

Theoretical Model for Load Distribution on Cylindric Gears: Application to Contact Stress Analysis

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[The statements and opinions contained herein are those of the author and should not be construed as an official action or opinion of the American Gear Manufacturers Association.]

Abstract

In this paper, the load sharing along the line of contact on spur and helical gear teeth is determined from the hypothesis of minimum elastic potential. From this non-uniform load distribution and Hertz's equation, a method for determining both the value and the location of the critical contact stress is described. Obtained results are compared with those given by ISO and AGMA standards.

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THEORETICAL MODEL FOR LOAD DISTRIBUTION ON CYLINDRIC GEARS: APPLICATION TO CONTACT STRESS ANALYSIS

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Nomenclature

C	operating center distance
E_1, E_2	modulus of elasticity
F	compressive load
F_n	normal load
F_{nt}	component of the load contained in the transverse plane
L	height of both cylinders
P	transmitted power
R_1, R_2	radii of both elastic cylinders
U	deformation potential
Z_E	elastic coefficient
Z_1, Z_2	numbers of teeth on pinion and wheel
b	face width
d_a, d_f	fractional parts of ε_α and ε_β
f	load per unit of length
l	linear coordinate along the line of contact
l_c	length of contact
$l_{c \min}$	minimum length of contact
l_{eff}	effective length of contact for ISO model
m	normal module
p_b	base pitch
r_{a1}, r_{a2}	pinion and wheel outside radii
r_{b1}, r_{b2}	pinion and wheel base radii
r_{c1}, r_{c2}	pinion and wheel contact radii
r_{m1}	pinion mean radius
u	unitary potential
v	inverse unitary potential
α_t	standard transverse pressure angle
α'_t	operating transverse pressure angle
β	standard helix angle
β_b	base helix angle

ε_β	axial contact ratio
ε_α	transverse contact ratio
μ_1, μ_2	Poisson's ratios
ρ	relative radius of curvature
ρ_{a1}, ρ_{a2}	pinion and wheel curvature radii at the tip
ρ_s	relative curvature radius at the lowest point of single tooth contact of the pinion
ρ_l	relative curvature radius at the lowest point of contact of the pinion
ρ_1, ρ_2	pinion and wheel curvature radii at the transverse section
ρ_{n1}, ρ_{n2}	pinion and wheel curvature radii at the normal section
σ_H	contact stress
ω_1	pinion rotational velocity
ξ	dimensionless parameter for the rotational position

Introduction

Both ISO [1] and AGMA [2,3] models for the contact stress between two cylindric gear teeth are based on Hertz's equation for the contact between two elastic cylinders [4]

$$\sigma_H = \sqrt{\frac{F}{L} \frac{\frac{1}{R_1} + \frac{1}{R_2}}{\pi \left(\frac{1-\mu_1^2}{E_1} + \frac{1-\mu_2^2}{E_2} \right)}} \quad (1)$$

where F is the compressive load, L the height of both cylinders, and R , μ and E the radius, the

Poisson's ratio and the modulus of elasticity, respectively, for each cylinder. Replacing F by the normal load F_n , L by the length of contact l_c , R_1 and R_2 by the curvature radii of both normal profiles of pinion and wheel at the contact point ρ_{n1} and ρ_{n2} , and doing the elastic coefficient Z_E

$$Z_E = \frac{1}{\pi \left(\frac{1-\mu_1^2}{E_1} + \frac{1-\mu_2^2}{E_2} \right)} \quad (2)$$

we have

$$\sigma_H = Z_E \frac{F_n}{l_c} \left(\frac{1}{\rho_{n1}} + \frac{1}{\rho_{n2}} \right) \quad (3)$$

Finally, taking into account that $F_{nt} = F_n \cos \beta_b$ and $\rho = \rho_n \cos \beta_b$, where F_{nt} is the component of the load contained in the transverse plane, ρ the radius of curvature of the transverse profile and β_b the base helix angle, we obtain

$$\sigma_H = Z_E \frac{F_{nt}}{l_c} \left(\frac{1}{\rho_1} + \frac{1}{\rho_2} \right) \quad (4)$$

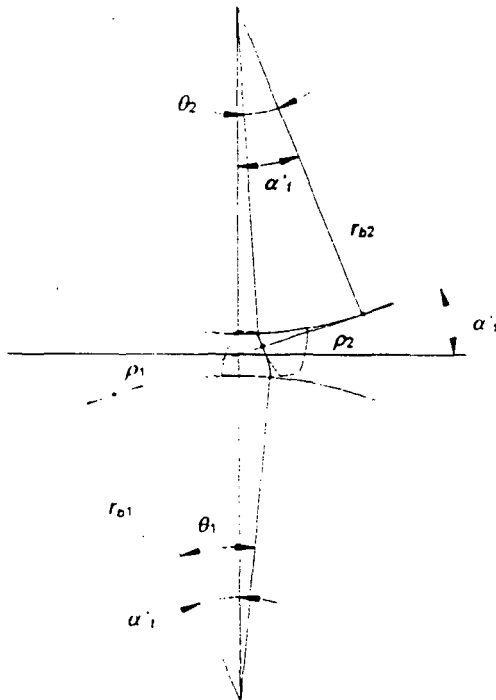


Figure 1. Curvature radii.

Subscript 1 and 2 in equation (4) denote the pinion and the wheel, respectively. F_{nt} can be computed from the transmitted power P , the pinion base radius r_{b1} and the pinion rotational velocity ω_1 as

$$F_{nt} = \frac{P}{\omega_1 r_{b1}} \quad (5)$$

while the curvature radii at the transverse section, as seen in Figure 1, may be derived from

$$\begin{aligned} \rho_1 &= \sqrt{r_{c1}^2 - r_{b1}^2} \\ \rho_2 &= \sqrt{r_{c2}^2 - r_{b2}^2} \end{aligned} \quad (6.a)$$

where r_c denotes the radius of the contact point. Figure 1 shows that ρ_1 and ρ_2 are related by

$$\rho_1 + \rho_2 = (r_{b1} + r_{b2}) \tan \alpha'_1 \quad (6.b)$$

where α'_1 is the transverse operating pressure angle.

To simplify the notation, if we do $\frac{1}{\rho} = \frac{1}{\rho_1} + \frac{1}{\rho_2}$, we

have

$$\sigma_H = Z_E \sqrt{\frac{F_{nt}}{l_c} \frac{1}{\rho}} \quad (7)$$

and, according to Equations (6), we can express ρ as a function of the radius of the pinion contact point r_{c1} .

Equation (7) gives the values for the nominal contact stress at any contact point and it is involved in both ISO and AGMA models for pitting calculations. From a theoretical point of view, the critical value, or the determinant value, of σ_H will be located at the contact point in which $(l_c \rho)^{-1}$ is maximum.

Figure 2 shows the graphic representation of the function ρ^{-1} along the interval of the line of action between both tangency points to the base circumferences of pinion and wheel. From this figure, ρ^{-1} will be maximum at the point closest to any of the limits of the interval, i.e., the point whose ρ_1 or ρ_2 is minimum. Usually this point is the lowest point of contact of the pinion, denoted by subscript 1, which is the point meshing with the tip of the wheel.

ρ_{a1} is smaller than ρ_{a2} , due to the curvature radius at the tip of the wheel ρ_{a2} is greater than that of the pinion ρ_{a1} , except for very special cases.

For spur gears, the length of contact l_c is equal to the face width b at the interval of single tooth contact, and equal to $2b$ outside this interval. Consequently, the critical contact stress will be located at the lowest point of contact or at the lowest point of single tooth contact of the pinion, denoted by subscript ls , according to $2\rho_1$ is smaller or greater than ρ_{ls} , respectively. If p_b is the base pitch and subscript a denotes the outside circumference, the above condition can be expressed as

$$2\rho_{2a}[(r_{b1} + r_{b2})\tan\alpha'_t - \rho_{2a}] \leq (\rho_{1a} - p_b)[(r_{b1} + r_{b2})\tan\alpha'_t - (\rho_{1a} - p_b)] \quad (8.a)$$

for critical stress at point l , and

$$2\rho_{2a}[(r_{b1} + r_{b2})\tan\alpha'_t - \rho_{2a}] \geq (\rho_{1a} - p_b)[(r_{b1} + r_{b2})\tan\alpha'_t - (\rho_{1a} - p_b)] \quad (8.b)$$

for critical stress at point ls . The corresponding values of the nominal contact stress will be, respectively,

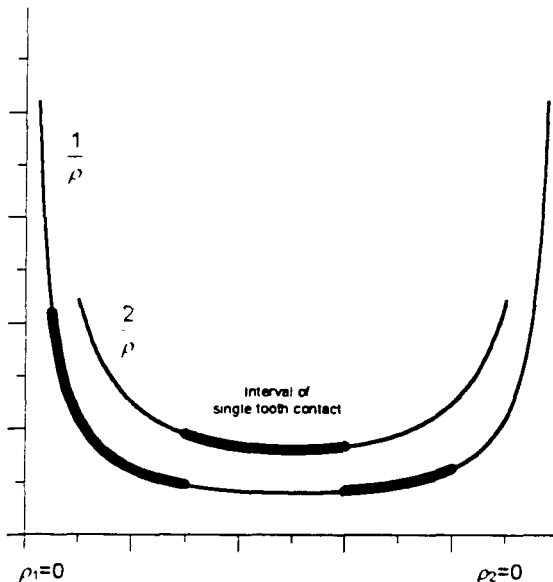


Figure 2. Graphic representation of function ρ^{-1} .

$$\sigma_H = Z_E \sqrt{\frac{F_{nt}}{2b} \frac{1}{\rho_l}} \quad (9.a)$$

and

$$\sigma_H = Z_E \sqrt{\frac{F_{nt}}{b} \frac{1}{\rho_{ls}}} \quad (9.b)$$

For helical gears, it is always possible to find transverse sections meshing at the pinion lowest point of contact during the interval of minimum length of contact. These points will be the critical ones, and the nominal contact stress for this case will be given by

$$\sigma_H = Z_E \sqrt{\frac{F_{nt}}{l_{cmin}} \frac{1}{\rho_l}} \quad (10)$$

The above theoretical results are not in good agreement with the experimental ones, which show that, for spur gears, pitting arises at the lowest point of single tooth contact of the pinion, even if Equation (8.b) is not verified, and at a point near the pitch point, for helical ones. To get a better agreement with experimental results, ISO and AGMA standards make some considerations. ISO model [1] suggests:

- To compute the nominal contact stress at the lowest point of single tooth contact or the pitch point (one giving greater stress) for spur gears, at the pitch point for helical gears with axial contact ratio ε_β greater than 1, and at a point giving the value of ρ^{-1} corresponding to a linear interpolation between the values of ρ^{-1} corresponding to $\varepsilon_\beta = 0$ and $\varepsilon_\beta = 1$.
- To consider an effective length of contact l_{eff} , given by

$$l_{eff} = \frac{b}{\cos\beta_b \left[\frac{4 - \varepsilon_a}{3} (1 - \varepsilon_\beta) + \frac{\varepsilon_\beta}{\varepsilon_a} \right]} \quad (11)$$

where ε_a is the transverse contact ratio and a value of $\varepsilon_\beta = 1$ should be considered if ε_β were greater than 1.

- To correct the value of the contact stress by the