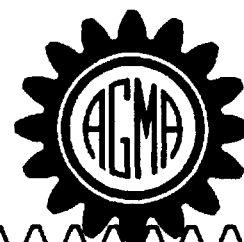


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Fatigue Analysis of Shafts for Marine Gearboxes

by: E. William Jones and Anying Shen, Mississippi State
University and Robert E. Brown, Caterpillar, Inc.

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**E. William Jones and Anying Shen, Mississippi State University and
Robert E. Brown, Caterpillar, Inc.**

[The statements and opinions contained herein are those of the author and should not be construed as an official action or opinion of the American Gear Manufacturers Association.]

Abstract

The design of shafts for marine gearboxes, which may include the effects of torsional vibration, is presented. The influence of the vibratory torque on the values of shaft diameter and safety factor is discussed. Use of the Finite Element Method to evaluate unknown stress concentration factors is illustrated. A program for the design of shafts, which are subjected to fatigue, has been developed.

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American Gear Manufacturers Association
1500 King Street, Suite 201
Alexandria, Virginia, 22314

October, 1994

ISBN: 1-55589-635-9

FATIGUE ANALYSIS OF SHAFTS FOR MARINE GEARBOXES

E. William Jones, Ph.D., P.E.

Anying Shen, Graduate Student

Department of Mechanical Engineering

Mississippi State University, Mississippi State, MS 39762

Robert E. Brown, Project Engineer--Gear Design

Caterpillar, Inc.

NOMENCLATURE

M_a alternating reversed-bending moment, lb in
 M_m mean bending moment, lb in
 T_a alternating shaft torque, lb in
 T_m mean shaft torque, lb in
 P_a alternating axial force, lb
 P_m mean axial force, lb
 V_a alternating shear force, lb
 V_m mean shear force, lb
 σ_a alternating normal stress, psi
 σ_m mean normal stress, psi
 τ_a alternating shear stress, psi
 τ_m mean shear stress, psi
 σ' von Mises equivalent stress, psi
 A cross section area, in²
 I moment of inertia of shaft, in⁴
 J polar moment of inertia of shaft, in⁴
 c radius of the shaft, in
 d_{os} shaft outside diameter, in
 d_{is} shaft inside diameter, in
 K_{fb} fatigue stress concentration factor in bending
 K_{ft} fatigue stress concentration factor in torsion
 K_{fp} fatigue stress concentration factor in tension
 k_f modifying factor for stress concentration
 K_b theoretical stress concentration factor in bending
 K_t theoretical stress concentration factor in torsion
 K_p theoretical stress concentration factor in tension
 q_b notch sensitivity factor for bending

q_t notch sensitivity factor for torque
 q_p notch sensitivity factor for tension
 k' shaft fatigue limit modification
 k_a shaft surface finish factor
 k_b shaft size factor
 k_c shaft reliability factor
 k_d shaft temperature factor
 k_e shaft life factor
 k_g shaft miscellaneous factor
 S_{fc} shaft fatigue (endurance) limit of polished, unnotched test specimen in reverse bending, psi
 S_u shaft ultimate tensile strength, psi
 S_f shaft fatigue limit, psi
 S_y shaft tensile yield strength, psi
 F_s^{sy} shaft factor of safety for fatigue
 F_s^y shaft factor of safety for yielding
 ω_s shaft rotation frequency
 ω_v vibration frequency
 ϕ phase angle

INTRODUCTION

The traditional method for designing shafts for marine gearboxes is to use the ABS formula^[4]. Although it is easy to use, it has some drawbacks. First, the vibratory torque is not included in the calculation even though it is often evaluated by torsional analysis and its magnitude is the major contributor to the value of the AGMA application factors, K_a and C_a . Second, the stress concentration factors, surface finish factor and size factor, which may vary significantly from shaft to shaft, are not treated as independent design variables.

This paper presents shaft design formula which explicitly includes vibratory torque and stress concentration factors. A sensitivity study of the influence of the vibratory torque on the shaft diameter and safety factor is presented. A computer program, which is written in C, is developed to facilitate the use of this method of shaft fatigue analysis. An example of stress concentration factor evaluation by the Finite Element Method is given to illustrate the modeling procedure.

SHAFT LOADING

The shaft load includes the bending moment (M), shaft torque (T), axial force (P), and horizontal shear force (V). If the forces due to longitudinal and transverse vibrations are neglected, the M , P , and V are linear functions of T .

The maximum shaft torque may be expressed as

$$T = T_m + T_a \quad (1)$$

The amplitude of the shaft torque, $T(t)$, at any time t is:

$$T(t) = T_a \sin(\omega_v t + \phi) + T_m \quad (2)$$

where

T_m = mean torque,
 T_a = alternating torque,
 ω_v = vibration frequency,
 ϕ = phase angle.

The value T_a may be primarily due to torsional vibrations, which are functions of the prime mover and driven machinery. The amplitude of the vibrating torque may be computed by torsional vibration analysis. The Military standard 167 [5] gives limiting values for vibratory torques, which suggests values for T_a for this class of vessels. The application factors C_a and K_a used in calculation of compressive and bending stresses for gear teeth are load modifiers, which account for the increase in tooth load due to external effects including torsional vibrations. According to Det Norske Veritas [6], "Generally, the torsional vibrations shall not cause the maximum cyclic torques to exceed the approved K_a times the approved rated torque. Further, the highest permissible application factor for gears is $K_a = 1.5$. (Note the application factor used in this connection only refers to the influence of torsional vibrations.)". Hence, when the gear designers select the values of C_a and

K_a , the values suggest magnitudes for the alternating torque T_a .

The torque due to torsional vibrations depends on system parameters and is not uniformly distributed across the system, but the system is usually ill defined when the gearbox is being designed.

For a stationary shaft, the bending moment is a linear function of torque

$$M = C_1 T \quad (3)$$

where C_1 is a constant.

Strictly speaking, Equation (3) is true only for a constant torque T . If the torque T includes vibratory torque, the value of the bending moment M is no longer the simple function of torque T , then C_1 will depend on the system characteristics and the frequency of the torque T_a .

For a rotating shaft, the moment may be expressed as

$$M = C_1 T(t) \sin \omega_s t \quad (4)$$

where

ω_s = shaft rotation frequency.

Substituting (2) into (4) yields:

$$M = C_1 (T_m + T_a \sin(\omega_v t + \phi)) \sin(\omega_s t) \quad (5)$$

The vibration frequency ω_v is usually 0.5 to 12 times that of shaft rotation frequency ω_s . ϕ is not always 0. M varies with frequency and phase angle. The following formula gives the bending moment for the worst possible combination.

$$M = C_1 (T_m + T_a) \sin(\omega_s t) \quad (6)$$

For the rotating shaft the mean component of moment, M_m , is usually zero.

$$M_m = 0 \quad (7)$$

Hence, the alternating moment, M_a , is:

$$M_a = C_1 T_m + C_1 T_a \quad (8)$$

Equation (8) implies that if vibratory torque T_a is 25% of mean torque T_m , then M_a will increase to 125% of the value produced by the mean torque acting alone.

If axial vibration is neglected, the

axial force P is also a linear function of torque T

$$P = C_2(T_m + T_a \sin(\omega_v t + \phi)) \quad (9)$$

where C_2 is a constant.

Therefore

$$P_m = C_2 T_m \quad (10)$$

$$P_a = C_2 T_a \sin(\omega_v t + \phi)$$

If vibratory torque T_a increases 25%, then P_a increases 25%.

DISTRIBUTION OF STRESSES IN SHAFTS

Figure 1 shows the distribution of four types of stresses over a cross section of the shaft. From left to right these stresses are: normal stress due to bending, normal stress due to axial force, shear stress due to torsion, shear stress due to bending.

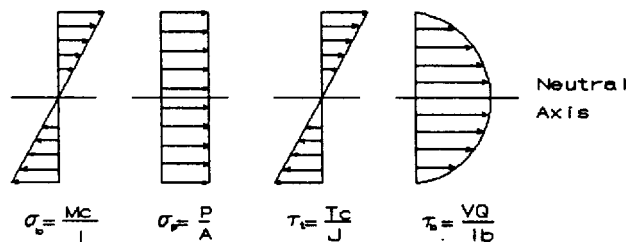


Figure 1. Stress Distribution in the Shaft

The value of Q and b for a solid circular-section shaft per Figure 2 are:

$$Q = \frac{2}{3} (R^2 - y_1^2)^{3/2} \quad (11)$$

$$b = 2\sqrt{R^2 - y_1^2}$$

For a hollow shaft per Figure 2 with $y_1 \leq r$,

$$Q = \frac{2}{3} [(R^2 - y_1^2)^{3/2} - (r^2 - y_1^2)^{3/2}] \quad (12)$$

$$b = 2(\sqrt{R^2 - y_1^2} - \sqrt{r^2 - y_1^2})$$

For a hollow shaft per Figure 2 with $y_1 > r$,

$$Q = \frac{2}{3} (R^2 - y_1^2)^{3/2} \quad (13)$$

$$b = 2\sqrt{R^2 - y_1^2}$$

where

R = shaft outside radius,
 r = shaft inside radius,
 y_1 = distance from the center to the point where the shear stress, τ_b , is evaluated. (see Figure 2).

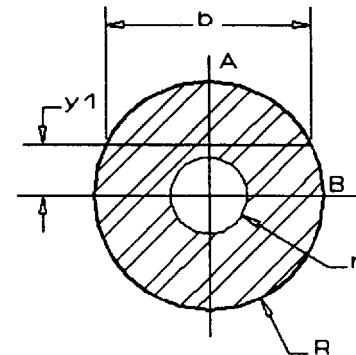


Figure 2. Shaft Cross Section

For this complex stress state, the distortion-energy theory will be used to obtain the equivalent von Mises stress. Generally, the maximum equivalent stress occurs at the point A in Figure 2 because the bending stress is dominant. However, for a very short shaft, the maximum equivalent stress may occur between point A and B. An example is given in Appendix A.

DERIVATION OF SHAFT DESIGN FORMULA

Given the shaft alternating bending moment M_a , mean bending moment M_m , alternating torque T_a and mean torque T_m , the stress components for a solid round shaft are:

$$\begin{aligned} \sigma_a &= \frac{M_a c}{I} = \frac{32}{\pi} \times \frac{M_a}{d_{os}^3} \\ \tau_a &= \frac{T_a c}{J} = \frac{16}{\pi} \times \frac{T_a}{d_{os}^3} \\ \sigma_m &= \frac{32}{\pi} \times \frac{M_m}{d_{os}^3} \\ \tau_m &= \frac{16}{\pi} \times \frac{T_m}{d_{os}^3} \end{aligned} \quad (14)$$