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The Effect of Manufacturing Errors on the Predicted Dynamic Factors of Spur Gears

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American Gear Manufacturers Association



TECHNICAL PAPER

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Abstract

This paper studies the effect of manufacturing errors on predicted dynamic factors of spur gears. Three types of dynamic factors are defined and studied: dynamic load factors, dynamic tooth force factors, and dynamic bending moment factors. Three different computer programs for predicting dynamic factors are introduced. These programs are a MATLAB forced vibration analysis using a six degree of freedom model, a multi-degree of freedom Dynamic Transmission Error Program (DYTEM) that uses a six degree of freedom model, and the Geared Rotor Dynamics Program (GRD) that uses a finite element method. After comparing the three programs' results with experimental data provided by National Aeronautics and Space Administration (NASA), the DYTEM program is used for dynamic factors prediction. The effects of different profile tolerances for AGMA quality 10, 12, and 14 gears are presented.

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American Gear Manufacturers Association
1500 King Street, Suite 201
Alexandria, Virginia, 22314

October, 1995

ISBN: 1-55589-651-0

The Effect of Manufacturing Errors on Predicted Dynamic Factors of Spur Gears

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INTRODUCTION

The dynamic factor used in gear design has been the subject of many studies, and has resulted in many formulations. The existing factors used by AGMA (American Gear Manufacturers Association) and ISO (International Standard Organization) are empirical in nature. Although accuracy is a consideration, it has not been studied to any great extent. The advent of many relatively simple models and available analysis procedures allows for dynamic factors to be predicted by a variety of methods. This paper uses three different modeling procedures that predict dynamic factors and then uses one of the procedures to compare the effects of profile accuracy with the values used in present AGMA standards.

The development of the dynamic factor, using either experimental and theoretical methods, can be traced back more than one century ago. Walker introduced a factor which was reformulated by Barth as the ratio of static load to dynamic load [1]. Buckingham has performed systematic studies in the prediction of dynamic tooth loads for designing gears [2]. The formulation of dynamic factor has changed as more factors, such as operational speeds, tooth precision, and gear geometry, are considered. The AGMA 2001 standard provides some standard equations for the computation of the dynamic factor based on the gear quality numbers of the gears [3]. The AGMA Standard recognizes that it does not consider system resonances and is empirical in nature.

Thus, more intensive investigations in the analytical prediction of dynamic factors have been performed by many investigators. Kubo [4] presented a method whose results compared favorably with his experimental data. Kubo's

measured results are presented by Dudley [5] in his fairly detailed discussion of dynamic factors. Lin, et al. [6] have conducted an investigation in predicting the effects of both linear and parabolic tooth profile modifications on the dynamic response using a four degree of freedom torsional model. Rebbechi, et. al. [7] have performed some experimental studies to predict the gear dynamic behavior. These experimental results were compared with the computer program developed at NASA (National Aeronautics and Space Administration) for a four degree of freedom model [8]. Wang [9] has also applied time domain torsional dynamic modeling for the prediction of dynamic factors. Although not applied directly to dynamic factor modeling, Kahraman and Singh [10] and Padmanabhan and Singh [11] have studied dynamic modeling procedures for gears that have non-linear effects near system resonances.

Ozguven and Houser [12] have developed a computer program, DYTE (Dynamic Transmission Error Program) that uses either measured or computed static transmission errors to excite a single degree of freedom non-linear time domain model. This program determines the dynamic factors, dynamic tooth forces, and dynamic transmission errors. This computer program has been extended to a six degree of freedom non-linear model called DYTEM (Dynamic Transmission Error Program for Multi-degree of freedom model that provides more complete results [13]. In addition to the dynamic factors, dynamic tooth forces, and dynamic transmission errors predicted by DYTE, dynamic bearing forces, bearing displacement, and torsional shaft motions are also predicted with this time domain program. A computer program called GRD (Gear Rotor Dynamics Program) has been developed by Kahraman, et. al. [14]. This procedure uses finite elements to analyze the dynamics of a system consisting of two shafts supported on bearings and coupled by a pair of helical gears.

mesh. This program uses harmonic components of transmission error to predict the dynamic loads in using a frequency domain approach.

The three programs can be used to determine several types of dynamic factors of gears in mesh. More specifically, the following dynamic factor effects may be predicted: Dynamic Load Factor, Dynamic Tooth Force Factor, Dynamic Bending Moment Factor, Dynamic Contact Stress Factor, and Dynamic Root Stress Factor. These factors are defined below.

The *Dynamic Load Factor* is the ratio of the static load carried by a gear pair to the peak dynamic load. This dynamic load usually does not consider load sharing between teeth so it is possible to have the highest dynamic load where two pairs of teeth are in contact, hence providing a conservative value. This is the factor used by AGMA in its standards.

The *Dynamic Tooth Force Factor* method considers each tooth pair as a separate entity and then computes the highest ratio of static force carried by one tooth relative to the highest dynamic force carried by the same tooth. This factor is close to being correct for surface durability since it is indicative of Hertzian contact stress amplitude. However, since it is not location dependent, it may or may not be indicative of the peak dynamic surface stress on a gear tooth.

The *Dynamic Bending Moment Factor* method takes the dynamic tooth force and multiplies it by the moment arm to the location of the critical root stress as obtained in the geometry factor calculation. This factor does not directly indicate root stress, but allows for the computation of a root stress multiplier that is essentially the same as computing the ratio of static root stress to dynamic root stress.

The *Dynamic Contact Stress Factor* is the most sensible dynamic factor for surface durability. It is computed by taking the ratio of the highest static contact stress on a gear tooth to the highest dynamic contact stress. In most instances this factor will differ only slightly from dynamic tooth force factor, but where the peak dynamic loads occur at a different location than the peak static loads, there will be differences between the two methods.

The *Dynamic Root Stress Factor* method calculates dynamic factor based on the root stresses of each element of a gear pair. Since the peak stress location at the root changes slightly with load location, this factor computes more precise results than the dynamic bending moment factor, but is much more difficult to implement.

In this paper, a study of three of the dynamic factor computation methods is conducted. They are the *Dynamic Load Factor* as computed by AGMA, the *Dynamic Tooth Force Factor*, and the *Dynamic Bending Moment Factor*. The models used have the capability of carrying the computations through the final step to calculate contact or root stresses, but this option has not yet been implemented. In fact, in this paper, the intent is to see the effects of profile errors on the dynamic factors and these effects show up equally well with each of the factors that is chosen. Spacing errors, which are another

significant input to dynamic factor are not included in this analysis. In this paper, the dynamic factor will always be considered as a dynamic stress multiplier in the fashion of Kubo [4] and later presented in Dudley [5]. The expression used can always be converted to an AGMA type factor by inverting the number that is computed. This approach implies that higher dynamic factors indicate higher stresses on a component.

MODEL DISCUSSION

As mentioned earlier, there are many simulation procedures for computing the dynamic loads on a gear pair. First, one must select an appropriate physical model and then use one of several equation solving techniques to evaluate the physical model.

Many of the different physical models that have been used for gear dynamics modeling have been discussed by Ozguven, et. al. [12 - 14]. Later papers that extend their literature review include works by Blankenship and Singh [15], and Vinayak and Singh [16]. Some of the physical models that have been used are presented below.

- a) The simplest gear dynamics model considers the gear pair as two rotary inertias coupled by a spring. This spring is usually the mesh stiffness of the gear pair. The mesh stiffness is time varying, but often is averaged to give a linear time invariant model. There is but one torsional natural frequency that is predicted with this model. Depending upon the implementation, this model may or may not include the nonlinear effects due to loss of contact.
- b) The next simplest model that is often used includes the translational motions of the gears by adding the stiffness of the supporting shafts, bearings, and housing. In its simplest form, this model has three natural frequencies each of which has various degrees of torsional and lateral coupling. Gregory, et. al. [17] was the first to identify the necessity to use this model as the absolute minimum for practical gears, since all gears tend to have some degree of lateral/torsional coupling and except for very symmetrical cases, each of the vibration modes is excited to some extent by the dynamic excitations of the mesh. The use of model "a" has been successful in modeling some test stands because efforts were made to decouple the lateral motions by using inordinately large gear wheel inertias or by having extremely compliant shafts and bearings.
- c) By adding additional shaft inertias to (b) depending upon the complexity of the actual system, one can have additional natural frequencies. However, these added masses seldom affect the mesh forces in the frequency range studied in this paper.
- d) A more complex model considers the shafts and bearings to be continuous and includes shaft bending separate from bearing stiffness. This approach is the most complex of the models discussed and usually requires the application of a finite element approach. This method is most appropriate when there are multiple masses on the same shaft and when pinions have diameters of the same order as its attached shaft.



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These physical models still require an excitation. In the time varying mesh stiffness models, one may use only the time varying stiffness as the excitation. However, this method may only be applied for "perfect involute" (no profile modification) gears and even then may not account for "off-line-of-action" contact that might occur. The effects of significant displacement excitations due to tooth deflections and profile modifications or due to errors in manufacture are usually not adequately considered in these models. Ozguven and Houser [12] developed a means of separately incorporating each of these excitations through the computation of the loaded transmission error of a gear pair. The transmission error that is used is computed at the mean operating load, so there are some potential errors due to the time varying loads that occur in operating gears.

Three different models of varying complexity were compared in this study to see how they differed in their prediction of dynamic factors.

1. Linear forced vibration analysis using sinusoidal transmission error as the excitation. In this procedure, a matrix analysis program called MATLAB [18] was used to perform the forced vibration analysis. The six degree of freedom model shown in Fig. 1 was used where I_D and I_L are the drive and load torsional inertias, respectively, and I_1 and I_2 are the respective inertias of the pinion and gear. The respective shafts connecting the drive and load to the gear pair are modeled as pure torsional springs, but there is the ability to include shaft damping. The combined effects of bearing compliance and shaft compliance as measured along the line of action are indicated by the stiffnesses, k_1 and k_2 , for the pinion and gear, respectively. The associated damping is modeled with discrete dampers, c_1 and c_2 , but may also be included as modal damping after the individual vibration modes are decoupled in the MATLAB analysis. Mesh stiffness is assumed to be an average value, k_m . The MATLAB model is solved using matrix methods that assume proportional damping and then modal decomposition and modal summation methods are used to solve for the response of any of the variables as a function of the sinusoidal transmission error excitation, $e(t)$. Six natural frequencies and their respective modes shapes are solved in the analysis. Only *Dynamic Load Factors* may be computed with this method since there is no means of computing the load sharing between individual tooth pairs.
2. A time domain simulation of the dynamic model of Fig. 1 was performed using a program called DYTEM (Dynamic Transmission Error Program for Multi-degree of freedom Model) [19]. This model uses a loaded transmission error excitation computed from a program called LDP (Load Distribution Program) [20]. The LDP program also computes the time varying mesh stiffness that is part of the excitation model. Parameters such as tip relief, spacing errors, or more general tooth profile and tooth lead errors may be simulated. The DYTEM model uses a fourth order Runge-Kutta technique for solving the set of differential equations. Each of the three chosen dynamic factor types may be computed with this method. Since this method keeps track of the load carried by each individual tooth pair,

not only are the *Dynamic Load Factors* computed, but also the *Dynamic Tooth Force Factors* on each tooth are computed. The *Dynamic Bending Moment Factors* are computed by multiplying the instantaneous tooth force by the moment arm to a critical radius at the tooth root.

3. This model uses a finite element procedure incorporated in a program called the Geared Rotor Dynamics Program (GRD) [21]. Typical models using this program is shown in Fig. 2. This program also uses a transmission error input computed by LDP, but treats the equations as linear time invariant and uses an average value for the mesh stiffness. Bearing stiffnesses are placed at the actual bearing locations, and shaft deflections that could only previously be incorporated into the bearing stiffness, are now independently included. Because this method does not divide the transmitted load into individual tooth forces, only the *Dynamic Load Factor* may be computed.

An unknown of each of these three models is the amount of damping to be included. Models 1 and 3 only allow modal damping, while model 2 allow discrete dampers to be placed at the same locations as the springs. In all cases, damping values were selected to give good comparisons with experimental data taken by Rebbechi, et. al. [7].

COMPARISON WITH EXPERIMENTAL DATA

Fig. 3 presents root strain gage data for two consecutive spur gears run by Rebbechi, et al. [7]. For each speed, the time axis is normalized to position along the line of action. In this case, the test rig has extremely compliant shafts so only a single torsional natural frequency predominates. The modes of vibration that have significant shaft bending are at low frequencies that do not show up in the strain data. By inverting the periods of the oscillations observed in the strain gage data, for the three lower speeds, one finds that the primary natural frequency of the system is roughly 3400 Hz. A single natural frequency is difficult to discern from 6000 rpm data because very few oscillations occur. Also, it is noted that the damping, as qualitatively assessed from the decay rate of the oscillations, is lowest at 800 rpm and then increases as the speed is increased.

Each of the three models discussed earlier was used to analyze Rebbechi's gear set using the data of Table 1 and Table 2a. Table 3 shows that the primary natural frequencies predicted by each of the models compare quite favorably with one another. The maximum dynamic load factors are 2.4, 2.8, 2.6, and 2.1 using MATLAB, DYTEM, GRD model A, and GRD model B, respectively. Adding the external load and driver into the GRD model A, the natural mesh frequency predicted is not affected. However, the model with external load and driver (GRD model B) has a lower dynamic factor at the major resonance but at all lower speed is nearly identical to the GRD model A. For general information, the next lower natural frequencies that are predicted by the MATLAB model are at 151 and 400 Hz., respectively. At these frequencies, the