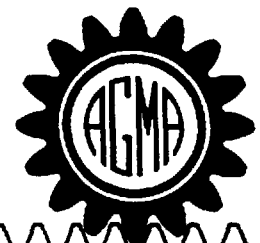


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An Investigation of Globoidal Wormgear Drives

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TECHNICAL PAPER

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[The statements and opinions contained herein are those of the author and should not be construed as an official action or opinion of the American Gear Manufacturers Association.]

Abstract

This paper investigates the following present globoidal wormgear drives: (1) original and modified Hindley wormgear drives; (2) Wildhaber wormgear drive with inclined plane teeth of wormgear; (3) inclined plane and cone enveloping globoidal wormgear drives; (4) plane, cone and inverted cone enveloping globoidal wormgear drives based on Sakai's theory. Meanwhile, a new approach for generation of Hindley wormgearing, and plane and cone enveloping globoidal wormgear drives is developed in this paper. Contact lines, dual and single contact ratios, relative curvature radii, meshing angles between tangents of contact lines and relative velocities, and sliding ratios of wormgear tooth surfaces of the above globoidal wormgear drives are studied by computerized simulation for numerical examples.

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1. INTRODUCTION

Hindley wormgearing as a main type of globoidal wormgear drives has been widely used in power transmissions since S. I. Cone modified it in the beginning of 1930's (Cone 1930, 1931 and 1932). In comparison with cylindrical wormgear drives, as well-known by theoretical and experimental research, the modified Hindley wormgear drives have the following advantages: dual contact lines, large contact ratio, large relative curvature radius and large meshing angle between tangent of contact line and relative velocity. The above characteristic causes better load distribution and lubrication condition, which means high load capability.

Since 1960's, however, ground(enveloping) globoidal worms have commanded much attention in research and then in industry because Hindley worms can not be ground which implies a constraint to use them for higher load and higher accuracy transmissions. Today, plane and cone enveloping globoidal wormgear drives are playing a more and more important role in globoidal wormgear drives. For plane enveloping wormgearing, its wormgear tooth surfaces are generated by such hobs that are made in the same or modified forms as Wildhaber worms (Wildhaber 1924) that are envelopes of families of planes, so the worm and hob thread surfaces can be ground by plane wheels (Sakai et al 1978, Sakai et al 1980, Ye et al 1990, Zhang and Qin 1988). The cone enveloping globoidal wormgear drives whose worm or hob thread sur-

faces are enveloped by conical surfaces of milling or grinding wheels are developed by German patent 1969, Pavlov et al 1975, Sakai and Maki 1980, Sakai et al 1980). Toroidal enveloping globoidal wormgear drives whose worm and hob thread surfaces are enveloped by surfaces of revolution with circular arc profiles of torus milling and grinding wheels are researched by Hu and Wang 1988, Hu et al 1988, Kobayashi et al 1993. A simpler approach to make the wormgear by two-tooth cutter is reported by Simon 1973 and 1988.

The purpose of this paper is: (1) to analyze the geometry of the present industrial Hindley and enveloping globoidal wormgear drives by computerized simulation; (2) to develop new globoidal wormgear drives with better geometry. The dual contact property, dual, single and total contact ratios, relative curvature radius, meshing angle between tangent of contact line and relative velocity and sliding ratio are investigated in the same numerical examples with 40 : 1 gear ratio and 100 mm center distance for different globoidal wormgear drives.

2. HINDLEY WORMGEAR DRIVES

Hindley wormgearing was invented in 1765 and improved by S. I. Cone in 1930's (Cone 1930, 1931 and 1932). Worm and hob thread surfaces of this type of globoidal wormgear drive are generated by straight lines with orthogonal rotation

axes between the cutter and worm/hob, and wormgear tooth surfaces are generated by hobbing, see Fig. 1. The tooth surface of the wormgear can be represented in the following equations:

$$\left. \begin{aligned} \mathbf{r}^{(g)}(u, \theta, \psi) &= \mathbf{M}_{gh}(\psi) \mathbf{r}^{(h)}(u, \theta) \\ \mathbf{v}^{(hg)} \cdot \mathbf{n} &= A \sin \psi + B \cos \psi + C = 0 \end{aligned} \right\} \quad (1)$$

in which $\mathbf{r}^{(g)}$, $\mathbf{r}^{(h)}$ are tooth surfaces of the wormgear and the hob; $\mathbf{n}$ and $\mathbf{v}^{(hg)}$ are surface normal and relative velocity between the hob and the wormgear; (u, θ) and ψ are surface parameters and rotation angle of the hob; $\mathbf{M}_{gh}$ is transformation matrix from the hob to the wormgear systems, and A , B and C are coefficients related with surface parameters of the hob and machine settings, respectively.

Substitution of the half-tangent equation into Eqn. (1) gets two solutions ψ_1 and ψ_2 for unknown ψ . So the wormgear tooth surface is represented in two equations in the following form:

$$\left. \begin{aligned} \mathbf{r}^{(g_1)}(u, \theta, \psi_1), \quad \psi_1 &= 2 \tan^{-1} \left(\frac{-A + \sqrt{A^2 + B^2 - C^2}}{C - B} \right) \\ \mathbf{r}^{(g_2)}(u, \theta, \psi_2), \quad \psi_2 &= 2 \tan^{-1} \left(\frac{-A - \sqrt{A^2 + B^2 - C^2}}{C - B} \right) \end{aligned} \right\} \quad (2)$$

here $A^2 + B^2 - C^2 \geq 0$.

Each of the above equations geometrically expresses a group of contact lines or a subsurface on the wormgear tooth surface. This means from Eqn. (2) there are two groups of contact lines that may exist simultaneously and the wormgear tooth surface consists of two subsurfaces $\mathbf{r}^{(g_1)}$ and $\mathbf{r}^{(g_2)}$.

Eqn. (2) can represent the wormgear tooth surfaces of such globoidal wormgear drives if their wormgears are hobbled by the hobs that have the same or modified thread geometry with that of the worms. In this paper Eqn. (2) can be used for all of the globoidal wormgear drives except Wilhaber wormgearing.

2.1 Original Hindley Wormgear Drive

The original Hindley wormgear drive (without modification) is practically useless because of ridge contact and undercutting. Fig. 2 shows the TCA results of the original Hindley wormgear drive with center distance $E_{wg} = 100 \text{ mm}$ and gear ratio $m = 40 : 1$. The ridge line $d-d$ on the middle of the wormgear tooth surface is the intersecting line of two subsurfaces of the wormgear tooth defined by Eqn. (2). One group of the contact lines meshes with the worm from 24° to -24° in which the ridge line takes the contact from -3° to -24° . Another group of the contact lines was undercut by the last edge of the hob (Litvin 1994) except the ridge line $d-d$ that

contacts with the worm from 22° to 0° . Theoretically, the tooth surfaces of the worm and the wormgear mesh in dual contact lines from 22° to 2° , one of which is the ridge line. However, since the ridge line $d-d$ keeps contacting with the worm thread surface and dwelling at the same position at the wormgear tooth surface for the whole meshing process, it will be rapidly worn away. This means that only a single contact line is in mesh practically for the original Hindley wormgear drive in a small area on the wormgear tooth surface.

2.2 Modified Hindley Wormgear Drives

The Hindley wormgear drives used today are the improved ones with different modification, such as modified center distance and/or tooth number of cutter $N_c \neq N_g$ (here N_c and N_g are tooth numbers of the cutter and the wormgear) (Litvin 1968, 1994); non-orthogonal axes of the hob and the wormgear (Wildhaber 1963), Hindley worms made in accordance with the so-called "basic member gear theory" in section 5 of this paper (Hose et al 1981) and varying rolling ratio between the worm and the cutter (Ye et al 1990). The approach with varying rolling ratio is discussed here since it is the most popular and powerful one.

In this approach rotation angle of the cutter is modified with a quadratic function in the following equation:

$$\psi_c = a_1 \frac{\psi_h}{m} + a_2 \left(\frac{\psi_h}{m} + a_3 \right)^2 \quad (3)$$

in which m , ψ_c and ψ_h are design gear ratio, rotation angles of the cutter and the hob/worm; (a_1, a_2, a_3) are coefficients of the quadratic function, respectively.

To compare contact ratios of different globoidal wormgear drives, two concepts are defined in this paper: (1) dual contact ratio c_d that is contact ratio with dual contact lines in mesh simultaneously at any moment, and (2) single contact ratio c_s that is the contact ratio with single contact line in mesh. Total contact ratio c_t is equal to the sum of the dual and single contact ratios, that is

$$c_t = c_d + c_s \quad (4)$$

The same numerical example with gear ratio $m=40:1$ and design center distance $E_{wg} = 100 \text{ mm}$ is used for the different globoidal wormgear drives in this paper for comparability. The results of this modified Hindley wormgearing are shown in Figs. 3 and 4. Fig. 3 shows the contact lines and the entrance and exit positions at 24° and -24° for one group of contact lines and at 24° and 3° for another. The contact ratios are: $c_d = 2.33$, $c_s = 3.0$ and $c_t = 5.33$ respectively. The total contact ratio of 5.33 means there are maximal 6 or minimal 5 wormgear teeth contacting with the worm thread surface at

a moment, among them some teeth contact in dual contact lines but some in single contact line. Number of contact lines N_{cls} for a meshing moment varies from 7 to 9 in the meshing process. It is also noted that about one-third of the wormgear tooth surface is free from the mesh. Fig. 4 shows the relative curvature radii ρ , the meshing angles q_{cv} between tangents of the contact lines and the relative velocities and sliding ratios of the wormgear ξ_g for the whole wormgear tooth surface, in which the maximum, minimum and average values of them are shown in legends for convenience. The average values of the above items are $\rho_{avg} = 3.4 \text{ in.}$, $q_{cv \text{ avg}} = 79.9^\circ$ and $\xi_{avg} = 0.05$ respectively. The equations to computerize the above items are shown in another paper of the author (Chen 1996) and are quoted in Appendix for reference.

There is a special treatment about the relative curvature radius in this paper: if its value is larger than 20.0 in. it is cut to 20.0 in. without loss of practical meaning. The reasons are that (1) curvature theory of differential geometry is based on local geometry and too large relative curvature radius is not in proportion to the dimension of the meshing area after tooth deformation; and (2) if the calculating grids are on or near to the intersecting line of two subsurfaces of the wormgear tooth that is a singular line, the relative curvature radii will be infinity or very large, which will increase their average value largely and loses its practical meaning.

The main weaknesses of the Hindley wormgear drives are: (1) the worm thread surfaces can not be ground since they are undevelopable ruled surfaces; and (2) part contact on the wormgear tooth surface.

3. WILDHABER WORMGEAR DRIVES

The Wildhaber wormgear drive was invented in 1924 by E. Wildhaber (Wildhaber 1924), in which hob and worm thread surfaces are generated by spur or inclined planes in orthogonal rotation axes (Figs. 5(a) and 6(a)), and wormgear tooth surfaces are spur or inclined planes milled by plane milling cutters (Figs. 5(b) and 6(b)). Modified Wildhaber wormgear drives are reported by Ishida et al 1978 and Ye et al 1990. The tooth surfaces of Wildhaber wormgear can not be represented by Eqn. (2) so they do not have the property of dual contact that is an exception for globoidal wormgear drives. However, the advantage of Wildhaber wormgear drives is that both tooth surfaces of the worm and the wormgear can be ground so the wormgear can be made of harder material and used for high-accuracy motion transmissions. The main disadvantage of Wildhaber wormgear drives is that only half of the wormgear tooth surface are in contact with the worm thread

surface at a single contact line, which means they are not suitable for power transmissions.

The Wildhaber's idea is modified in the following globoidal wormgear drives: (1) The wormgear with cone tooth surfaces milled by a cone milling cutter meshes with the cone enveloping globoidal worm whose thread surfaces are envelopes of the families of conical surfaces (Popov 1977 and Umazeno and Maki 1991). (2) The globoidal worm with the thread surfaces enveloped by spur or helical involute surfaces meshes with spur or helical gear (Buckingham 1963, Ye et al 1990). Since their geometry property is similar to that of the plane Wildhaber wormgearing, here we only discuss the plane Wildhaber wormgear drives.

The spur Wildhaber wormgearing has serious undercutting and pointing problems on the worms so it is only used for high gear ratio sets ($m \geq 40$). The example is designed for an inclined Wildhaber wormgear drive with an inclined tooth angle $\beta = 6^\circ 50'$. The results are shown in Figs. 7 and 8. The wormgear enters the mesh with the worm at 20° and leaves at -10° after contacting only half of the wormgear tooth surface in single contact line (Fig. 7). The contact ratios are: $c_t = c_s = 3.33$ and $c_d = 0.0$, and the number of contact lines N_{cls} for a meshing moment varies from 3 to 4. The average relative curvature radius ρ_{avg} and average meshing angle $q_{cv \text{ avg}}$ are 2.7 in. and 69° that are good enough, but average sliding ratio is $\xi_{avg} = 1.5$ and sliding ratios change very sharply that results in non-uniform wear (Fig. 8).

4. INCLINED PLANE AND CONE ENVELOPING GLOBOIDAL WORMGEAR DRIVES

4.1 Inclined Plane Enveloping Globoidal Wormgear Drive

If hobs are made in the same approach as the inclined Wildhaber worms (Fig. 6(a)) whose thread surfaces are envelopes of the families of planes and then are used to cut wormgears, see Fig. 9, this type of wormgear drive is called inclined plane enveloping globoidal wormgear drive (Zhang et al 1988, Ye et al 1990).

The wormgear tooth surface of this wormgearing can be represented by Eqns. (1) and (2) so it possesses the dual contact property. To get better tooth geometry, the worm and hob are modified usually in cutter tooth number $N_c \neq N_g$, center distances $E_{cw} \neq E_{wg}$ and inclined tooth angle β , here E_{cw} and E_{wg} are center distances between the cutter and the worm and between the worm and the wormgear, see Fig. 9. The results of the inclined plane enveloping globoidal wormgearing are shown in Figs. 10 and 11. Two subsurfaces defined

by Eqn. (2) intersect about the middle of the wormgear tooth surface and enter and leave the mesh with the worm from 22° to -10° and from 21° and -10° respectively (Fig. 10). The contact ratios are $c_d = 3.44$, $c_s = 0.11$ and $c_t = 3.55$. The number of contact lines N_{cls} for a meshing moment varies from 6 to 9 in the meshing process. The average relative curvature radius ρ_{avg} , average meshing angle $q_{cv avg}$ and average sliding ratio ξ_{avg} are 6.1 in., 58° and 0.019 respectively (Fig. 11).

4.2 Inclined Cone Enveloping Globoidal Wormgear Drive

To reduce the grinding cost of the worm thread surfaces of the plane enveloping globoidal worms (single flank grinding), cone enveloping globoidal wormgear drives (duplex grinding) are developed in German patent 1969, and modified by Pavlov et al 1975). The same machine settings (Fig. 9) are used and the generating surfaces for the worm or hob thread surfaces are replaced by conical surfaces of cone milling cutter or cone grinding wheel, see Fig. 15(b).

The results of the inclined cone enveloping globoidal wormgearing are shown in Figs. 12 and 13. The wormgear tooth surface enters and leaves the mesh with the worm thread surface at 23° and -8° for one group of contact lines and 22° and -8° for another (Fig. 12). The contact ratios are $c_d = 3.33$, $c_s = 0.11$ and $c_t = 3.44$. The number of contact lines N_{cls} for a meshing moment varies from 6 to 9 in the meshing process. The average relative curvature radius ρ_{avg} , average meshing angle $q_{cv avg}$ and average sliding ratio ξ_{avg} are 5.3 in., 75.7° and 0.027 respectively (Fig. 13).

5. ENVELOPING GLOBOIDAL WORMGEAR DRIVES BASED ON SAKAI'S THEORY

5.1 Sakai's Basic Member Gear Theory

Sakai's "basic member gear theory" was developed for hypoid gearing in Sakai 1955 and was firstly utilized in globoidal wormgearing in Sakai et al 1978. The basic idea is that the contact lines between the tooth surfaces of the cutter and the worm/hob will coincide with the contact lines between the tooth surfaces of the worm and the wormgear at any moment for one group of contact lines if the following three equations are satisfied:

$$\left. \begin{aligned} N_c &= N_g \cos \delta - N_w \sin \delta \\ E_{cw} &= E_{wg} \cos^2 \delta \\ l_c &= E_{wg} \psi_c \sin \delta \cos \delta \end{aligned} \right\} \quad (5)$$

here N_c , N_g and N_w are tooth numbers of cutter, wormgear and worm; δ , E_{wg} and E_{cw} are tilt angle of cutter axis z_c

from wormgear axis z_g , center distances between the worm and the wormgear and between the worm and the cutter, ψ_c and l_c are rotation angle and translation of the cutter about and along its axis z_c , respectively (Fig. 14).

There are two differences of Sakai's approach from others: (1) The cutter performs screw motion about and along its axis z_c ; (2) Among the four design parameters (N_c , E_{cw} , δ , l_c), only one, say tilt angle δ , is independent but other three are determined by Eqn. (5).

Sakai's "basic member gear theory" are used for Hindley wormgearing (Hose 1981), plane enveloping globoidal wormgearing (Sakai et al 1978), cone enveloping globoidal wormgearing (Sakai et al 1980), toroidal enveloping globoidal wormgearing (Kobayashi et al 1993), and inverted cone enveloping globoidal wormgearing (Horiuchi 1995). In this section, plane, cone and inverted cone enveloping globoidal wormgear drives are discussed.

5.2 Sakai's Plane Enveloping Globoidal Wormgear Drive

Sakai's plane enveloping globoidal wormgear drive uses spur planes with zero pressure angle as the cutter tooth surfaces, see Fig. 15(a) (Sakai et al 1978). Since the spur planes of the cutter tooth surfaces are parallel to its rotation axis z_c , the translation of the cutter is useless and the cutter only performs rotation motion, which is only one exception for Sakai's approach.

The same numerical example is designed for Sakai's plane enveloping wormgear drive for design parameter $N_c = 39$ (Sakai et al 1978), and the results are shown in Figs. 16 and 17.

From Fig. 16 two subsurfaces of the wormgear tooth defined by Eqn. (2) intersect at one-fourth of the face width and enter and leave the mesh with the worm at -6° and -24° simultaneously. The contact ratios are $c_t = c_d = 2.0$ and $c_s = 0.0$. The number of contact lines N_{cls} is 4. The average relative curvature radius ρ_{avg} , average meshing angle $q_{cv avg}$ and average sliding ratio ξ_{avg} are 9.1 in., 59° and 0.017 respectively (Fig. 17).

An undesirable characteristic of the Sakai's plane enveloping globoidal wormgear drives is lower contact ratio, especial for low gear ratio sets.

5.3 Sakai's Cone Enveloping Globoidal Wormgear Drive

Sakai's cone enveloping globoidal wormgearing is developed in 1980 (Sakai et al 1980) to improve the geometry of the plane enveloping one. It keeps the "basic member gear theory" and the machine settings (Fig. 14) but with the following improvement: (1) the tooth surfaces of the cutter are conical surfaces; and (2) the tooth surfaces of the wormgear are