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Dynamic Distribution of Load and Stress on External Involute Gearings

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[The statements and opinions contained herein are those of the author and should not be construed as an official action or opinion of the American Gear Manufacturers Association.]

Abstract

The influence of additional dynamic loads on the distributions of load, flank pressure and tooth root stress are shown. The additional dynamic loads excited by the variations of mesh stiffness and influenced by tooth deviations and modifications are calculated and used as input for the calculation of the distributions of load and stresses in the plane of action. The programs used for these calculations were developed at the Institute of Machine Elements at the Dresden University of Technology. Very efficient calculations are obtained by using specially developed calculation algorithms. An extensive graphic presentation of the calculation results is included. The same connection of the calculation of load distribution with the calculation of dynamic loads is also possible in principle on the basis of very similar programs (LDP & DYTEM) developed at the Ohio State University.

The calculation results are compared with dynamic loads and transmission errors measured on test rigs in the gear lab at the Ohio State University.

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Dynamic Distribution of Load and Stress on External Involute Gears

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1. Introduction

Rating standards use a nominal load, that through the use of additional factors, is transformed into an existing maximum tooth load. Derating of the input torque is achieved with an application factor and internal additional loads are derated with a dynamic factor and a load distribution factor. The load distribution factor, which is due to deformations and misalignments, is split into two factors, one in the lead or face width direction and one in the transverse direction. These factors are often the focus of discussions about the question: how exactly can the maximum load be predicted through their use? The values of the maximum load predicted by rating standards sometimes differ considerably from those obtained from an extensive numerical analysis. But for some applications the simple equations of the rating standards allow only a raw approximation. More exact calculations are necessary to obtain a good approximation of the real load distribution. These calculations usually require high calculation and data handling times. Therefore, their use has been accepted only in special situations.

The ability to predict load distributions and gear dynamics have been investigated by many researchers. Conry and Seireg [4] were the first to develop a comprehensive approach to the analysis of load distribution of gears. Subsequent work by numerous investigators including Oehme [17], Krenzer [11], Mudd [16], Houser [10], Kubo [12], Pennell [18], have used iterative techniques similar to Conry's to evaluate load distribution. A more sophisticated finite element approach that not only predicts load distribution but also directly predicts contact stresses has been developed by Vijayakar [23]. Numerous dynamics analysis procedures that predict dynamic factors have been

developed and including the works of Lin, et. al. [13], Harianto and Houser [7], Rebbechi [20], and Velez [22].

In this paper, a very efficient calculation of the load distribution which needs efficient calculation and data handling is introduced. Both programs LVR [26] for the calculation of load distribution and GDA [25] for the analysis of gear dynamics are used in these calculations. These programs were developed in the Institute of Machine Elements and Machine Design at the Dresden University of Technology with the goal to reduce the calculation expense to a minimum without significantly reducing accuracy. They contain special calculation methods which fulfill this goal. The basis for these methods was created in numerous investigations about load distribution [1, 6, 8, 9, 17] and gear dynamics [2, 5, 14, 19] at the Dresden University of Technology.

The combination of both programs LVR and GDA goes a step further than other programs used for the calculation of load distribution. The combination allows the calculation of dynamic load distributions which consider the real additional dynamic loads in the obtained load distribution. At first, the calculation methods and the content of the programs will be explained and then some applications of the programs will be presented.

2. Load distribution calculation method

A new method is used for the calculation of load distribution in the program LVR. This method based on the use of deflection influence coefficients. The starting point is the dividing of the load along the line of contact into single loads. Each contact line is divided into separated sections with equal face width b_i with the single loads F_i being

applied in the center of each section. The deformation along the line of action is described by influence numbers a_{ik} . The deformation value v_i on the i -th section results from:

$$v_i = \sum_{k=1}^n a_{ik} \cdot F_k \quad (1)$$

This statement leads to the following system of equations for the calculation of the load distribution

$$\underline{A} \underline{F} = \underline{f} + \underline{e} x \quad (2)$$

with, $\underline{A}$ matrix of deformation influence numbers
 $\underline{F}$ vector of single loads
 $\underline{e}$ unity vector (all elements $e_i = 1$)
 $\underline{f}$ vector of deviations from the theoretical involute
 x tooth deformation

The amount of deformations along the line of contact dies away at a distance of 6 times tooth module from the point of a single applied load. The resultant band range n_b of the matrix $\underline{A}$ is:

$$n_b = \frac{12 m_n}{b_i} \quad (3)$$

As far as possible, a low section width b_i is needed for exact investigations of local influences on the load distribution, but this leads to a high calculation effort caused by resultant large band ranges.

The method of LVR uses an inversion of stiffness matrices to reproduce the deformation behavior. Specially dimensioned coupling stiffnesses are applied between the sections which guarantee a good reproduction of the real deformation curves. The approximation shown in Fig. 1 can be reached when the deformation function is based on results of extensive investigations at the Dresden University of Technology (a modified cosine function).

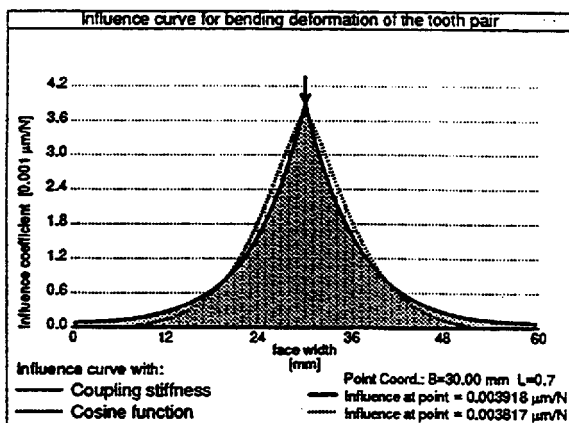


Fig. 1 Deformation due to an applied single load resultant from coupling stiffness method in comparison to the influence function

The stiffness matrices $\underline{K}$ used in the calculation have the following structure:

$$\underline{K} = \begin{bmatrix} k_1 + \lambda_1 & -\lambda_1 & 0 & \dots & 0 \\ -\lambda_1 & \lambda_1 + k_2 + \lambda_2 & -\lambda_2 & \dots & 0 \\ 0 & -\lambda_2 & \lambda_2 + k_3 + \lambda_3 & \dots & 0 \\ \dots & \dots & \dots & \dots & -\lambda_{n-1} \\ 0 & 0 & 0 & -\lambda_{n-1} & -\lambda_n + k_n \end{bmatrix} \quad (4)$$

These stiffness matrices are used to transform the system of equation (2) from the level of deformation influence into the level of force influence. This leads to a very low band range of the matrices used for the calculation of load distribution. The increased calculation speed is clearly visible during LVR applications.

The matrix of influence coefficients $\underline{A}$ can be separated into the proportions of bending deformations $\underline{A}_{b1}$ on tooth 1 and $\underline{A}_{b2}$ on tooth 2 and the Hertzian deformation of the tooth pair $\underline{A}_H$. The bending deflections include not only bending and shear of the teeth, but also the deflections of the tooth base.

$$\underline{A} = \underline{A}_{b1} + \underline{A}_{b2} + \underline{A}_H \quad (5)$$

The substitution of $\underline{A}$ by $\underline{K}^{-1}$ leads to the following system of equations.

$$(\underline{K}_{b1}^{-1} + \underline{K}_{b2}^{-1} + \underline{K}_H^{-1}) \underline{F} = \underline{f} + \underline{e} x \quad (6)$$

Two methods are possible for the transformation of system (6) into the force influence level. A simplified transformation can be used to retain the tridiagonal structure of the matrices $\underline{K}$ with the lowest possible band range of 3. The exact transformation leads to matrices with a larger but still low band range.

The simplification used in the first method is reached by a substitution of the separated bending matrices for each tooth by a tooth pair bending matrix, which is built up with the resultant section stiffness of the tooth pair k_b :

$$\frac{1}{k_b} = \frac{1}{k_{b1}} + \frac{1}{k_{b2}} \quad (7)$$

with, k_{b1} section bending stiffness of tooth 1
 k_{b2} section bending stiffness of tooth 2

The system of equations is thus reduced as follows:

$$(\underline{K}_b^{-1} + \underline{K}_H^{-1}) \underline{F} = \underline{f} + \underline{e} x \quad (8)$$

This system is then transformed with the matrix $\underline{K}_b$:

$$\underline{K}_b (\underline{K}_b^{-1} + \underline{K}_H^{-1}) \underline{F} = \underline{K}_b (\underline{f} + \underline{e} x) \quad (9)$$

$$\underline{E} (\underline{K}_b \underline{K}_H^{-1}) \underline{F} = \underline{K}_b (\underline{f} + \underline{e} x) \quad (10)$$

tridiagonal system of equations allows an extremely high speed calculation.

The exact transformation of the system (2) without the simplification of equation (7) leads to the following system of equations:

$$(\underline{E} + \underline{K}_{b1} \underline{K}_{b2}^{-1} + \underline{K}_{b1} \underline{K}_H^{-1}) \underline{F} = \underline{K}_{b1} (\underline{f} + \underline{e} x) \quad (11)$$

The product $\underline{K}_{b1} \underline{K}_{b2}^{-1}$ is a fully filled matrix and the inversion of $\underline{K}_{b2}$ is needed. But the resultant matrix from $\underline{K}_{b1} \underline{K}_{b2}^{-1}$ is not balanced. The number of elements decreases rapidly with increasing distance from the diagonal. Because of this, the band range used in the calculation of load distribution can be reduced to $n_b = 3 m_n / b_i$ without a reduction of accuracy. The calculation is still much faster than those based on deformation influence coefficients with the band range $n_b = 12 m_n / b_i$.

3. Method of the calculation of vibrations

The calculation of vibrations is carried out in the frequency domain with the program GDA and uses as a basis, the Fourier series for all time-varying parameters. The dependence of tooth stiffness, tooth deviations and transmission errors is transformed into a time dependence on the assumption that deviations from the running speed caused by vibrations are small in comparison to the running speed. The damping to be constant. The following system of differential equations has to be solved.

$$\underline{M} \ddot{\underline{\varphi}} + \underline{D} \dot{\underline{\varphi}} + \underline{K}(t) \underline{\varphi} = \underline{r}(t) + \underline{h} \quad (12)$$

with, $\underline{\varphi}$ vector of the degrees of freedom

$\underline{M}$	matrix of mass inertias
$\underline{D}$	damping matrix
$\underline{K}(t)$	stiffness matrix
$\underline{r}(t)$	vector of excitation
$\underline{h}$	vector of static loads

The system of equations (12) contains only linear parameters. That means that all time-varying parameters are determined due to the nominal load $\underline{h}$ and existing load dependencies are neglected. The nonlinearity caused by flank separation cannot be considered with this calculation method. This flank separation occurs only in the case of very high vibration amplitudes which occur during resonant operation, a condition that is usually avoided in practice. The main focus is on the reduction of the high amplitudes at resonances by changing the gear design or to avoid operating in such regions. The calculation without the influence of flank separation generally results in higher amplitudes than with the influence of flank separation if flank separation occurs (approximately if the dynamic factor exceeds 2). The method does not include the critical resonance ranges but does not consider flank separation. Because of the uncertainty of the damping models, it is unlikely that including flank separation of heavily loaded gears would give a result with a higher degree of accuracy.

The linear time-varying stiffness can be divided into the constant part $\underline{K}_m$ and the time-varying part $\underline{K}_v(t)$:

$$\underline{K}(t) = \underline{K}_m + \underline{K}_v(t) \quad (13)$$

The eigenfrequencies ω_k and the eigenmodes $\underline{v}_k$ are determined in GDA on the basis of the non-damped time-independent system derived from system (12):

$$(\underline{K}_m + \underline{M} \omega_k^2) \underline{v}_k = 0 \quad (14)$$

Using the generalized coordinates $\underline{q}$, the system of equations (12) is transformed with the modal matrix $\underline{V}$ (filled with the eigenmodes $\underline{v}_k$):

$$\ddot{\underline{q}} + \underline{\Delta} \dot{\underline{q}} + (\underline{\Lambda} + \underline{\Gamma}(t)) \underline{q} = \underline{\rho}(t) + \underline{\eta} \quad (15)$$

with,

$$\begin{aligned} \underline{\varphi} &= \underline{V} \underline{q} \\ \underline{V}^T \underline{M} \underline{V} &= \underline{E} ; \quad \underline{V}^T \underline{D} \underline{V} = \underline{\Delta} \\ \underline{V}^T \underline{K}_m \underline{V} &= \underline{\Lambda} ; \quad \underline{V}^T \underline{K}_v \underline{V} = \underline{\Gamma} \\ \underline{V}^T \underline{r}(t) &= \underline{\rho}(t) ; \quad \underline{V}^T \underline{h} = \underline{\eta} \end{aligned} \quad (16)$$

The following basic equation is used to solve the system of equations (15):

$$\underline{q} = \underline{q}_m + \varepsilon \underline{q}_v \quad (17)$$

The factor ε used in this calculation method marks only the condition $\underline{q}_v < \underline{q}_m$. The following calculation [2] can be used assuming the stiffness values fulfill the condition (17), i. e. $\underline{C}_v(t) < \underline{C}_m$:

$$\ddot{\underline{q}}_{i+1} + \underline{\Delta} \dot{\underline{q}}_{i+1} + \underline{\Lambda} \underline{q}_{i+1} = -\underline{\Gamma}(t) \underline{q}_i + \underline{\rho}(t) \quad (18)$$

There exists no closed analytical solution for the system (15) excited by the tooth stiffness. The parametrically excited system is approximately transformed into a linear system (18) with excitations only on the right-hand side for which a direct analytical solution exists. The excitation resultant from the time-varying stiffness is determined for the calculation of the $(i+1)$ -th step with the use of the solution $\underline{q}_i$ from the i -th step and the matrix $\underline{\Gamma}(t)$. The iteration starts with the static solution:

$$\underline{q}_0 = \underline{q}_m = \underline{\Lambda}^{-1} \underline{\eta} \quad (19)$$

which is used to determine the excitation for the first iteration step for $\underline{q}_1$. The number of harmonics used in the solution statement for $\underline{q}_i$ increases greatly in every iteration step caused by the product of the Fourier series of $\underline{q}_i$ and $\underline{\Gamma}(t)$.

But many new excitation harmonics are very low and they can be neglected. The intensity of excitation is analyzed in the program GDA and all harmonics with amplitudes less 0.5 % of the static tooth load are neglected. An unlimited