + \text{inv} \phi \quad (1)$$

$$C_r = \frac{N_1 + N_2}{2 \cos \psi} m_n + y m_n \quad (2)$$

$$D_{o1} = \left(\frac{N_1}{\cos \psi} + 2 \right) m_n + 2 (y - x_{g2}) m_n \quad (3)$$

$$D_{o2} = \left(\frac{N_2}{\cos \psi} + 2 \right) m_n + 2 (y - x_{g1}) m_n \quad (4)$$

$$h = (2 + k) m_n - (x_{g1} + x_{g2} - y) m_n \quad (5)$$



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where

$$\tan\phi = \frac{\tan\phi_n}{\cos\psi}$$

$$y = \frac{N_1 + N_2}{2\cos\psi} \left(\frac{\cos\phi}{\cos\phi_n} - 1 \right)$$

$$y_c = \frac{1}{2\sin\phi\sqrt{1-\sin^2\psi\cos^2\phi_n}} \left(\frac{\Delta C_n}{m_n} \right)$$

Design Equations of Helical Gears with x , the US Conventional Shift Factor

$$\text{inv}\phi_r = 2 \tan\phi_n \left(\frac{x_1 + x_2}{N_1 + N_2} \right) + \text{inv}\phi \quad (6)$$

$$C_r = \frac{N_1 + N_2}{2 \cos\psi} m_n + y m_n \quad (7)$$

$$D_{o1} = \left(\frac{N_1}{\cos\psi} + 2 \right) m_n + 2 (y - x_2) m_n + 2 k_2 \quad (8)$$

$$D_{o2} = \left(\frac{N_2}{\cos\psi} + 2 \right) m_n + 2 (y - x_1) m_n + 2 k_1 \quad (9)$$

$$h = (2 + k) m_n - (x_1 + x_2 - y) m_n + (k_1 + k_2) \quad (10)$$

$$\Delta C_n = \Delta C_{n1} + \Delta C_{n2} \quad (11)$$

where

$$k_1 = \frac{\Delta C_{n1}}{2\sin\phi_n} \quad k_2 = \frac{\Delta C_{n2}}{2\sin\phi_n}$$

Nakada's Equation :

Nakada [3] first published the criteria which establishes equal tooth thickness for mating gears on the base circle. The equation is stated as:

$$\frac{x_{g2} - x_{g1}}{N_2 - N_1} = - \frac{\text{inv}\phi_n}{2\tan\phi_n}$$

Derivation of this equation is given in Appendix B.

As an example of designing equal tooth thickness gears consider the following problem.

Example 1

Given: $N_1 = 14$, $N_2 = 15$, $\phi_n = 20^\circ$,
 $x_{g1} = 0.1812$

Find the gear shift factor, x_{g2} , which makes the gear's tooth thickness equal to the pinion's tooth thickness on the base circle. Using Nakada's equation the solution is readily found.

Solution:

$$x_{g2} = x_{g1} - \frac{\text{inv}\phi_n}{2\tan\phi_n} = 0.1607$$

Although, the tooth thicknesses are the same for these mating gears, it does not follow that the bending stresses under operation will be the same.

Designing for Equal Bending Strength :

Method 1: In case of same material properties for the pinion and gear.

Mabie et. al. [4] assume that the gear tooth is acting as a cantilever beam and that the load is applied at the tip of the tooth and carried by a single pair of teeth. Using traditional cantilever beam theory the bending stress is calculated, and then the addendum modification needed to balance the bending strengths are found by an iterative process. The first method presented here extends this work by making use of the AGMA formulae for calculating the bending stress which considers both the tangential and radial components of the loading, as well as the stress concentration factor, and by solving for the required addendum modification directly from a describing nonlinear equation.

The AGMA formula for bending stress (without the appropriate stress factors) is given by the equation:

$$\sigma = \frac{W_t}{F m_n J} \quad (12)$$

Rewriting this stress equation into the form introduced in reference [4] so that the load and face width need not be considered, results in

$$\frac{\sigma F}{W_n} = \frac{\cos\beta}{m_n J} \quad (13)$$

The J -factor is calculated using the method described in AGMA paper p139.03 [6]. The method used here for solving the