

The Multi-Objective Optimization of Nonstandard Gears Including Robustness

by: D.R. Houser, A.F. Luscher, The Ohio State University
and I.C. Regalado, CIATEQ

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**Donald R. Houser and Anthony F. Luscher, The Ohio State University and
Isaias C. Regalado, CIATEQ**

[The statements and opinions contained herein are those of the author and should not be construed as an official action or opinion of the American Gear Manufacturers Association.]

Abstract

The design of gears involves the optimization of different objectives, and the presence of errors in the manufacturing and operating conditions affects the performance of the gears; therefore, a robust optimization procedure applying Taguchi methods was used as a tool in the design of nonstandard cylindrical gears. This paper presents an outline of the procedure and discusses some of the results.

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Houser Donald, R.
Mechanical Engineering Department, The Ohio State University
Luscher Anthony, F.
Mechanical Engineering Department, The Ohio State University
Regalado Isaias C.
Mechanical Transmissions Department, CIATEQ A. C.

INTRODUCTION:

Design optimization methods have often been applied to gear design [4] [6] [11] [12] Most of these attempts were one or two variable optimizations and did not always take into account "real" design considerations such as hob shifting and only one has included noise and/or transmission error in the design criteria [11]. In this paper, an eight variable optimization procedure is presented. The procedure not only varies conventional design variables such as number of teeth, pressure angle, face width, center distance, and helix angle, but also allows for hob shift, different sized hobs and uses actual load distributions for stress computations. In addition robustness techniques are applied so that the optimum design is one that is least sensitive to manufacturing errors.

NOMENCLATURE:

F	Face width (mm)
m	Normal module (mm)
N_g	Number of teeth in the gear
N_p	Number of teeth in the pinion
$[S/N]$	Signal to noise ratio
T	Torque
U_i	Utility function for the objective i ; where:
$i = 1$	-Balanced pitting and bending life
$i = 2$	-Balanced bending life pinion and gear
$i = 3$	-Minimum transmission error (noise)
$i = 4$	-Maximum efficiency
$i = 5$	-Minimum volume
I'_{σ}	Profile tolerance
I'_{ω}	Lead tolerance
X_g	Coefficient of tool shift in the gear
X_p	Coefficient of tool shift in the pinion

α_i	Weighting for the objective i
ϕ	Normal pressure angle
ψ	Helix angle
BENP, BENG	Ratio between actual and design bending stresses
CDR	Ratio operating vs theoretical center Distance
CRP	Profile contact ratio
CRF	Axial contact ratio
HERTZ	Ratio between actual and design Hertzian stress
LOSS	Factor of power loss
PPTE	Peak to peak transmission error (μm)
SI	Sensitivity index (Criterion for robust optimum)
VOL	Volume (lt)
PPTE*	Peak to peak transmission error without modification (μm)

PROBLEM OUTLINE:

At the beginning of this study, sponsors of Ohio State's Gear Dynamics and Gear Noise Research Laboratory [<http://gearlab.eng.ohio-state.edu/>] were polled regarding both geometric variable that they vary in gears design and the factors that they would like considered in comparing designs. Based on their responses, the following design variables and objective functions were included in the analysis.

$x(1)$	Pinion teeth number
$x(2)$	Pressure angle
$x(3)$	Helix angle
$x(4)$	Face width
$x(5)$	Gear teeth number
$x(6)$	Ratio between center distances
$x(7)$	Pinion tool shifting

x/8) Normal module

The range used for each variable is an input and tolerances are placed on such parameters as center distance so that that variable has a nominal value and a percentage range about that value. Gear tool shifting also occurs in a manner such that the working depth always has the same parameter. At this time only a cutting rack definition is applied, but in the future it is anticipated that rack addenda and dedenda could be changed. The analysis also allows the use of lead crowning and profile modifications in the gear tooth design.

Five objective functions were considered, namely:

- (F_1) Balanced pitting and bending life
- (F_2) Balanced bending life in pinion and gear
- (F_3) Minimize transmission error
- (F_4) Improve efficiency
- (F_5) Minimize volume

Each of the above variables is evaluated at the design load for the gears. However, when minimizing transmission error, the load for minimization is often considerably lower than the design load. Hence, the procedure allows the designer to select a load other than the design load for evaluation of transmission error.

For each of the objective functions, a Single Attribute Utility Function (U) was defined [10], and these utility functions were combined into a Multi-Attribute Utility function given as:

$$MAU = \sum_{i=1}^5 \alpha_i (U_i)^{\beta_i} \quad (1)$$

From a preliminary analysis [10], the following manufacturing errors and variations in operating conditions were found to be relevant in the performance of the gears:

- Profile error (Pressure angle error)
- Misalignment (Lead error)
- Torque variations

It was also observed that the sensitivity of the response to these errors may be non-linear. Therefore, during this investigation, three levels in the noise factors were used, allowing a nonlinear (second degree) approximation of the response. Under the requirement of minimization in the attributes, the sensitivity analysis was based on the

signal to noise ratio type 's' (Smaller is better) given

$$as[9]. \quad [S/N]_s = -10 \log \left(\frac{1}{n} \sum f_i^2 \right) \quad (2)$$

The design for robustness must include the optimization of the performance and the minimization of the sensitivity to parametric noise factors. Using the sensitivity index (SI) Only the argument in the logarithm of equation (2) proposed by Sundaresan [11], equations (1) and (2) give the objective function as the minimization of:

$$S = \gamma \cdot U + (1 - \gamma) \cdot \left(\frac{1}{N} \sum U_j^2 \right); \quad j = 1, 2, \dots, N \quad (3)$$

Where N is the number of replications at each candidate point for the robustness analysis.

The constraints are of two types, geometrical and mechanical as shown in Table 1. In order to guarantee the performance of the gears regardless of the existence of parametric noise factors, all of these constraints must be inactive or statistically active [11].

Geometric	Mechanical
Tip tooth thickness	Allowable bending stress pinion
Undercut	Allowable bending stress gear
Gear ratio*	Allowable pitting stress pinion
Root clearance*	Allowable pitting stress gear
Interference*	
Limits of full recess & full approach action*	

* Defined only in assembled gears

Table 1 Constraints involved in the optimization of gears.

The performance of the candidate points as well as the mechanical constraints are evaluated using LDP* instead of the traditional AGMA procedure [1]. In this way, the bending and pitting stresses are calculated using the actual load and point of application for all the points in the mesh cycle, and the stresses are far more realistic predictions that can take into account profile and lead modifications. However, in order to use LDP, the gear-set must meet all of the geometrical constraints, and there is the need of using these constraints as a filter for the candidate points.

* Load Distribution Program

SELECTION OF CANDIDATE POINTS AND REPLICATIONS:

The candidate points may be selected applying different optimization techniques using the objective function in equation (3). During this investigation, a grid search method using Orthogonal Arrays (OA) was used to select the candidate points. This approach was used because it was observed [10] that the valid range in the independent variables is different for each combination of the parameters. Besides, the evaluation of constraints like pointed teeth and interference requires an iterative analysis, making it difficult to implement a higher order method for the selection of the candidate points.

Orthogonal array for the design variables

The requirement for a second degree approximation is rotatability. Montgomery [8] recommends using full factorials or Central Composite Designs (CCD). A full factorial with eight variables and three levels implies performing $3^8 = 6561$ experiments per iteration. This number of experiments is prohibitive due to the computing time required to perform each experiment; therefore, a fractional factorial was used.

The analysis of the geometrically feasible region [10] showed that this region is not suitable for the application of OA's or CCD's. For instance, Fig. 1 shows a typical geometrically feasible region in the space of (N_p, N_g, CDR) .

In order to handle this odd-shaped design region, and taking into account the requirement of making the variance-covariance matrix $(X^T X)^{-1} \sigma^2$ as close to diagonal as possible, the following approach was used:

A - Using an orthogonal array, generate a geometrically feasible point in the space of the four independent variables (N_p, ϕ, ψ, F_w) . During the implementation, standard $L_9(3^4)$ and $L_{27}(3^{13})$ OA's were used allocating the variables in the way shown in Table 2.

With this designation of variables, the effects of ϕ and ψ are confounded with the interactions of N_p & F_w in the L_9 array; and the effect of ϕ is confounded with the interactions of N_p , F_w & ψ in the L_{27} array.

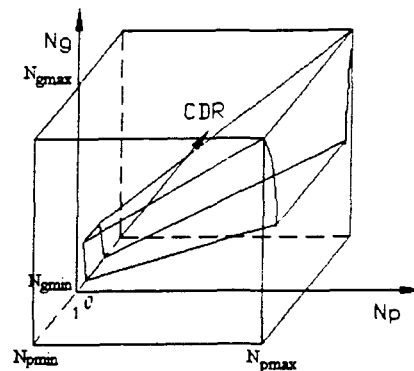


Fig. 1 Geometrically feasible region in the space (N_p, N_g, CDR) .

Variable	Column No. in L_9	Column No. in L_{27}
N_p	1	1
ϕ	3	13
ψ	4	2
F_w	2	5

Table 2 Allocation of variables in the Orthogonal Arrays.

B - At each of these points in the (N_p, ϕ, ψ, F_w) space, a geometrically feasible combination of the other independent variables (N_g, CDR, X_p, M_s) was selected instead of the one given at that location assuming an Orthogonal Array would be applicable (see Fig. 2). During this investigation, several points selected randomly in the (N_g, CDR, X_p, M_s) space were used in order to generate enough candidate points.

C - Each of the geometrically feasible candidate points was considered as a row in an inner-outer array configuration for the robustness analysis.

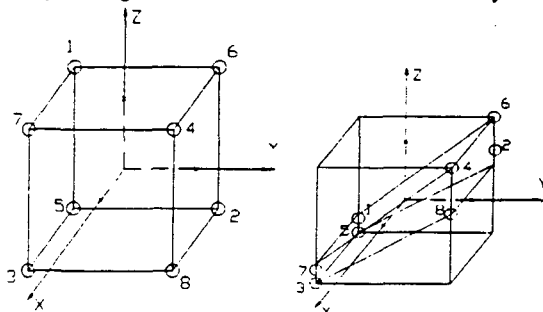


Fig. 2 Substitution of unfeasible points with feasible points in the OA.

Orthogonal array for the noise factors

In order to analyze robustness, each of the candidate points must be replicated by perturbing it with the parametric noise factors. In the gear problem, the manufacturing errors are limited by the AGMA quality specified by a tolerance with limits

$$-\frac{V_{\sigma T}}{2} \leq \text{Profile error} \leq \frac{V_{\sigma T}}{2} \quad (4)$$

$$-\frac{V_{\psi T}}{2} \leq \text{Lead error} \leq \frac{V_{\psi T}}{2} \quad (5)$$

And the variation in torque is specified by the limits:

$$T_{\min} \leq T \leq T_{\max} \quad (6)$$

It was assumed that the noise factors have a uniform distribution within these limits; therefore, an OA is applicable. The smaller OA for three noise factors at three levels is the $L_9(3^4)$, and the assignment of the columns was as follows:

Variable	Column No.
Profile error	1
Lead error	2
Torque variation	4

With this assignment, the effect of torque variation is confounded with the interaction of profile error and lead error.

Final procedure

With the approach used, each of the candidate points was geometrically feasible and should be analyzed for its level of performance and violation of the mechanical constraints. Fig. 3 shows the flow diagram of the outlined design process.

Two examples were performed giving a broad range in the independent variables, under the assumption that the designer has no previous experience about what the optimum for each particular objective will be. In this way, it is possible to see how the program locates the optimum in the design space depending on the aimed objective.

The convergence criterion used the total range in the independent variables in order to assure that all the design space is searched for candidate points. The range in the independent variables was reduced in each iteration using:

$(X_{i\max}^{j+1} - X_{i\min}^{j+1}) = 0.8(X_{i\max}^{j-1} - X_{i\min}^{j-1})$ until the total range of each variable were within the following limits:

$$N_{p\max} - N_{p\min} \leq 3$$

$$\phi_{\max} - \phi_{\min} \leq 0.2^\circ$$

$$\psi_{\max} - \psi_{\min} \leq 0.2^\circ$$

$$F_{w\max} - F_{w\min} \leq 2.5 \text{ mm} \quad (0.1 \text{ in})$$

The value of 0.8 may be increased in order to perform a more refined analysis in the design space. However, this implies an increase in the number of iterations and calculation time. For better results, it is recommended to run a first approximation with a big reduction of the range (0.6) to approximate the optimum, and then, use a smaller range of reduction (i.e. 0.9) to refine the solution starting with smaller ranges.

In order to simulate what a real manufacturing procedure would produce, the profile modifications were performed using parabolic approximations of the tip, root and lead modifications. These approximations were based on the maximum deflection during the meshing of the gears as is shown in Fig. 4, where the magnitude of profile modification at the tip of the tooth was given as:

$$T_{pm} = 1.25 \cdot TE_{\max} \quad \text{for high contact ratio gears (HCR)}$$

$$T_{pm} = 1.40 \cdot TE_{\max} \quad \text{for low contact ratio gears (LCR)}$$

and the start of profile modification (point A) was located at the highest point of single tooth contact for LCR gears and the middle point between the pitch point and the highest point of double tooth contact for HCR gears. The parabolic lead modification used was:

$$L_{pm} = 0.5 \cdot TE_{\max}$$