

97FTMS1

Coordinate Measurement and Reverse Engineering of ZK Type Worm Gearing

by: Xiaogen Su and Donald R. Houser, Ohio State University



TECHNICAL PAPER

Coordinate Measurement and Reverse Engineering of ZK Type Worm Gearing

Xiaogen Su and Donald R. Houser, Ohio State University

[The statements and opinions contained herein are those of the author and should not be construed as an official action or opinion of the American Gear Manufacturers Association.]

Abstract

A comprehensive model for the measurement, inspection, performance prediction and reverse engineering of ZK type of worm gearing is developed. The measurements and the best fit processes both for the worm thread and for the gear tooth flank are discussed in detail. A CMM measurement strategy free of tip compensation which applies to many types of tools and parts in the gear industry is proposed. A real case of ZK type of worm gearing with the parabolic profile modification on the hob is studied to illustrate the reverse engineering process.

Copyright © 1997

American Gear Manufacturers Association
1500 King Street, Suite 201
Alexandria, Virginia, 22314

November, 1997

ISBN: 1-55589-710-X

Coordinate Measurement and Reverse Engineering of ZK Type Worm Gearing

X. Su (Research associate) D. R. Houser (Professor)
Department of Mechanical Engineering
The Ohio State University

April 20, 1997

Nomenclature

Σ_c	the hob surface.
Σ_w	the worm surface.
Σ_g	the worm gear surface.
E_h	center distance between the worm-like hob axis and gear axis while hobbing
E_m	center distance between the worm axis and gear axis while meshing
E_c	center distance between the hob axis and the grinder while grinding the hob.
E_w	center distance between the worm axis and the grinder while grinding the worm.
γ_h	cross angle between the hob axis and gear axis while hobbing.
γ_m	cross angle between the worm axis and gear axis while meshing.
γ_c	cross angle between the hob axis and the grinder axis while grinding the hob.
γ_w	cross angle between the worm axis and the grinder axis while grinding the worm.
α	grinder's profile angle, subscript c or w may be applied for hob or worm grinders
a	distance from the apex of the V shaped grinder cone to its base (see Fig. 2).
s	double-V shaped grinder's dimension (see Fig. 2).
ϕ_h	the rotation angle of the hob while hobbing the gear.
ϕ_m	the rotation angle of the worm while meshing.
N_c	thread number of worm-like hob
N_w	thread number of worm
N_g	worm gear teeth number.
m_h	the gear ratio while hobbing. $m_h = N_c/N_g$.
m_m	the gear ratio while meshing. $m_m = N_w/N_g$.

l_c	hob lead
l_w	worm lead
ψ_c	rotational curvilinear ordinate of hob thread.
ψ_w	rotational curvilinear ordinate of worm thread.
ρ_c	screw parameter of the hob ($\rho_c = l_c/2\pi$).
ρ_w	screw parameter of the worm ($\rho_w = l_w/2\pi$).
λ_h	swivel angle while hobbing ($\lambda_h = \pi/2 - \gamma_h$).

1 Introduction

ZK type worm gearing is probably the most widely used worm gearing in this country. The merit of ZK type of worm gearing is that both the worm and the worm-like hob can be ground by a disc grinder so that a high quality topography can be achieved. The complexity of the geometry involved in the manufacturing of the worm, hob and the wormgear still challenges the quantitative measurement, inspection, performance prediction and reverse engineering of these mechanical parts. Janninck[1] first presented a method to predict the contact pattern of ZA type of worm gearing. Colbourne[2] proposed his method to design the hob with extended life span and improved contact patterns, but these patterns were only approximate. More simulations were done by Litvin et al[3][4] to study the effect of various mismatches on transmission errors and bearing contacts. Yoshimura[5] tried to inspect an hourglass worm through coordinate measurement. Hu and Pennell[6] made their effort to do the lead inspection on the worm gears. The goal of our research is to provide an alternative tool for worm gearing quality control other than the traditional rolling test, and to formulate a model for the reverse engineering of ZK type of worm gearing.

In ZK type of worm gearing[7][8], the worm sur-

face Σ_w and the hob surface Σ_c are generated by the relative screw motion between the double-V shaped tool (a milling cutter or a grinder) and the part. Fig. 1 shows the process. Imagine the tool in the process of generation is at rest, the worm (or the hob) being generated performs the screw motion about its axis with the screw parameter ρ_w ; the axes of the tool and the worm are crossed forming the angle γ_w , which equals the lead angle on the worm pitch cylinder. In reality, the tool rotates about its axis to provide the necessary cutting speed, but this rotation will be ignored in our geometry model. Some gashes are applied to the tool's surface to produce the blades if the tool is a milling cutter.

Litvin[7] presents the equations of the worm surface (the hob surface equations have the same format) when the tool has straight generatrix. It resembles spur gears in profile modification. We think the worm and/or the gear do not have a profile modification if the grinder's generatrix is straight. Most likely, some kind of profile modification is applied to the tool to produce a better contact pattern, for example, to locate the contact point, or to eliminate edge contact or cornering. A model with a parabolic profile modification is presented here.

Referring to Fig. 2, in the coordinate system attached to the grinder, a point on its cone surface with straight generatrix can be expressed as

$$\mathbf{r} = \begin{bmatrix} u \cos(\alpha) \cos(\theta) \\ u \cos(\alpha) \sin(\theta) \\ u \sin(\alpha) - a \end{bmatrix} \quad (1)$$

The inward normal is

$$\mathbf{n} = \begin{bmatrix} -\sin(\alpha) \cos(\theta) \\ -\sin(\alpha) \sin(\theta) \\ \cos(\alpha) \end{bmatrix} \quad (2)$$

With the parabolic profile modification, the straight generatrix is replaced by a parabola with its apex at $u = u_m$, the amount of modification (material removed in the inward normal direction) is $c_2(u - u_m)^2$. Now the point on the modified pseudo-cone surface is

$$\mathbf{r}_0 = \mathbf{r} + c_2(u - u_m)^2 \mathbf{n} \quad (3)$$

and the outward normal is

$$\mathbf{n}_0 = \frac{\partial \mathbf{r}_0}{\partial \theta} \times \frac{\partial \mathbf{r}_0}{\partial u} \quad (4)$$

Transforming the point on the grinder surface to the system attached to the worm and applying transformations to $\mathbf{r}_0$ and $\mathbf{n}_0$

$$\mathbf{r}_w = P_{screw} P_{10} \mathbf{r}_0 \quad (5)$$

$$\mathbf{n}_w = N_{screw} N_{10} \mathbf{n}_0 \quad (6)$$

where P_{10} is the point transformation matrix from the grinder system to the system 1. N_{10} is the normal transformation matrix from the grinder system to the system 1. P_{screw} is the point transformation matrix for the screw motion, N_{screw} is the normal transformation matrix for the screw motion. They are

$$P_{10} = \begin{bmatrix} 1 & 0 & 0 & E_w \\ 0 & \cos(\gamma_w) & -\sin(\gamma_w) & 0 \\ 0 & \sin(\gamma_w) & \cos(\gamma_w) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (7)$$

$$P_{screw} = \begin{bmatrix} \cos(\psi_w) & \sin(\psi_w) & 0 & 0 \\ -\sin(\psi_w) & \cos(\psi_w) & 0 & 0 \\ 0 & 0 & 1 & \rho_w \psi_w \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (8)$$

If E_w and ρ_w are set to be 0, then P_{10} and P_{screw} become N_{10} and N_{screw} , where ψ_w is the rotation parameter of the worm screw thread.

To generate a point on the worm surface, the following equation must be satisfied.

$$\mathbf{n}_0 \cdot \mathbf{v}^{w0} = 0 \quad (9)$$

where $\mathbf{n}_0$ is the normal in the grinder system, and $\mathbf{v}^{w0}$ is the velocity at a certain point of the worm relative to the grinder.

The expanded form of the above equation is presented in [7]. In the case of a parabolic profile modification, insert the above derived new equations of $\mathbf{r}_w$ and $\mathbf{n}_w$ in the equations of (18.7.7-8)[7], we arrive at

$$e[-\cos(\theta) \sin(\gamma) f + d] - g[\sin(\gamma) h - \rho \cos(\gamma)] = 0 \quad (10)$$

$$\begin{aligned} d &= -\sin(\theta) \cos(\gamma) E - \sin(\theta) \rho \sin(\gamma) \\ e &= \sin(\alpha) + 2c_2(u - u_m) \cos(\alpha) \\ f &= u \sin(\alpha) - a + c_2(u - u_m)^2 \cos(\alpha) \\ g &= \cos(\alpha) - 2c_2(u - u_m) \sin(\alpha) \\ h &= u \cos(\alpha) \cos(\theta) - c_2(u - u_m)^2 \sin(\alpha) \cos(\theta) + E \end{aligned}$$

This equation implicates the relation $\theta = f(u)$ which traces out a curve on the grinder surface. After we get the hob thread surface (point and normal), the surface of the gear being hobbled can be obtained by solving the hobbing equation[7]

$$\begin{aligned} & [z_c \cos(\phi_h) + E_h \cot(\gamma_h) \sin(\phi_h)] n_{zc} + \\ & [-z_c \sin(\phi_h) + E_h \cot(\gamma_h) \cos(\phi_h)] n_{yc} - \\ & \{ [x_c \cos(\phi_h) - y_c \sin(\phi_h) + E_h] \\ & - \rho_c \frac{1 - m_{21}^h \cos(\gamma_h)}{m_{21}^h \sin(\gamma_h)} \} n_{zc} = 0 \end{aligned} \quad (11)$$

where $\mathbf{r}_c = (x_c, y_c, z_c)^T$ is the point on the hob surface. ϕ_h is the hob rotation angle while hobbing the gear. For a certain point on the hob thread, the equation (11) is iteratively solved to get the ϕ_h . Again, The point on the hob is transformed into the gear system, the gear tooth can be found out point by point.

Now, in the gear reference frame, a point and its normal on the gear tooth flank can be expressed as

$$\mathbf{r}_g = \mathbf{f}_p(u_c, \psi_c, \phi_h) \quad (12)$$

$$\mathbf{n}_g = \mathbf{f}_n(u_c, \psi_c, \phi_h) \quad (13)$$

$$f_{\text{hobbing}}(u_c, \psi_c, \phi_h) = 0.0 \quad (14)$$

Among u_c, ψ_c and ϕ_h , u_c, ψ_c are the 2 curvilinear coordinates of the hob thread surface. We use ϕ_h and u_c as the two curvilinear coordinates to define a point on the gear tooth.

2 Find the Gear Reference Frame

The gear reference frame used while the gear is being hobbled is very important for our measurement purpose. Firstly, we need the hobbing axis of the gear. The bore axis can be easily located on a CMM, but it is not enough. Significant eccentricity and wobble may exist there. They add difference from tooth to tooth if not excluded. Secondly, the special enveloping shape of the worm gear teeth (single enveloping) makes its axial reference (or the actual mid-face position) critical too. Finally, the hobbing center distance E_h is required to derive each point on the tooth surface. It is a machine setup which changes after every re-sharpening of the hob.

2.1 Measurement of the Bottom Land Surface of Revolution

Even though the bottom lands between consecutive teeth are not important and thus not controlled qualitatively during hobbing, it is useful for our purpose. All the bottom lands rest on a surface of revolution. Experiments prove the measurement of the bottom land provides us with good and repeatable results about the gear reference frame because of its simple geometry and vast area. Fig. 3 shows the generation process of the bottom land surface of revolution. Theoretically it is cut by the hob's top blades during the hobbing process. Usually the top blades are straight. Therefore the bottom land surface of revolution is decided by the following 3 parameters,

E_h	hobbing center distance
d_o	hob outside diameter
λ_h	swivel angle (due to oversize, 2 axes are not perpendicular $\lambda_h = \frac{\pi}{2} - \gamma_h$)

The problem is that the effects of these 3 parameters on the shape of the bottom land surface of revolution are nearly coupled together. Fortunately, λ_h is the difference between the lead angles of the worm and the hob at their pitch diameters, and usually it is less than 0.5° . Simulation proves that, for a gear of average size, a difference of 0.5° in λ_h creates a difference of $0.00025''$ in E_h (Fig. 4c shows both the real case and simulation results), which are close to the accuracy of the CMM machine. Thus if we simply assume $\lambda_h = 0$, the other two parameters E_h and d_o can be obtained through best fit. Or by assuming a series of different λ_h between 0° and 0.5° and finding the 'best' best fit, all three parameters can be found through best fit.

The first step of this process is the measurement of the bottom land surface of revolution. There exist as many narrow strips of bottom land as the teeth number of the gear. On each bottom land between 2 consecutive teeth, certain number of points are measured and recorded by a computerized CMM. A theoretical bottom land surface of revolution will be reconstructed through best fitting the measured one. This process not only tells us E_h, d_o and the mid-face position of the gear, but also gives us the hobbing axis of the gear (repeatable with good accuracy).

The result of a real example is listed in section 6