

Effect of Radial Runout on Element Measurements

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ABSTRACT:

Radial runout in a gear can contribute substantially to measured values of Profile, Pitch, Index, and in helical gears, Tooth Alignment (Lead) Variations. This paper takes each of these variations in turn and for AGMA Standard measurement methods, gives the equation that relates the measured values to the amount of radial runout in an otherwise ideal gear.

To demonstrate these relationships, the results of actual measurements are compared to values calculated from these equations. The test gear used was a specially designed master gear, itself without appreciable runout, but with a pre-set amount of runout introduced by mounting on an eccentric arbor. For these comparisons, both CNC and other types of inspection equipment were used. Measurements are also provided for the master gear without the pre-set runout.

There is also discussion of how the guidance provided by the equations can be used to aid interpretation of gear inspection data, to explain differences between measurement methods, and to encourage revision of element tolerances.

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Introduction

Radial Runout is recognized as one of several element variations that together largely define the geometric quality of a gear. Others identified by the AGMA Gear Classification and Inspection Handbook, AGMA 2000-A88, are Profile, Pitch (and closely related Index), and Tooth Alignment (Lead). In this Handbook, as in other references, each element variation is treated separately, with its own measurement method and with its own set of tolerances. This leaves the impression that all variation measurements are independent of each other, that the presence of one variation will not affect the measurements of any of the others.

On the other hand, actual inspection records often give evidence of the presence of Runout in measurements of other element variations. In Figure 1, a set of Profile measurements, Runout is recognized by the differences in slope of the individual curves from the four equally spaced teeth. In Figure 2, showing Pitch measurements, Runout is revealed by the general shape, resembling a sine-wave, of the successive plotted values for all the teeth on the gear. The same effect is shown in the Index measurements of Figure 3. Finally, the Tooth Alignment (Lead) measurements on a helical gear in Figure 4 show the Runout by the differences of slope.

This paper will describe the mathematical relationships between the amount and direction of Runout and the resulting effects on the measurements of these other element variations. Such relationships should prove valuable in at least the following ways:

- To lead to methods of using any of the element measurements to determine the exact magnitude and location of Runout on the gear. Such information can then be used for correction when a more direct measurement of Runout is not available.
- To help establish a revised set of element tolerances which more accurately account for the contribution of Runout to Profile, Pitch, and Lead.
- To encourage the recognition and tolerancing of new element variations corresponding to Profile, Pitch, and Lead from which the effect of Runout has been removed.

Mathematical Relationships

Each of these are derived with the following conditions:

- The gear is an external spur or helical gear. (Equations for helical gears can be reduced to spur gears by setting the helix angle to zero.)
- The gear has ideal geometry except for Runout caused by the eccentricity of the gear tooth pattern with respect to the central datum. This excludes Runout due to out-of-round.
- The amount of Runout is very small in relation to the tooth size. This means that the Runout does not substantially change the angle of the tooth profile at the measuring probe location as the measurement moves from tooth to tooth.
- The measurement method is as described in the AGMA Handbook.
- The probe has either a sharp edge, a properly oriented cylindrical edge, or a spherical radius. (On some types of measuring equipment, a compensation for probe size is included in the reported measurement. When using the equations to predict measurements on such equipment, a zero radius is used.)

The terminology used in these mathematical relationships is as follows:

- The tooth identification terminology used is the same as in the Handbook (see Figure 5). (At least one make of inspection equipment reports its measurements using an opposite system of identifying tooth number and tooth flank. This identification must be corrected before the equations are used.)
- The symbols and corresponding terms are listed in Table 1. Some of the symbols are also shown in Figure 6 and are used on the figures associated with each type of measurement. Table 1 also identifies the usual source of the data item for each symbol. Where needed, it also supplies the simple equation for calculating the data item from other data.
- Symbols for the element variations, wherever possible, are identical to those used in the Handbook. The definitions of these variations, in terms of direction of measurement, are also identical.

Profile

Figure 7(a) shows the left flank of a tooth on an ideal, that is, concentric, gear. Also shown is the same flank displaced due to Runout. The direction and amount of displacement corresponds to the eccentricity indicated at the gear center. The Profile measurement at a point on the involute is in the direction normal to the involute curve. The figure identifies the various angles that combine to relate the measurement direction to the eccentricity direction. Figure 7(b) shows an enlarged detail at the point of measurement. It contains a small right-triangle which relates the measurement to the flank displacement. This triangle and the angles associated with its sides give the following equation for the Profile measurement on the left flank:

$$M_{\phi L} = e \cdot \sin[(\theta_{e|L} - \alpha_{PBt}) + (k-1)\tau + \epsilon_M] \quad [\text{Eq 1}]$$

Note that this equation is a sine function in which the angle has three components. The first, $(\theta_{e|L} - \alpha_{PBt})$, is a constant for the gear being measured since it contains the direction of the eccentricity and a measure of the tooth thickness adjusted for the probe size. The second, $(k-1)\tau$, is constant for a single tooth but changes as measurements are moved to other teeth spaced around the gear. The third component is the roll angle at the point along the involute at which the measurement is being made. This changes as the point is moved along the tooth being measured, from point S at the Start of Measured Profile (SMP) to point E at the End of Measured Profile (EMP).

A plot of Equation 1 is shown in Figure 8(a). The four indicated portions of the sine-wave represent the four Profile measurements made at four teeth equally spaced around the gear. These measurements are grouped together in Figure 8(b) as they would appear in an inspection chart. What are customarily taken as straight lines of varying slope are actually small portions of a sine-wave taken at spaced intervals. For this otherwise ideal gear with Runout, the Profile Variation at any one tooth is generally equivalent to this 'slope', taken as the difference between the measurements at the start and end of the measured portion of the involute. This Profile Variation at any one tooth is closely expressed by the following equation in which the eccentricity is replaced by half the Runout Variation:

$$V_{\phi L} \approx \frac{V_r}{2} (\epsilon_E - \epsilon_S) \left(\frac{\pi}{180^\circ} \right) \times \cos[(\theta_{eIM} - \alpha_{PBt}) + (k-1)\tau + \left(\frac{\epsilon_E + \epsilon_S}{2} \right)]$$

$$\text{for: } (\epsilon_E - \epsilon_S) \leq 30^\circ \quad [\text{Eq 2}]$$

The quality of the gear with respect to Profile is generally defined by the 'worst' Profile Variation among those measured at the several teeth (usually 3 or 4). This equation and Figure 8(a) show that the maximum possible slope or Profile Variation depends on: (1) the amount of Runout, (2) the roll angle interval, which is generally inversely proportional to the number of teeth in the gear, and (3) the usually accidental choice of which tooth is used to start the inspection. In regards to the last item, the maximum variation will appear if a measurement is made at a tooth corresponding to the 0 or 180 degree location along the sine-wave, as shown in Figure 8(c). The magnitude of this maximum Profile Variation is given by the following equation:

$$V_{\phi(\max)} \approx \frac{V_r}{2} (\epsilon_E - \epsilon_S) \left(\frac{\pi}{180^\circ} \right) \quad [\text{Eq 3}]$$

The smallest 'worst' variation, approximately 30 percent smaller, will be obtained if the teeth are selected to correspond to the 45, 135, 225, and 315 degree locations, as shown in Figure 8(d).

The corresponding equations for Profile measurements and Profile Variation for the right flank, given below, are very similar to those for the left. There are sign differences in the first two components of the angle of the trigonometric function. These sign differences translate into a 'phase-shift' difference which, in turn, changes the locations along the sine-wave at which the measurements are positioned and explains why the 'slopes' on the measurement traces for the two flanks of the same tooth are generally not the same.

$$M_{\phi R} = e \times \sin[(-\theta_{eIM} - \alpha_{PBt}) - (k-1)\tau + \epsilon_M] \quad [\text{Eq 4}]$$

$$V_{\phi R} \approx \frac{V_r}{2} (\epsilon_E - \epsilon_S) \left(\frac{\pi}{180^\circ} \right) \cos[(-\theta_{eIM} - \alpha_{PBt}) - (k-1)\tau + \left(\frac{\epsilon_E + \epsilon_S}{2} \right)]$$

$$\text{for: } (\epsilon_E - \epsilon_S) \leq 30 \quad [\text{Eq 5}]$$

Pitch and Index

These two variations are closely related, as explained in the Handbook. Since each Pitch measurement can be derived from two successive Index measurements, the Index relationship will be developed first.

Figure 9(a) shows the left tooth flank in the concentric and displaced eccentric positions. The measurement is in the transverse plane and in the tangential direction as specified in the Handbook. Figure 9(b) shows the geometric relationship connecting this measurement and the eccentricity. The measurement is expressed in the following equation for the left flank.

$$M_{xL} = \frac{e}{\cos \phi_{Mt}} \times \sin[(\theta_{eIM}) + (\phi_{Mt} - \alpha_{PMt}) + (k-1)\tau] \quad [\text{Eq 6}]$$

This equation is also a sine function. The angle is expressed in three components. The first two are constant for the gear and the measurement set-up and the third varies with the number of the tooth at each measurement. The equation is plotted in Figure 10(a). The x-axis represents the rotation angle going around the gear in the direction of increasing tooth number. The measurement trace made by inspection equipment would not have the form of this smooth curve. The trace, instead, would be a series of line segments, as also shown in the figure, or bars, as in Figure 3. The tips of such lines or the centers of the tops of the bars, however, would lie along this smooth curve.

Figure 10(a) shows the Total Index Variation as the total amplitude of the sine curve. The relationship to the Runout Variation is expressed in the following equation.

$$V_{x(tot)} = \frac{V_r}{\cos \phi_{Mt}} \quad [\text{Eq 7}]$$

This shows that the Total Index Variation and the Runout Variation are not exactly equal. The difference is in the cosine of the transverse pressure angle at the point of measurement. This effect of this cosine term is generally small, a little over 6 percent for a 20 degree transverse pressure angle and about 10 percent for 25 degrees. Some inspection equipment make the measurement differently from the specification in the Handbook, still in the transverse plane but in the direction normal to the involute instead of tangential to the measurement circle. The Modified Total Index Variation reported by this method equals the Runout Variation.

The equation for the Index measurement for the right flank is as follows.

$$M_{xR} = \frac{e}{\cos \phi_{Mt}} \times \sin[-(\theta_{eIM}) + (\phi_{Mt} - \phi_{PMt}) - (k-1)\tau] \quad [\text{Eq 8}]$$

This equation is plotted in Figure 10(b). Note that the curves for the left and right flanks are the same except for the sine-wave starting points, or phase-shift. It is often mistakenly assumed that the tooth with the peak Index measurement is located 90 degrees from the direction of the eccentricity. The equations and Figure 10 show that this is not precisely true.

Inspection equipment typically supplies the measurement traces for the two flanks in an 'opposing' arrangement, as in Figure 3, rather than in the 'parallel' arrangement of Figure 10. The two curves plotted in this arrangement appear in Figure 11. The 'peaks' of the two curves still do not line up with each other but, when extended as shown, symmetrically straddle the location of the eccentricity. The spacing between these 'peaks' equals twice the second component in the angle portion of the sine function in Equations 6 and 8, as shown in the figure.

As noted previously, each Pitch measurement is equivalent to the difference between the Index measurements at adjacent teeth. The left flank Index and Pitch measurement traces in Figures 12(a) and 12(b) show how the Pitch values are derived from the Index values. The left flank Pitch values can also be found from the following equation by using 'one-half k' values. For example, to calculate the pitch value for the adjacent teeth numbered 1 and 2, use $k=1.5$.

$$M_{pL} = \frac{e}{\cos \phi_{Mt}} \left(\frac{2\pi}{N} \right) \times \cos[(\theta_{eIM}) + (\phi_{Mt} - \phi_{PMt}) + (k-1)\tau] \quad [\text{Eq 9}]$$

The corresponding equation for the right flank is as follows:

$$M_{pR} = \frac{e}{\cos \phi_{Mt}} \left(\frac{2\pi}{N} \right) \times \cos[180^\circ - (\theta_{eIM}) + (\phi_{Mt} - \phi_{PMt}) - (k-1)\tau] \quad [\text{Eq 10}]$$

The maximum plus or minus Pitch Variation is related to the Runout Variation in the following equation. Note that this variation is determined in part by the number of teeth.

$$V_P(\max) \approx \frac{V_r \pi}{N \cos \phi_{Mt}} \quad [\text{Eq 11}]$$

Another way to present Pitch data is by Accumulated Pitch over some selected number of teeth. If the number of teeth consists of the entire gear, this measurement, made by adding successive Pitch measurements, is essentially the same as the Index measurement. The Total Accumulated Pitch Variation is therefore the same as the Total Index Variation.