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# Computer–Aided Numerical Determination of Hofer, Lewis, Niemann and Colbourne Points

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George Washington University

**American Gear Manufacturers Association**



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## TECHNICAL PAPER



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**George Washington University**

[The statements and opinions contained herein are those of the author and should not be construed as an official action or opinion of the American Gear Manufacturers Association.]

## **ABSTRACT:**

In rating the bending strength of the hobbled gear teeth, the critical point located within the root fillet, where the fracture occurs due to the tensile bending stress, is necessary to be determined. Hofer, Lewis, Niemann, and Colbourne propose their methods to find this point approximately, in which a numerical iteration method is needed to solve nonlinear one-variable equations established to find their critical points. This paper presents equations expressed easily and in a similar way for finding these four critical points as well as the general gear tooth profile equations derived based on the vector analysis method. These equations are assuredly solved by the finite difference Newton-Raphson iteration algorithm. Position comparison of their points was achieved with computer-aided graphical and numerical output.

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## Abstract

In rating the bending strength of the hobbled gear teeth, the critical point located within the root fillet, where the fracture occurs due to the tensile bending stress, is necessary to be determined. Hofer, Lewis, Niemann, and Colbourne propose their methods to find this point approximately, in which a numerical iteration method is needed to solve nonlinear one-variable equations established to find their critical points. This paper presents equations expressed easily and in a similar way for finding these four critical points as well as the general gear tooth profile equations derived based on the vector analysis method. These equations are assuredly solved by the finite difference Newton-Raphson iteration algorithm. Position comparison of their points was achieved with computer-aided graphical and numerical output.

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## 1. List of Symbols

|            |                                                                                    |
|------------|------------------------------------------------------------------------------------|
| N          | number of teeth in pinion or gear                                                  |
| $\alpha_c$ | tool pressure angle, radians                                                       |
| x          | addendum modification coefficient                                                  |
| $C_k$      | clearance factor for standard gear pair                                            |
| dev        | deviation from reference thickness of tool tooth for one side                      |
| $r_T$      | radius of tool tooth tip arc                                                       |
| d          | half of tool tooth top land length, 0 for full-round                               |
| $R_o$      | radius of reference pitch circle                                                   |
| $R_b$      | radius of base circle of involute                                                  |
| $\theta$   | parametric variable (roll angle) of involute vector, radians                       |
| $\theta_o$ | origin angle of involute, radians                                                  |
| $\theta_i$ | starting angle of involute profile, radians                                        |
| $\theta_f$ | end angle of involute profile, radians                                             |
| $\theta_t$ | slope angle of involute, radians                                                   |
| $\theta_l$ | load point parameter, radians                                                      |
| $\phi_L$   | load angle, radians                                                                |
| $I_x$      | x-component of involute vector                                                     |
| $I_y$      | y-component of involute vector                                                     |
| $\rho$     | parametric variable (roll angle) of root fillet and tool trochoid vectors, radians |
| $\rho_o$   | reference angle of root fillet and tool trochoid vectors, radians                  |
| $\rho_i$   | starting angle of root fillet and tool trochoid, radians                           |
| $\rho_f$   | end angle of root fillet and tool trochoid, radians                                |
| $\phi_t$   | slope angle of root fillet and tool trochoid, radians                              |
| $H_x$      | x-component of tool trochoid vector                                                |
| $H_y$      | y-component of tool trochoid vector                                                |
| $R_x$      | x-component of root fillet vector                                                  |
| $R_y$      | y-component of root fillet vector                                                  |
| p          | of pinion                                                                          |
| g          | of gear                                                                            |
| inv        | involute function, $\text{inv}\alpha = \tan\alpha - \alpha$ , radians              |
| SQRT       | square root of                                                                     |
| arctan     | inverse tangent function, $-\pi/2 \sim \pi/2$                                      |

## 2. Introduction

There is a similarity in determining Hofer, Lewis, and Niemann critical points on the root fillet of a gear tooth. These three critical points are commonly associated with the slope angle of the root fillet. Alternatively, Colbourne [1] introduces a different approach to identify a critical point.

Hofer method, known as 30 degree tangent method, determines its critical point using the slope angle of the root fillet only. This point has been used to evaluate the tooth form factor in Deutsches Institut für Normung (DIN) Standard 3990 [2].

Lewis simplified bending stress analysis by using a parabola which makes a gear tooth an equal strength cantilever. Lewis point is found where an upward parabola inscribes with the root fillet. This means the slope angles of the parabola and the root fillet are equal to each other at Lewis point. This point has been used formally in the American Gear Manufacturers Association (AGMA) Standard [3]. Also, AGMA introduces Errichello's [4] numerical procedure for finding this point. He uses the Newton-Raphson method as a numerical iteration method to find Lewis point from his iteration function.

Niemann point is located where a tangent line to the root fillet passes through its specific point. In finding this point numerically, it is easily understood that with replacing the quadratic equation of parabola for Lewis point by its linear tangent line equation, it can be determined in a similar way as Lewis point is determined

Consequently, the above-mentioned three points are all obtainable independently of stress analysis. However, Colbourne finds the maximum stress value iterating his fairly complete stress function described based on the gear tooth cantilever stress analysis. It is reasonable that Colbourne point, where the maximum stress occurs, can be a good estimated critical point.

Equations established to find these four critical points are formed into nonlinear one-variable equations. The Newton-Raphson numerical iteration method may be best suited to solve these equations. In general, however, the Newton-Raphson method suffers from obtaining failed or undesired solutions and requiring the first order derivatives of iteration functions as iteration functions become complicated. Two special strategies for finding the four critical points are described in this paper when the Newton-Raphson method is used. One is finding a good initial guess point and the other is simplifying the numerical iteration algorithm by using the finite difference method.

For convenience, throughout this paper, all variables or dimensions having length units, inches or millimeters, are

normalized at 1 diametral pitch (D.P.) for the D.P. design system or at 1 module for the module design system. The very common full-depth and external gear tooth system is used with an assumption that the generating pitch line of the cutting tool is higher than the center of its tip arc, which is a usual cutting condition ( $h > x$  in Figure 3). The origin of the coordinate system is located at the center of the gear blank. Also, nonundercut spur gear teeth are assumed to be generated by nonprotuberance rack-type cutting tools. For helical gear teeth, designers can refer to DIN 3990 [2] and AGMA Standard [3] where equivalent spur gear teeth methods are introduced.

### 3. Involute Profile Equations

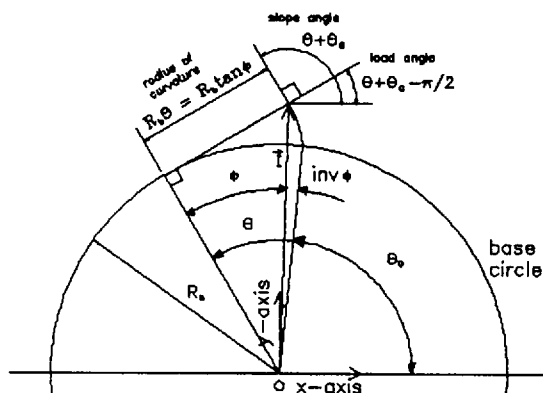


Figure 1 Involute profile curve

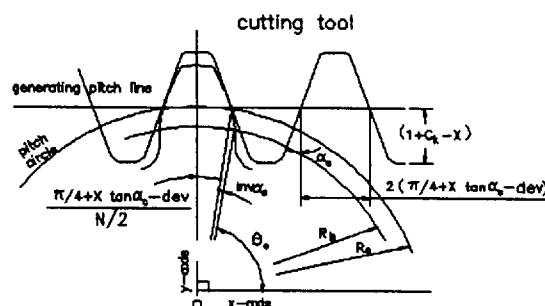


Figure 2 Origin angle of the involute profile

Figure 1 defines illustratively the involute vector and some fundamental mathematical properties of the involute. A term  $\theta_0$  in Figures 1 and 2, the origin angle of the involute, is used efficiently for various profile analyses. Figure 2 shows the reference positions of gear tooth and tool tooth space which their center lines are coincident with the y-axis and how to determine  $\theta_0$ . The following equations stand for the basic mathematical properties of the involute profiles in gear teeth generated by nonprotuberance rack-type cutting tools.

Origin angle of the involute, radians:

$$\theta_0 = \pi/2 - \text{inv}\alpha_c - 2 (\pi/4 + x \tan\alpha_c - \text{dev})/N \quad (3.1)$$