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Non-Dimensional Characterization of Gear Geometry, Mesh Loss and Windage

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TECHNICAL PAPER

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[The statements and opinions contained herein are those of the author and should not be construed as an official action or opinion of the American Gear Manufacturers Association.]

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New relationships for involute spur gear geometry are introduced and integrated with new methods of correlating lubricant traction and windage test data. Compact math models for lubricant density and viscosity under contact pressure are proposed. A modern approach to dimensional analysis is introduced to characterize lubricant traction data and gear windage data with dimensionless terms which apply to gear systems which may have a configuration and/or size different from those tested. Finally, system considerations for optimal gearbox efficiency are proposed.

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Abstract

New relationships for involute spur gear geometry are introduced and integrated with new methods of correlating lubricant traction and windage test data. Compact math models for lubricant density and viscosity under contact pressure are proposed. A modern approach to dimensional analysis is introduced to characterize lubricant traction data and gear windage data with dimensionless terms which apply to gear systems which may have a configuration and/or size different from those tested. Finally, system considerations for optimal gearbox efficiency are proposed.

Introduction

"An important link in the chain that ties the prime mover to the products of man is the GEAR. In the past, gears were only wheels with teeth chiseled in stone, carved in wood ... as the gears ground on through their daily tasks, the teeth took on a curvature at the points of wear ... running a little smoother than they did when they were brand new."

T.W. Khiralla

In his book (1), Khiralla traces the history of the ubiquitous spur gear. We learn that Phillipe de La Hire proposed an involute shape for spur gears near the end of the 17th century. However, the mathematics of the involute spur gear were first developed by Euler in the middle of the 18th century. In today's technology, the role of the spur gear takes on increasing importance. Consequently, we should continually strive to improve our characterizations of its geometry, motion, and power loss. For a high-speed gearbox, such characterizations must include the potentially dominant effects of windage in the presence of airborne oil. These topics are addressed herein.

1.0 Spur Gear Geometry and Motion

In this section, we review the well-known equations for involute geometry and introduce additional geometry and velocity characterizations which, although fundamental, appear to have been previously unpublished. These will be integrated, in later sections, with new methods for the calculation of mesh loss and windage.

1.1 Involute Geometry

Figure 1.1 shows the familiar representation of pinion and gear base circles connected by an imaginary string. The gear and pinion, at rotational speeds (ω_1 and ω_2), transmit the normal force (F) at constant velocity ($V_b = \omega_1 r_1 = \omega_2 r_2$) along a line of action which is inclined at the pressure

angle (ϕ_p). The companion Figure (1.2) describes involute geometry, showing the contact radius (r) which can be "normalized" as a ratio with the base radius (r_b). Similarly, rolling and sliding speeds can be normalized with (V_b), gear traction force (T) can be normalized with (F), and the mesh power loss can be normalized with the mesh power (FV_b).

Figure 1.1 Base Velocity and Radii
References for Non-Dimensionalization

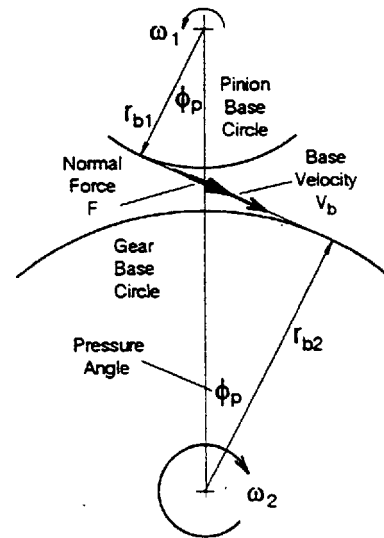
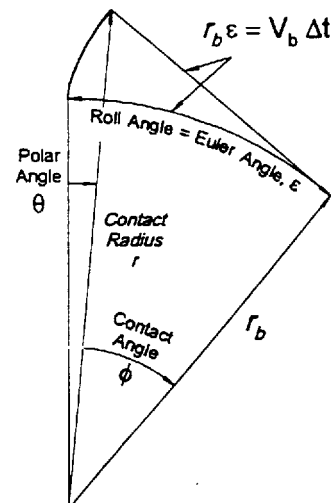


Figure 1.2 Involute Geometry
Euler Angle (ϵ) Varies linearly with time (t)



Equations (1.1) and (1.2), corresponding to Figure 1.2, are given by Dudley (2). We propose the name "Euler Angle" for that angle which varies linearly with time throughout the mesh. We also designate a dimensionless contact radius as (ρ) and relate it to the contact angle (ϕ) with EQs (1.3) and (1.3a). Note that the contact and pressure angles are equal only at the pitch point.

$$\text{Euler Angle} \quad \varepsilon = \tan \phi \quad (1.1)$$

$$\text{Polar Angle} \quad \theta = \varepsilon - \phi \quad (1.2)$$

Dimensionless

$$\text{Contact Radius} \quad \rho = r/r_b = 1/\cos \phi \quad (1.3)$$

$$\text{Contact Angle} \quad \phi = \cos^{-1}(1/\rho) \quad (1.3a)$$

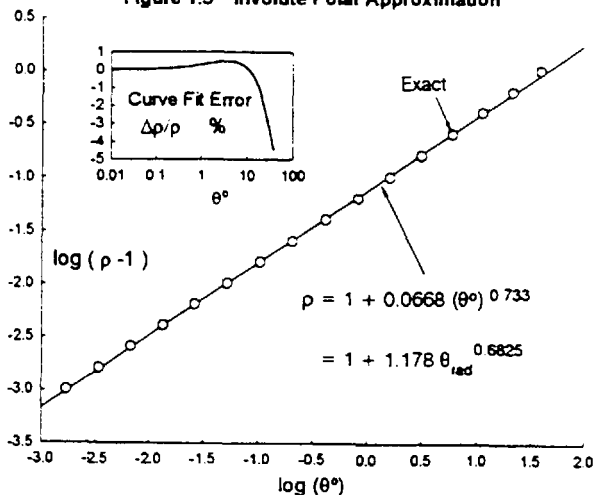
Given well-known relations between $\sin \phi$, $\cos \phi$, and $\tan \phi$, we can manipulate and combine EQs (1.1) and (1.3) to obtain a new geometric relation in the form of EQ (1.4). Next, we can combine EQs (1.4) with EQ (1.2) to directly relate the polar angle (θ) and dimensionless contact radius (ρ) with EQ (1.5). An inverse, but transcendental, relation is provided by EQ (1.6), where we can obtain (ρ), given both (θ) and (ρ). Since ($\rho-1$) is nearly proportional to (θ) raised to an exponent, we have almost a straight line on the log-log plot of Figure 1.3. The corresponding curve fit provides a very close first guess for a rapid and stable iterative solution to EQ (1.6) over the full range shown in the figure.

$$\varepsilon = \sqrt{\rho^2 - 1} \quad (1.4)$$

$$\theta = \sqrt{\rho^2 - 1} - \tan^{-1} \sqrt{\rho^2 - 1} \quad (1.5)$$

$$\rho = \left[\theta + \cos^{-1}(1/\rho) \right] / \sqrt{1 - (1/\rho)^2} \quad (1.6)$$

Figure 1.3 Involute Polar Approximation



1.2 Addendum and Dedendum Circles

To characterize rolling and sliding velocities, mesh power loss, and gear windage in non-dimensional form as shown later herein, we will need to determine the dimensionless gear tooth height and addendum radius. The addendum radius will be normalized to the base radius, but the tooth height (h) will be normalized to pitch diameter (D).

The addendum circle is shown, together with other circles, in Figure 1.4. First, we designate the number of teeth as (N) and the diametral pitch as (N/D). Next, we adopt a common AGMA standard whereby the addendum is given by ($a=D/N$) and the dedendum is given by ($b=1.25 D/N$) to obtain the dimensionless parameters of EQs (1.7 - 1.9).

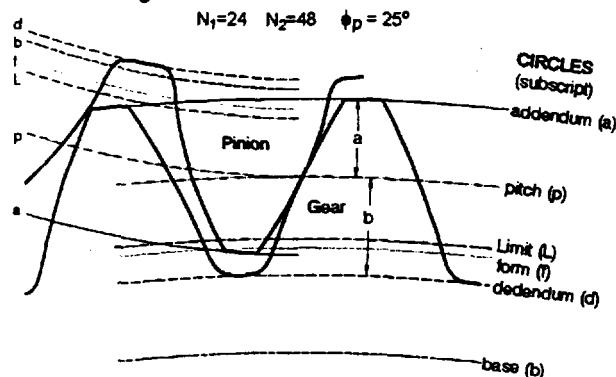
$$\rho_a = r_a / r_b = [r_p + (2r_p / N)] / r_b = [1 + (2 / N)] / \cos \phi_p \quad (1.7)$$

$$\rho_d = r_d / r_b = [1 - (2.5 / N)] / \cos \phi_p \quad (1.8)$$

$$h/D = 2.25 / N \quad (1.9)$$

For the pinion of Figure 1.4, the base circle lies outside the dedendum circle. For the gear, however, the base circle lies inside. The number of gear teeth below which the base circle lies outside is given by $2.5 / (1 - \cos \phi_p)$.

Figure 1.4 Gear and Pinion Circles



1.3 Limit Radius

The gear and pinion limit radii (Figure 1.4) represent the lower limits of involute action with exact centers. These radii are required to calculate sliding and rolling velocities. Also, the limit radius of each is affected by the geometry of the other. Figure 1.5 shows the pinion and gear teeth at the start and end of mesh. The law of cosines can be applied to solve for the limit radii as obtained in EQs (1.10-1.11), where (C) designates the center distance ($r_{p1} + r_{p2}$).

$$r_{L1} = \sqrt{r_{a2}^2 + C^2 - 2r_{a2}C \cos(\phi_{a2} - \phi_p)} \quad (1.10)$$

$$r_{L2} = \sqrt{r_{a1}^2 + C^2 - 2r_{a1}C \cos(\phi_{a1} - \phi_p)} \quad (1.11)$$

Figure 1.5 Limit Radius Calculation

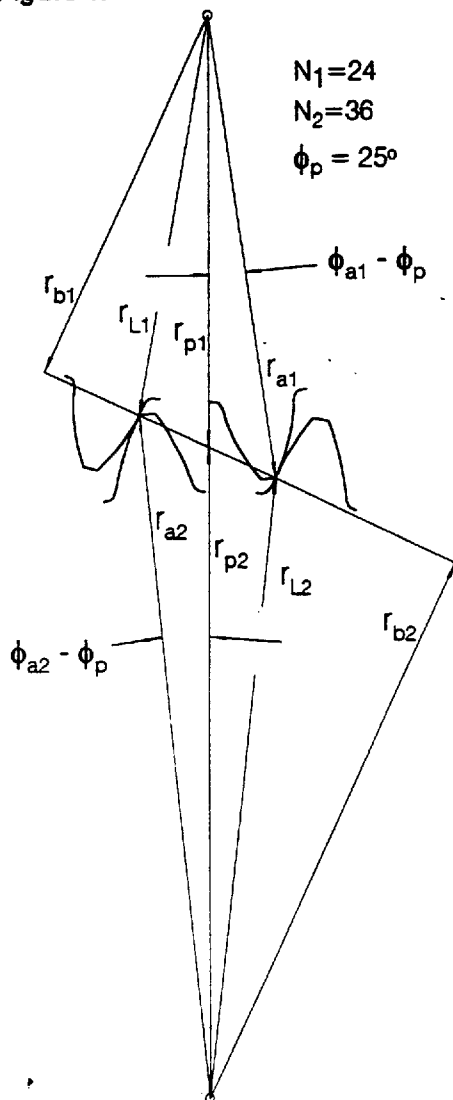
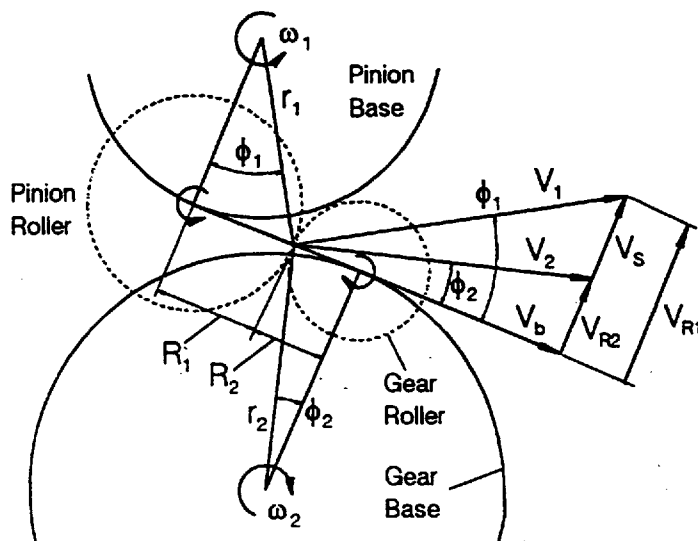


Figure 1.6 Rolling and Sliding Diagram



The average of the two roller surface speeds represents the common rolling velocity. At the pitch point, the rollers are in pure rolling contact, and they share a common surface speed ($V_{R1} = V_{R2}$). Therefore, the author advises against the practice of designating the sum ($V_{R1} + V_{R2}$) as a "rolling velocity." Here, "sum velocity" would be more appropriate, and this term has been adopted by many authors.

For the pinion roller, the surface speed, normalized to the base velocity, is given by ($V_{R1} / V_b = \tan \phi_1 = \epsilon_1$). That of the gear is given by (ϵ_2). Thus, at any point along the pressure line, the two rollers share a normalized rolling speed which is given by the average Euler angle (EQ 1.12). Likewise, the normalized sliding speed is given by the difference in Euler angles (EQ 1.13). Since, for either gear, the Euler angle varies linearly with time throughout the mesh, the rolling and sliding speeds are also linear with time. The sliding velocity changes sign at the pitch point.

$$\text{Normalized rolling speed } V_R / V_b = (\epsilon_1 + \epsilon_2) / 2 \quad (1.12)$$

$$\text{Normalized sliding speed } V_s / V_b = \epsilon_1 - \epsilon_2 \quad (1.13)$$

1.4 Rolling and Sliding Velocities

The rolling and sliding diagram of Figure 1.6 shows the true velocities (V_1 and V_2) of the pinion and gear tooth contact points at an arbitrary position along the pressure line. Note that V_1 is perpendicular to r_1 and V_2 is perpendicular to r_2 . For either velocity, the component along the pressure line equals the base velocity ($V \cos \phi = \omega r \cos \phi = \omega r_b = V_b$).

In the same figure, the dashed circles of radii ($R_1 = r_1 \sin \phi_1$) and ($R_2 = r_2 \sin \phi_2$) represent imaginary rolling cylinders. As the mesh progresses, the rollers change in relative size, but with a constant center distance as the contact point translates with the direction and velocity (V_b). Since only those velocity components normal to the pressure line are of interest here, we imagine an observer, traveling on the pressure line at speed (V_b), who sees the roller surface speeds (V_{R1} , V_{R2}). The observer also sees the sliding velocity (V_s), which is the difference between the two surface speeds.

2.0 Modern Dimensional Analysis

In this section, toward our goal of efficiently characterizing gear mesh loss and windage, we will review Rayleigh's classical method of dimensional analysis and improve upon it by proposing a far less arbitrary method of arranging the physical variables and by introducing matrix and computer methods to expedite the analysis.

Dimensional analysis is a powerful method which finds application in fluid dynamics, heat transfer, electrical circuits, orbital dynamics, and endless other fields. The method allows us to dramatically reduce the number of variables which must be carried in a calculation or test matrix. The resulting dimensionless form can then be "re-dimensionalized" with any consistent set of units.