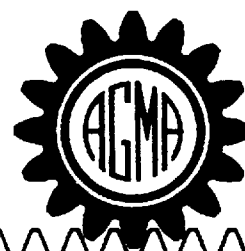


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Determination of the Dynamic Gear Meshing Stiffness of an Acetal Copolymer

by: Connie P. Marchek

American Gear Manufacturers Association



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Connie P. Marchek

[The statements and opinions contained herein are those of the author and should not be construed as an official action or opinion of the American Gear Manufacturers Association.]

Abstract

The dynamic gear meshing stiffness is an important parameter for designing plastic gearing, thus the objective of this research was to determine the dynamic gear meshing stiffness of an acetal copolymer, specifically Celcon® M90. A theoretical model was developed to simulate the torsional vibration and resonance of an operating gear pair. The resonant speed of the torsional system was determined experimentally. Using the theoretical model, it was possible to determine the dynamic gear meshing stiffness from the experimental resonant speed. The dynamic gear meshing stiffness was compared to the values calculated from available empirical formulas.

Disclaimer

The conclusions and opinions expressed in this research are those of the writer and do not necessarily represent the position of Worcester Polytechnic Institute or Hoechst Celanese, or any of its directors, officers, agents or employees with respect to the matters discussed.

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Preface

This paper, researched and conducted at Hoechst Celanese Advanced Materials Group (Wood Dale, Illinois) with the assistance of Packer Engineering (Naperville, Illinois), was submitted to the American Gear Manufacturers Association and will be presented at the Fall 1995 Technical Conference. Sufficient knowledge was acquired through course work at Worcester Polytechnic Institute and experience at Hoechst Celanese and Packer Engineering, to successfully complete this research.

I would like to thank God for giving me the courage and strength to continually accept the necessary risks to fulfill my dreams. I would like to express my appreciation to Hoechst Celanese Advanced Materials Group and Worcester Polytechnic Institute for providing me the opportunity to complete a graduate Thesis and Mechanical Engineering Degree. I would like to extend a special thanks to Professor W. W. Durgin for his continued guidance, assistance and support during my graduate career at Worcester Polytechnic Institute. Sincere thanks to Stuart Cohen and Maribeth Fletcher for providing the opportunity to complete a thesis at Hoechst Celanese. Many thanks to Michael J. Clemens of Packer Engineering for being a lifesaver. Your continued guidance, assistance, enthusiasm and overwhelming support made the completion of this research and thesis possible. In addition, I would like to thank the following people who provided me with support and assistance: Zan Smith, Ken Gitchel, ABA/PGT and the Faculty at WPI. Many thanks to my special friends: Barb, Beth, Stimpy, Ira, Brian, Lucille, Ginny, Joe and Sandy. Thanks for standing by me during both the good and the difficult times.

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1. Mathematical Model

The torsional system to be theoretically modeled consists of a parallel shaft gear apparatus depicted in Fig. 1.1. The system consists of a pair of plastic gears connected to a motor and inertia disk by steel shafts. The objective of this theoretical model is to determine the dynamic gear meshing stiffness at the experimental resonant speed.

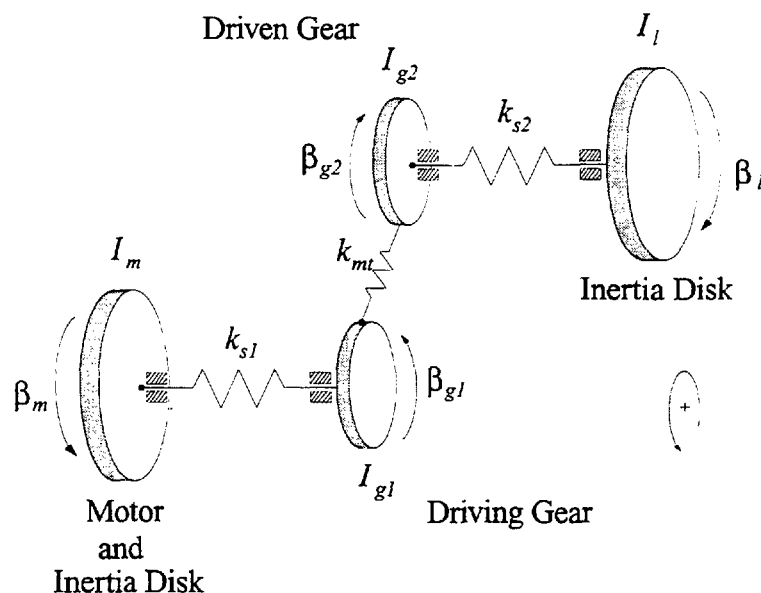


Fig. 1.1 Torsional System

To simplify the mathematical model, the motor, gears and inertia disks are assumed to be the rotating inertias. The shafts and gear mesh function as the springs of the rotating system. This model is concerned with only the rotational motion of the gears. The gear teeth are assumed to maintain contact along the theoretical line of action and the dynamic gear meshing stiffness is assumed to be constant. The effects of backlash and mean torque were not considered in this model.

1.1 Nomenclature

β_{g1} = angular displacement of the driving gear (*rad*)

β_{g2} = angular displacement of the driven gear (*rad*)

β_I = angular displacement of the inertia disk (*rad*)

β_m = angular displacement of the motor (*rad*)

$\Delta\beta_{g1\&g2}$ = angular displacement between the driving and driven gears (*rad*)

$\Delta\beta_{g2\&I}$ = angular displacement between the driven gear and inertia disk (*rad*)

$\Delta\beta_{m\&g1}$ = angular displacement between the motor and driving gear (*rad*)

I_m = equivalent motor mass moment of inertia (*in-lb / sec²*)

I_{g1} = equivalent driving gear mass moment of inertia (*in-lb / sec²*)

I_{g2} = equivalent driven gear mass moment of inertia (*in-lb / sec²*)

I_I = equivalent inertia disk mass moment of inertia (*in-lb / sec²*)

k_{mI} = linear dynamic gear meshing stiffness (*lb / in*)

k_{mI} = torsional dynamic gear meshing stiffness (*in-lb / rad*)

k_{s1} = stiffness of shaft connecting the motor and driving gear (*in-lb / rad*)

k_{s2} = stiffness of shaft connecting the inertia disk and driven gear (*in-lb / rad*)

ω = system's natural frequency (*rad / sec*)

T_{g1} = inertia torque on the driving gear (*in-lb*)

$T_{g1\&g2}$ = torque across the gear mesh (*in-lb*)

T_{g2} = inertia torque on the driven gear (*in-lb*)

$T_{g2\&I}$ = torque on the shaft between the driven gear and the inertia disk (*in-lb*)

T_I = inertia torque on the inertia disk (*in-lb*)

T_m = inertia torque on the motor (*in-lb*)

1.2 The Holzer Method

The Holzer Method [1] was developed from the principle that during free torsional vibration the sum of the inertia torques acting on a system must equal zero. The Holzer Method iterates the equations of motion to solve the eigenvalue problem. The following section develops the equations of motion, using the Holzer Method, for the torsional system shown in Fig. 1.1.

The inertia torque acting on the motor, gears and inertia disk is determined from the following equation:

$$T_i = I_i \beta_i \omega^2 \quad \text{where: } i = m, g1, g2, l$$

The relative angular displacements of the inertias are determined from the following equations:

$$\Delta \beta_{i \& j} = \beta_i - \beta_j$$

and

$$\Delta \beta_{i \& j} = \frac{T_{i \& j}}{k_{i \& j}} \quad \text{where: } i = m, g1, g2 \text{ and } j = g1, g2, l$$

As stated above, the principle of the Holzer Method is based on the concept that the sum of the inertia torques within the system is zero.

$$\sum_{i=1}^n T_i = 0 \quad \text{where: } n \text{ is the number of masses in the system}$$

To develop the Holzer Method for the four degrees of freedom free vibration torsional system shown in Fig. 1.1., the inertia torque acting on the motor is determined to be:

$$T_m = I_m \beta_m \omega^2 \quad (1.1)$$

The relative angular displacement between the motor and driving gear is:

$$\Delta\beta_{m\&g1} = \frac{T_m}{k_{s1}} \quad (1.2)$$

The angular displacement of the driving gear is determined as:

$$\beta_{g1} = \beta_m - \Delta\beta_{m\&g1} \quad (1.3)$$

The inertia torque acting on the driving gear is determined to be:

$$T_{g1} = I_{g1}\beta_{g1}\omega^2 \quad (1.4)$$

The total torque acting across the gear mesh is calculated as shown:

$$T_{g1\&g2} = T_m + T_{g1} \quad (1.5)$$

The relative angular displacement between the driving gear and the driven gear is:

$$\Delta\beta_{g1\&g2} = \frac{T_{g1\&g2}}{k_{mt}} \quad (1.6)$$

The angular displacement of the driven gear is determined as:

$$\beta_{g2} = \beta_{g1} - \Delta\beta_{g1\&g2} \quad (1.7)$$

The inertia torque acting on the driven gear is determined to be:

$$T_{g2} = I_{g2}\beta_{g2}\omega^2 \quad (1.8)$$

The total torque acting on the shaft between the driven gear and the inertia disk is calculated as shown:

$$T_{g2\&l} = T_m + T_{g1} + T_{g2} \quad (1.9)$$

The relative angular displacement between the driven gear and the inertia disk is:

$$\Delta\beta_{g2\&l} = \frac{T_{g2\&l}}{k_{s2}} \quad (1.10)$$

The angular displacement of the inertia disk is determined as:

$$\beta_l = \beta_{g2} - \Delta\beta_{g2\&l} \quad (1.11)$$

Finally, the inertia torque acting on the inertia disk is determined to be:

$$T_l = I_l \beta_l \omega^2 \quad (1.12)$$

The total torque acting on the system is calculated as follows:

$$\sum_{i=1}^4 T_i = 0$$

$$T_m + T_{g1} + T_{g2} + T_l = 0 \quad (1.13)$$

To initiate the Holzer Method, the angular displacement of the motor, β_m , was assumed to be one radian. The system's natural frequency was initially guessed and the total torque of the system calculated. The process of iterating the natural frequency was continued until the total torque of the system converged to zero. When the system's total torque was zero, the natural frequency was obtained. The respective angular displacement of the motor, driving gear, driven gear and inertia disk represented the mode shape or eigenvector corresponding to the natural frequency.

1.3 Predicted Dynamic Gear Meshing Stiffness

Four methods for predicting the dynamic gear meshing stiffness were investigated. In the first method, the average linear dynamic gear meshing stiffness was predicted using the cantilever beam theory [2]. The second method was an empirical formula developed by Hoechst, the German parent company of Hoechst Celanese [3]. Experiments were performed on plastic gears to determine the data necessary to develop the dynamic gear tooth deflection formula. To predict the linear stiffness of a gear tooth, using the third method, a cantilever beam with uniform cross section was assumed [4]. Finally, in the fourth method, the linear flexibility of a gear tooth with variable cross section was investigated and the dynamic gear tooth stiffness predicted [5].

1.3.1 Cantilever Beam Theory - Spotts

The average linear dynamic gear meshing stiffness for a pair of gears was determined using the cantilever beam theory, as developed in Equation 1.14 [2].

$$k_{ml} = \frac{b}{9} \left(\frac{E_{g1} E_{g2}}{E_{g1} + E_{g2}} \right) \text{ lb/in} \quad (1.14)$$

where: b is the gear face width (in), and E_{g1} , E_{g2} are moduli of elasticity (lb/in^2) for the driving and driven gears respectively.