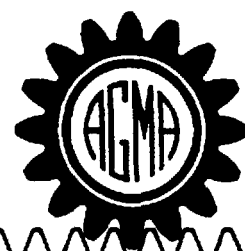


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# A Computer Based Approach Aimed at Reproducing Master Spiral Bevel and Hypoid Pinions and Gears

by: Claude Gosselin, Laval University, Yoshio Shiono, Yutaka  
Seimitsu Kogyo, Ltd., Tetsuya Nonaka, Kyoto University, and  
Aizoh Kubo, Kyoto University



## TECHNICAL PAPER

## A Computer Based Approach Aimed at Reproducing Master Spiral Bevel and Hypoid Pinions and Gears

Claude Gosselin <sup>(1)</sup>, Yoshio Shiono <sup>(2)</sup>, Tetsuya Nonaka and Aizoh Kubo <sup>(3)</sup>

- <sup>(1)</sup> Department of Mechanical Engineering  
Laval University, Québec, QC, Canada, G1K-7P4
- <sup>(2)</sup> Yutaka Seimitsu Kogyo Ltd.  
Seto Shi, Aichi Ken, Japan 489
- <sup>(3)</sup> Department of Precision Engineering  
Kyoto University, Kyoto, Japan 606-01

### 1. INTRODUCTION

In the spiral bevel and hypoid gear manufacturing industry, master pinions and gears are usually developed from the machine settings provided by a computer software such as The Gleason Works' TCA. Actual pinion and gear members are then cut from the TCA-proposed machine settings. Given the distortion of the tooth surfaces caused by heat treatment or lapping, or the variable state of the cutting machines, the calculated TCA bearing pattern is generally not obtained and the pinion machine settings must be modified until a satisfactory bearing pattern is obtained, a process called bearing pattern development. In such conditions, the master pinion or gear tooth surfaces will not correspond to those of the theoretical model.

In production, corrective machine settings are frequently applied where a Coordinate Measuring Machine (CMM) is used to calculate the differences between the target - real or simulated - and actual tooth surfaces; the surface differences are then used to evaluate by how much the various pinion or gear machine settings must be changed for the actual and target tooth surfaces to match. Therefore, if a known master pinion or gear is to be reproduced, corrective machine settings can be applied effectively only if the exact target tooth surface is known.

The currently available corrective machine settings software packages, such as Gleason's GAGE, calculate correction data based either on the TCA-defined or actual target tooth surface. However, if the latter case is used, the theoretical machine settings are not known, an unfavourable situation since the knowledge of the theoretical tooth surface is usually a desirable feature if only as a general process reference.

This paper presents a general method to bypass such a limitation, when the basic cutting method and the target tooth flank form are known, but the corresponding theoretical tooth surface is not exactly known. A computer based algorithm is used to find the theoretical machine settings which will produce a simulated tooth surface matched to that of a measured surface within a given tolerance range, where the calculated theoretical surface is technologically equivalent to the measured surface. The newly found theoretical surface is then used as a reference to calculate corrective machine settings by applying the same surface matching algorithm, but in the opposite direction. The algorithm can match, within a few  $\mu\text{m}$ , theoretical and measured tooth surfaces up to 2<sup>nd</sup> order differences for generated pinions and to 1<sup>st</sup> order differences for non-generated gear members, which is usually sufficient to obtain satisfactory tooth surfaces.

A test case is presented in the paper: a target master pinion is initially measured on a CMM. The theoretical pinion is then matched to the measured target master pinion using 2<sup>nd</sup> order surface matching to define the reference tooth surfaces. After surface matching, the new theoretical tooth surfaces are used as references to calculate corrective machine settings.

### 2. LIST OF MAIN SYMBOLS

|                |                              |
|----------------|------------------------------|
| Ra             | work to cradle ratio of roll |
| L <sub>1</sub> | cradle rotation angle        |
| DRatio         | decimal ratio                |
| Mctb           | work machine center to back  |
| Offset         | work offset                  |
| SlBase         | work sliding base            |

|                     |                                                    |
|---------------------|----------------------------------------------------|
| $\bar{N}$           | tooth flank normal unit vector                     |
| $\bar{V}_r$         | relative speed vector                              |
| $\bar{V}_\alpha$    | speed vector of a point on the cutter blade edge   |
| $\bar{V}_{wz}$      | speed vector of a point on the work tooth flank    |
| $\bar{X}_{wz}$      | position vector of a point on the work tooth flank |
| $\bar{\omega}_{wz}$ | work angular speed vector                          |
| $\bar{\omega}_{cw}$ | cradle angular speed vector                        |
| $\gamma_m$          | machine root angle                                 |
| $\alpha_c$          | cutter angular position                            |
| $\alpha_3$          | work roll angle                                    |
| $\tau$              | cutter tilt angle                                  |
| $\kappa$            | cutter swivel angle                                |

### 3. SIMULATION OF THE TOOTH CUTTING PROCESS

This section will briefly introduce the main aspects considered in the simulation of most current tooth cutting processes [1, 2, 3].

Both the generating process and operation of gear sets are based on the same basic concept, that of meshing elements. In the manufacturing process, meshing takes place between a cutter blade whose rotation represents the shape of one tooth of a crown gear, and the work itself; in operation, meshing occurs between the contacting teeth of the pinion and gear. The fundamental equation of meshing is:

$$\bar{N} \cdot \bar{V}_r = 0 \quad (1)$$

which states that the relative speed vector  $\bar{V}_r$  between contacting surfaces must be in a plane tangent to the meshing surfaces at any contact point.

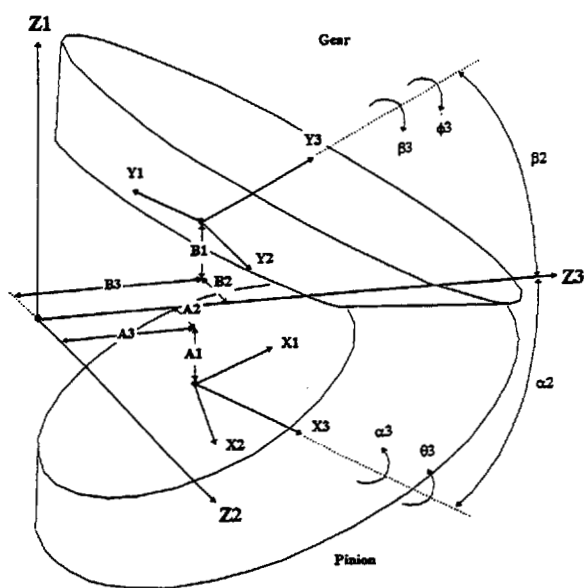


Figure 1: General Simulation Reference Frames

At the cutting level, let us consider a contact point common to both the generating surface described by a rotating cutter blade edge and the generated surface attached to the work, and state at this point the expression for the relative speed vector  $\bar{V}_r$  in a common general reference frame Z (figures 1 and 2):

$$\bar{V}_r = \bar{V}_\alpha - \bar{V}_{wz} \quad (2)$$

where  $\bar{V}_r$  is the relative speed vector between cutter and work,  $\bar{V}_\alpha$  is the speed vector of a point belonging to the cutter attached to a cradle rotating about its axis  $W_1$ , and  $\bar{V}_{wz}$  is the speed vector of a corresponding point belonging to the work rotating about its axis  $X_3$ .

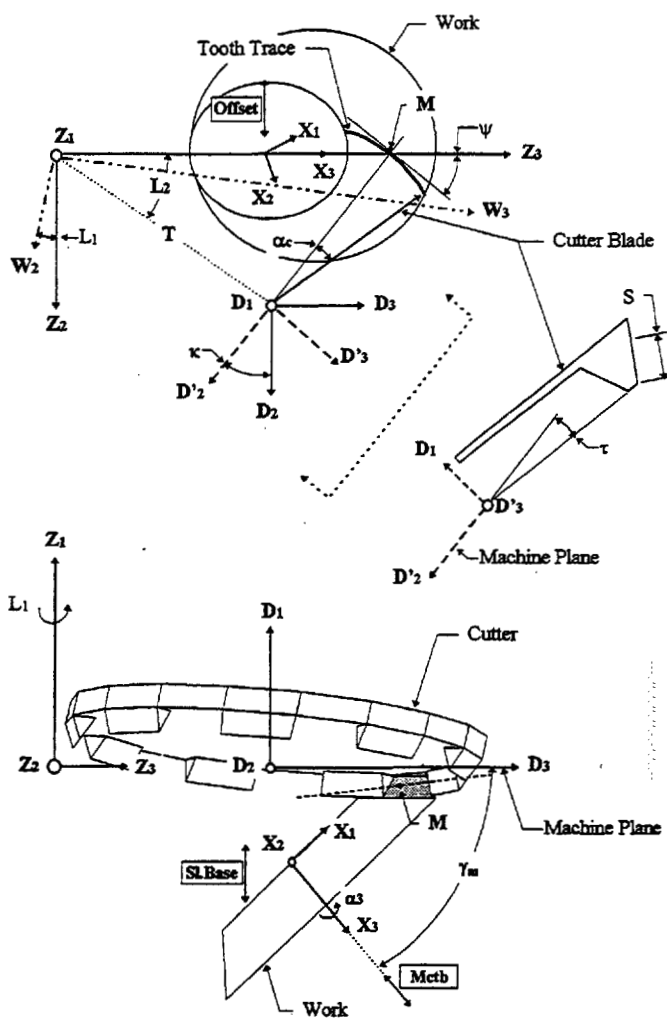


Figure 2: Simulation Reference Frames for Cutting Processes

The work speed vector  $\bar{V}_{wz}$  in reference frame X, rigidly connected to the work, is:

$$\bar{V}_{wz} = \bar{\omega}_{wz} \times \bar{X}_{wz} \quad (3)$$

where  $\bar{X}_{wx}$  is the position of a common point between cutter and work, given in the work reference frame X, and  $\bar{\omega}_{wx}$  is the work rotation vector given by :

$$\bar{\omega}_{wx} = \begin{bmatrix} 0 \\ 0 \\ -Ra \end{bmatrix} \quad (4)$$

where Ra is the ratio of roll between the work and the cradle, such that:

$$Ra = \frac{\alpha_3}{L_1} \quad (5)$$

where  $\alpha_3$  is the work roll angle and  $L_1$  is the cradle angle of rotation.  $\bar{V}_{wz}$ , the speed vector in general reference frame Z, is obtained by transforming the speed vector  $\bar{V}_{wx}$  from reference frame X to reference frame Z:

$$\bar{V}_{wz} = [R_{ym}]_2 \bar{V}_{wx} \quad (6)$$

where  $[R_{ym}]_2$  denotes a rotation of the work of machine root angle  $\gamma_m$  about axis  $Z_2$ .

Similarly, the cutter speed vector  $\bar{V}_{cw}$  in reference frame W, rigidly connected to the cradle, is:

$$\bar{V}_{cw} = \bar{\omega}_{cw} \times \bar{W}_{cw} \quad (7)$$

where  $\bar{\omega}_{cw}$  is the cradle rotation vector given by :

$$\bar{\omega}_{cw} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad (8)$$

and  $\bar{W}_{cw}$  is the position of the same common point between cutter and work, this time given in the cradle reference frame W.  $\bar{V}_{cz}$ , the speed vector in general reference frame Z, is obtained by transforming speed vector  $\bar{V}_{cw}$  from reference frame W to reference frame Z :

$$\bar{V}_{cz} = [R_{L1}]_1 \bar{V}_{cw} \quad (9)$$

where  $[R_{L1}]_1$  denotes a rotation of the work by cradle rotation angle  $L_1$  about axis  $W_1$ .

The unit normal vector  $\bar{N}_{cz}$  on the cutter blade edge is also normal to the work in any contact point; therefore, the expression of the cutter normal  $\bar{N}_{cz}$  in fixed reference frame Z is sufficient to complete equation (1).

Hence, eq. (1) yields the equation of the generated surface in the general reference frame Z. The obtained surface equation is a function of three variables  $\alpha_c$ ,  $\alpha_3$  and S, respectively the cutter angular position, the work roll angle and the position of a point along the cutter blade edge:

$$S = f(\alpha_c, \alpha_3) \quad (10)$$

The position of any point P on the generated tooth surface is therefore defined by a combination  $(\alpha_c, \alpha_3)$ . The solution of equation (10) is a series of contact points between the cutter blade edge and the work describing a line along the path of the cutter edge defined by its position angle  $\alpha_c$ . The bounded envelope, along the work roll angle  $\alpha_3$ , of a series of such lines in the work reference frame X gives the generated pinion tooth shown in figure 3. Figure 3 also shows a non-generated gear tooth whose peculiarity is that no work to cradle rolling motion occurs during cutting. A Newton-Raphson iterative method is used to numerically solve equation (10) [1, 2].

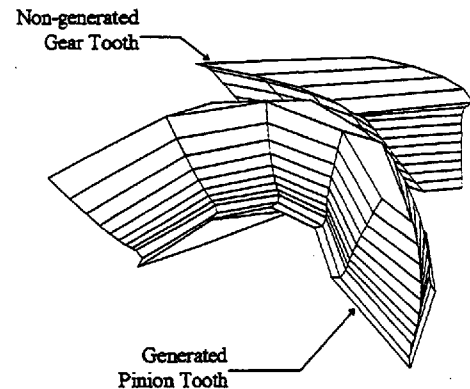


Figure 3: Contacting Pinion and Gear Teeth

The above presented simulation includes adjustments and movements found in most existing generators or Formate/Helixform machines. These adjustments can be classified in four broad categories:

- i) Cutter helical advance motion, such as that found in non-generating Helixform machines, where the cutter is advanced while cutting one tooth space.
- ii) Cutter tilt and swivel, respectively angles  $\tau$  and  $\kappa$  in figure 2. Cutter tilt is used when the available cutter blades do not match the desired pressure angle at the mean point M. It is also used when generating hypoid pinions meshing with non-generated gear members, as the cutter blades must duplicate the shape and orientation of the mating non-generated tooth at the mean point M. Swivel is used to change the direction of the cutter tilt axis,  $D'$ , in figure 2, such that the desired pressure angle is obtained at the mean point M, and that the tooth rootline is parallel to its design value.
- iii) Work position relative to reference frame Z. Such work positional changes are normally called Offset, Sliding Base