

Guide

Guide to Modeling Earth's Trapped Radiation Environment

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Guide

Guide To Modeling Earth's Trapped Radiation Environment

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Abstract

This Guide serves as both an introduction to the phenomena of radiation in the space environment and the product engineering issues facing spacecraft designers. Emphasis is on the trapped radiation environment of the Earth which is known as the Van Allen Belts. The leading empirical models are described and the problems in using them are identified. Current radiation modeling efforts are also discussed, along with shielding design and optimization. The Guide is intended for students, designers, mission planners, and others who need a ready understanding of this critical issue affecting spacecraft performance in Earth orbit.

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Foreword

With the increasing sophistication and high level of physical integration of electronics and electronic components, radiation effects have taken on a new significance in spacecraft design. The push toward "cheaper, better, faster" spacecraft has exacerbated the trend toward increasingly more radiation sensitive parts. Indeed, trapped radiation effects on microelectronics have been and are continuing concerns for all spacecraft designers, builders, and users. The availability of new radiation data coupled with long term changes in the Earth's magnetic field and trapped radiation fluxes have generated the requirement for newer, more comprehensive tools for modeling the Earth's trapped radiation environment and its effects on space systems. Realizing this need, the AIAA Atmospheric and Space Environments Committee on Standards in February of 1997 commissioned the preparation of a reference guide that would describe the current status of radiation belt studies and review methods for attacking the issues associated with modeling the trapped radiation environment in a systematic, practical fashion. The intent was to point the way to increasingly better methods of testing, designing, and flying reliable spacecraft systems in the Earth's radiation environment. The guide was to serve as a review of the key concepts associated with modeling the radiation environment and its effects. It would describe the principle models of the trapped radiation environment currently available and recommend a long range plan for enhancing the capabilities in this important environmental area. The document presented here represents the primary product of the Committee's commission. As such, it is hoped that it will be widely disseminated and discussed in the space community with the intent of encouraging further debate on this complex technical issue. Ultimately, the end result should be more robust, survivable spacecraft systems capable of addressing the stringent needs of the 21st Century!

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The guide was prepared by Dr. Henry B. Garrett of the Jet Propulsion Laboratory, the California Institute of Technology, Pasadena, CA for the AIAA Atmospheric and Space Environments Committee on Standards, Dr. Shu Lai, Chairman. This work was carried out at the Jet Propulsion Laboratory under a contract with NASA. Special thanks go to those who participated in the review of the document. In particular, Drs. Alfred Vampola and William Vaughan provided extremely useful and pertinent corrections.

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The Standards Executive Council accepted the Guide for publication in August 1999.

1. Introduction

Increasing concerns over trapped radiation effects on microelectronics, coupled with the availability of new data, long-term changes in the Earth's magnetic field, and variations in the trapped radiation fluxes, have generated the need for better, more comprehensive tools for modeling the Earth's trapped radiation environment and its effects on space systems. The objectives of this guide are to describe the current status of those efforts and to review methods for attacking the issues associated with modeling the trapped radiation environment in a systematic, practical fashion. The ultimate goal will be to point the way to increasingly better methods of testing, designing, and flying reliable spacecraft systems in the Earth's radiation environment.

To set the stage for these discussions, a review of the key concepts associated with modeling the radiation environment and its effects will be presented first. The review will include a description of the principal models of the trapped radiation environment currently available. Recent results from radiation experiments on spacecraft such as CRRES, SAMPEX, and Clementine will then be described. The report will close with a detailed discussion of the current status of the modeling of the radiation environment and recommend a long-range plan for enhancing capabilities in this important environmental area.

Because of the increasing sophistication and high level of physical integration of electronics and electronic components, radiation effects have taken on a new significance in spacecraft design. For example, the rapid drop in power and voltage levels and the associated drop in feature size for integrated components (ICs) have greatly enhanced the ICs' sensitivities to single event effects (SEEs). The push toward commercial off-the-shelf parts has often led to parts that are much less radiation tolerant although this is not always true. Overall, the requirements for "cheaper, better, faster" spacecraft have exacerbated this trend toward parts that are increasingly more radiation sensitive. The result is that, far from going away with time, radiation effects—both total ionizing dose and single event effects—are increasingly coming to dominate the design concerns for satellite manufacturers across the board. Add to this the desire of many new multisatellite communications providers to place their constellations in the middle of the harshest part of the radiation belts, and accurate modeling of the trapped radiation environment and its effects becomes a very real, long-term problem for the spacecraft community in general.

Solving the problems of trapped radiation effects on spacecraft is not as simple as just developing better models or more shielding. Although these are solutions in many situations, in general, most commercial spacecraft designers cannot afford either the large uncertainties in the current models or the extra mass necessary to cover the required design margins. Rather, proper design of radiation resistant systems requires complex trade-offs among parts, shielding, software, operations, redundancy, and orbit configuration. Each of these "solutions" is subject to uncertainty and has a cost impact on the final design. Unfortunately, the key component, the trapped radiation environment itself, is not well defined (observations and predictions that vary by factors of two for 5–11 year missions in Earth orbit are considered to be in excellent agreement; for shorter missions, factors approaching 10–100 are easily possible). Even given an accurate "average" description of the environment, short-term variations of several orders of magnitude in dosage and single event upset (SEU) rates have been seen in the span of hours (e.g., the 1989 solar proton events). Complicating the practical application of the radiation environment to spacecraft design, radiation transport codes and estimates of the effects of radiation damage are often inaccurate. Comparisons between ground tests and in situ measurements have shown significant disagreement. Furthermore, the parts used on the spacecraft can show variations in sensitivity of factors of 2–10, even within the same parts lot. Often, how a system is actually used can mask, or hopefully limit, the effects of radiation damage.

Thus, to a degree, mitigating radiation effects is a black art and, increasingly, a very expensive art for which any imprecision in the knowledge of the trapped radiation environment becomes a critical component. However, the ultimate solution is a comprehensive process that treats all uncertainties.

2. The Space Radiation Environment: Basic Concepts

This section provides an overview of the basic physical concepts and definitions that will be used throughout the guide. In particular, the concepts of energy, flux, fluence, and dosage will be briefly described. The reader is referred to the many excellent texts on space physics or astronomy for more detailed explanations.^{1,2}

First consider the concept of energy. In the case of particles that have a rest mass, the fundamental equation relating particle mass and velocity to kinetic energy is:

$$E = (\gamma - 1)m_0c^2 \quad \text{Relativistically} \quad (1)$$

$$= \frac{1}{2}m_0V^2 \quad \text{Non-Relativistically}$$

where

$$\begin{aligned} m_0 &= \text{particle rest mass} \\ V &= \text{particle velocity} \\ c &= \text{speed of light} \\ E &= \text{particle kinetic energy} \\ \gamma &= \left(1 - \frac{V^2}{c^2}\right)^{-1/2} \end{aligned}$$

For photons (which have no rest mass), the equivalent equation is:

$$E = h\nu \quad (2)$$

where

$$\begin{aligned} h &= \text{Planck's constant} \\ \nu &= \text{frequency of the light} \end{aligned}$$

Closely coupled to the concept of energy is that of dose. Simply put, dose is the total energy accumulated in a given volume element of a specific material due to incident radiation. It is typically given in units of rads or "radiation absorbed dose" for a particular material (the material *must* be specified because energy absorption is dependent on the material). As an example, for silicon, 1 rad (Si) = 10^{-2} J/kg (Si). It must be emphasized that, for the same incident flux, different materials will be affected differently depending on the composition of the radiation and the composition of the absorbing material.

In addition to the energy and composition of a particle or photon, it is also necessary to describe how many of them there are. This is usually done in terms of intensity or flux and, when speaking in terms of a time interval, fluence. Confusion arises over the concepts of intensity/flux and fluence because there are many different ways to define these quantities. Here, we will define the quantity "unidirectional differential intensity" $j(E, \theta, \phi, t)$ as:

The flux (number of particles or photons per unit time) of a given energy E per unit energy interval dE in a unit solid angle ($d\Omega = 2\pi \cos \theta d\theta d\phi$) about the direction of observation (in the θ, ϕ direction), incident on unit of surface area (dA) perpendicular to the direction of observation.

This is illustrated in Fig. 1.² Typical units are $\text{particles} \cdot \text{cm}^{-2} \cdot \text{s}^{-1} \cdot \text{sr}^{-1} \cdot \text{keV}^{-1}$ for protons or electrons and $\text{particles} \cdot \text{m}^{-2} \cdot \text{s}^{-1} \cdot \text{sr}^{-1} \cdot (\text{MeV} \cdot \mu^{-1})^{-1}$ for heavy ions (where μ is nucleon). A typical spectrum for iron cosmic rays is presented in Fig. 2.³ In the figure, the solid curves are for solar maximum (lower) and solar minimum (upper). The dashed curve is the 90% worst case iron spectrum, which is implied by comparison with the cosmic ray helium spectrum. The “unidirectional integral intensity” (or flux) is defined as the intensity of all particles with energy greater than or equal to a threshold energy E :

$$j_{\geq E} = \int_E^{\infty} j dE \quad (3)$$

with units of $\text{particles cm}^{-2} \text{s}^{-1} \text{sr}^{-1}$.

We define the “omnidirectional flux” J as:

$$J = \int_{4\pi} j d\Omega \quad (4)$$

Fluence I is the integral of the flux over a given time interval (e.g., one hour, one year):

$$I = \int_{\delta t} j dt \quad (5)$$

Here, when we refer to omnidirectional fluence $I(\geq E)$, we will normally mean the “omnidirectional integral (in energy) fluence” such that:

$$I_{\geq E} = \int_E^{\infty} dE \int_{4\pi} d\Omega \int_{\delta t} j dt \quad (6)$$

The units of this quantity are $\text{particles} \cdot \text{cm}^{-2}$ for some specified threshold energy E (typically 1 MeV or higher for radiation effects) and for a specified time interval (often one year).

FLUX

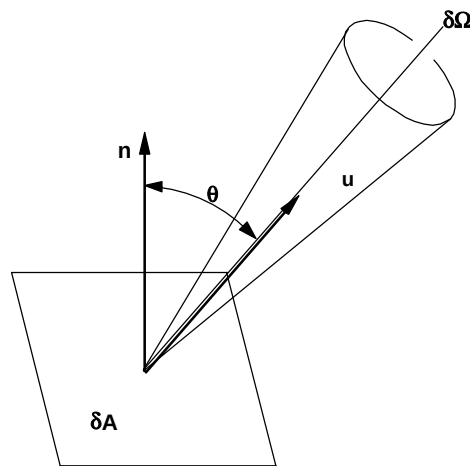


Figure 1 — The flux of a given energy E per unit energy interval dE in a unit solid angle about the direction of observation (Copyright by and used by permission of Springer-Verlag, New York)

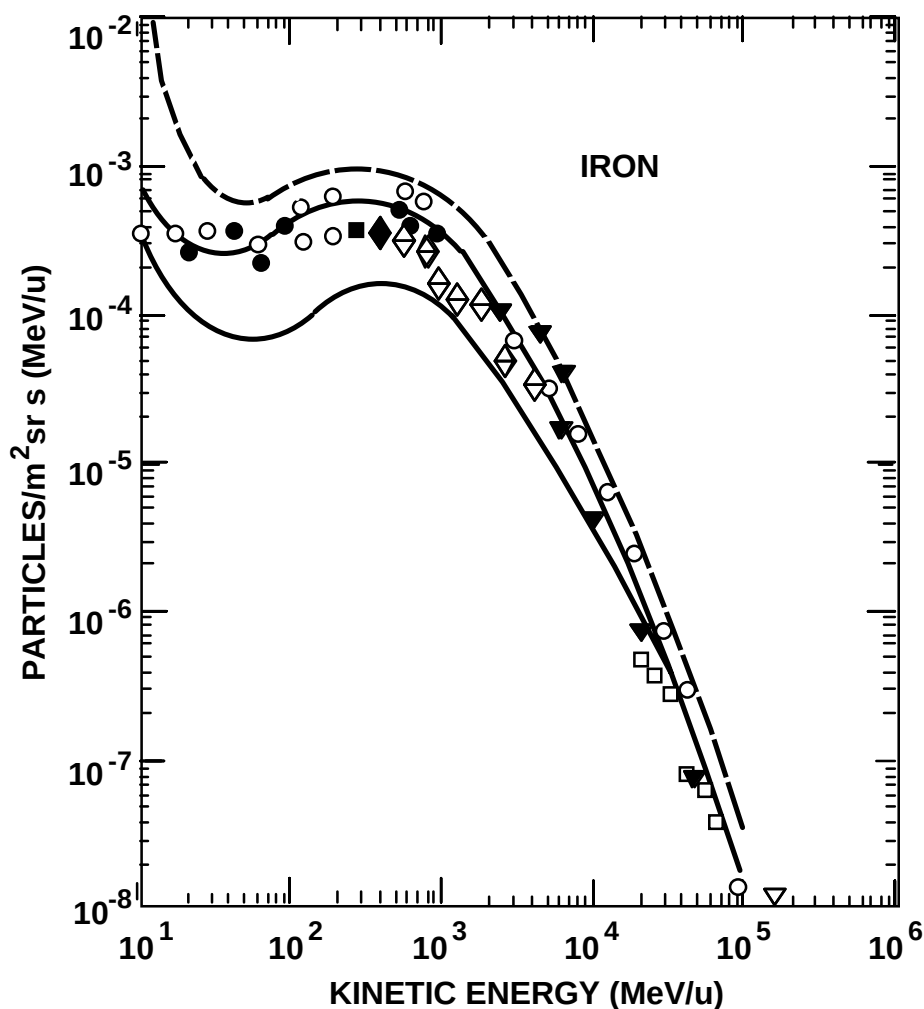


Figure 2 — The iron cosmic ray spectrum

To allow comparisons among different energies, particle types, and dosages, it is common practice to talk in terms of "1-MeV equivalent" (typically 1-MeV electrons or 1-MeV neutrons in silicon). First, the energy dependence of the damage and energy content of the spectra for the environment to be considered are used to determine what fluence of 1-MeV particles (electrons or neutrons) would produce the same amount of damage or dose in the material (typically silicon or aluminum). A curve for neutrons, in units of MeV-mb (where b is a barn or 10⁻²⁴ cm² and the relative displacement damage is defined in terms of the cross section times the energy of the incident particle), is given in Fig. 3.⁴ As an illustration, for 14 MeV neutrons, the 1-MeV neutron dose damage equivalent is given by multiplying the 14 MeV dose by 2.5 (obtained from Ref. 4). (Note: because of variations in the damage parameter with material and property, it should always be kept in mind that the use of a damage equivalent is not exact but an approximation for comparison purposes.)

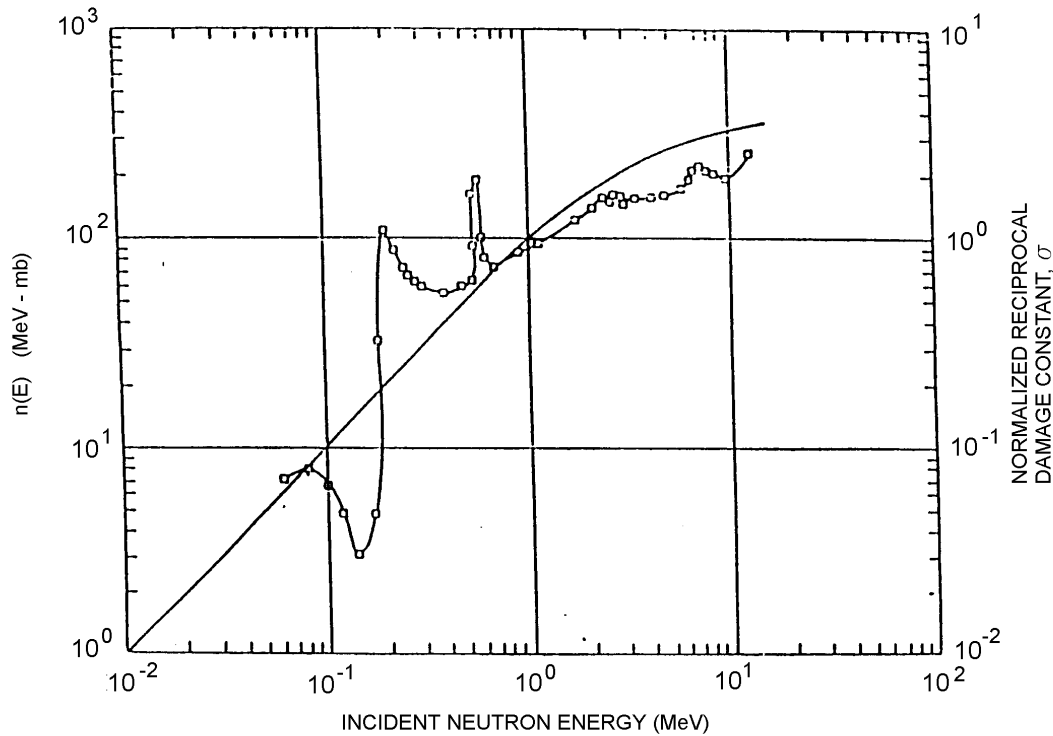


Figure 3 — Neutron displacement damage equivalence curve (Copyright by and used by permission of Institute of Electrical and Electronics Engineers)

A final quantity related to energy absorption and flux is the Linear Energy Transfer (LET). The LET is the energy transferred by radiation per unit length of absorbing material. That is, LET is proportional to dE/dx (note: there is in fact a slight difference between “energy transferred” and “energy lost per unit length” but that will be ignored here). For ionization and excitation effects, LET is often expressed in $\text{MeV}/\mu\text{m}$ of the primary particle track length or, if the density of the material is known, $\text{MeV}\cdot\text{cm}^2\cdot\text{mg}^{-1}$ (this is typically the unit when the reference is to an LET between 1 and 30 and is given by: $\frac{1}{\rho} \frac{dE}{dx}$).

The concept of LET is particularly important when discussing single event upsets (SEUs) or “soft errors.” These occur when a particle, typically an ionized, high energy atomic nucleus, deposits enough energy in the sensitive region of an electronic device to cause a change in the logic state of the device. Upsets occur only when the energy deposited exceeds a critical level in the sensitive region of the device. This is often computed in terms of LET. When viewed as a function of LET, the probability of upset is, in its simplest form, a threshold phenomenon: any particle with a minimum LET of L_0 or greater will cause an upset. This behavior is illustrated in Fig. 4 where the energy deposited per unit length (LET) is plotted vs incident particle energy—note how the curve has a peak rate. L_0 corresponds to a constant value of LET. As illustrated, there can be multiple values of energy (E_1 and E_2 here) that correspond to the same value of LET. A useful way of presenting the environment in terms of LET is the Heinrich curve. The Heinrich curve gives the integral flux as a function of LET rather than particle energy. The Heinrich flux F_H is the flux of particles for a single species with a (threshold) LET of LET_0 or greater:

$$F_H(LET_0) = \int_{E_1}^{E_2} f_i(E) dE \quad (7)$$

where f_i is the particle flux for the species i as a function of energy and E_1 and E_2 are the energies between which the LET is greater than or equal to the threshold LET_0 (a representative integral Heinrich curve for iron is plotted in Fig. 5⁵). The LET depends not only on particle energy, but on the target material as well because the LET vs energy curve will be different for all particle species. Experiments have shown, however, that to the first order it is the LET that is important for determining upsets and not the particle energy or its species. The Heinrich flux vs LET plot is the principal means of presenting radiation data for use in SEU calculations just as the particle flux vs energy is the main means of presenting radiation data for use in dosage calculations.

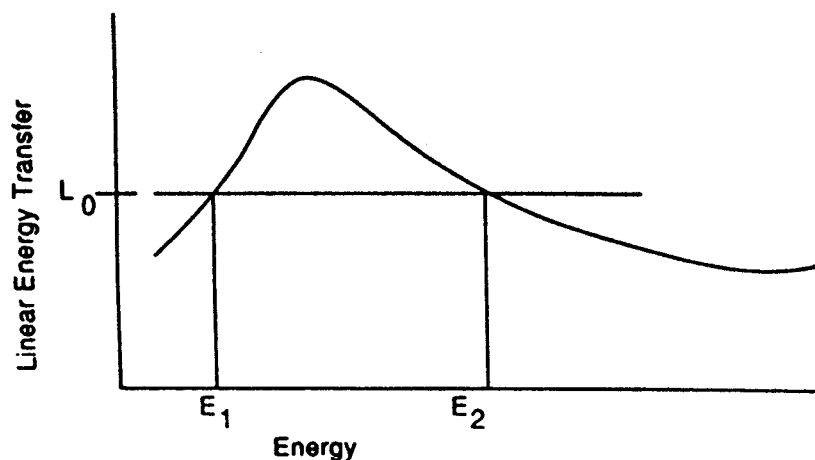


Figure 4 — Linear Energy Transfer Function (LET) vs Energy

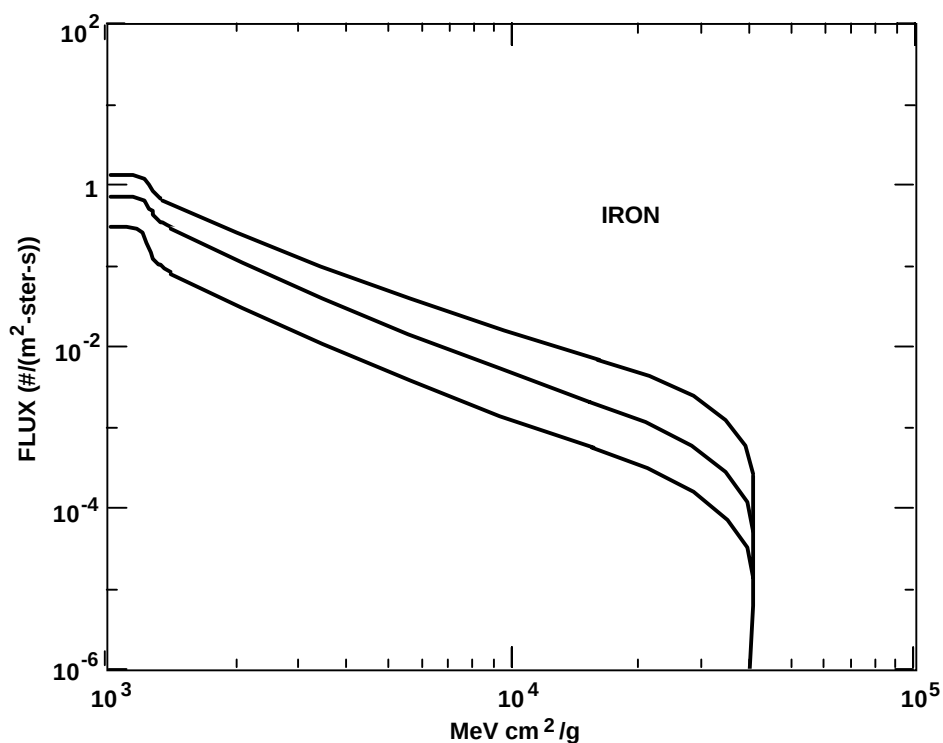


Figure 5 — Heinrich curves for iron cosmic rays

To summarize, this section has defined the basic terminology used to describe the radiation environment—dose, flux/intensity, fluence, LET, and 1-MeV equivalent. The reader is referred to books and articles by Roederer² and others for more complete descriptions of these concepts.

3. The Trapped Radiation Environment

3.1 Overview

By definition, the high energy particle radiation environment in space consists of electrons with energies greater than 40 KeV, protons or neutrons with energies greater than 1 MeV, and heavy ions with energies above 1 MeV/nucleon. Lower energy electrons, protons, and ions are ubiquitous but are considered as plasma. The populations are characterized in terms of their kinetic energy, charge state (or lack thereof), and composition. Unlike photons, which travel uniformly at the speed of light, particles can vary in velocity from a few m/s to a sizable fraction of the speed of light as in the case of cosmic rays. The high energy radiation population can be roughly divided into four families based on these characteristics:

- 1) Galactic cosmic rays, which consist of interplanetary protons, electrons, and ionized heavy nuclei.
- 2) Trapped radiation (for the Earth, the Van Allen belts).
- 3) Protons and heavy nuclei associated with solar proton events.
- 4) Neutrons (primarily cosmic ray albedo neutrons [CRAN] particles).

The first population changes relatively slowly (cyclically with the solar cycle). It is included here because it is believed to contribute to the CRAN and the trapped heavy ion population (see Section 6.2). The second population, the Earth's trapped radiation environment, can be divided into zones. Typically, the inner zone is populated with very high energy protons produced by CRAN decay and lower energy electrons, which also vary on a solar cycle timescale. The outer zone consists primarily of more energetic electrons and lower energy protons, which vary rapidly on a timescale that can be as short as one day, or less, in response to magnetic storms. The third population is highly time dependent, being associated with infrequent coronal mass ejections (CMEs). The fourth is a secondary population because the relatively short lifetime of neutrons severely limits any solar-produced fluxes at 1 AU or beyond. Each type of radiation has a characteristic spectrum and preferred interaction mode with matter that supports this simple division. Here, the discussion will focus primarily on the trapped radiation environment, the Van Allen belts.

First discovered by James Van Allen and his collaborators on Explorer I, trapped radiation at the Earth consists principally of energetic protons and electrons, with lesser percentages of heavy ions such as O^+ , contained by the Earth's magnetic field in toroidal belts. Commonly known as the "Van Allen belts,"⁶ these toroidal regions consist of (at least) two zones or belts. The inner belt extends from approximately hundreds of kilometers to ~6000 km in altitude and is populated by high energy (approximately tens of mega electronvolts) protons and medium energy (50–1000 keV) electrons, while the outer belt, up to 60,000 km in altitude, is predominately populated by high energy electrons. Schematics of the radiation flux contours for the Van Allen belts are illustrated in Fig. 6 where the average omnidirectional integral fluxes above energy thresholds are shown for 1-MeV electrons and 10-MeV protons.⁷ The detailed mechanism by which particles are entrapped in the belt regions is not well understood nor is the primary source clearly identified (albedo neutrons are considered an important source of the intense proton and electron fluxes in the inner belt, while the outer belt may be primarily due to the geomagnetic field entrapping low energy solar wind plasma followed by local acceleration) although observations of abundance ratios imply both terrestrial and interplanetary sources.

PROTON ($E > 10$ MeV) AND ELECTRON ($E > 1$ MeV) INTEGRAL FLUXES

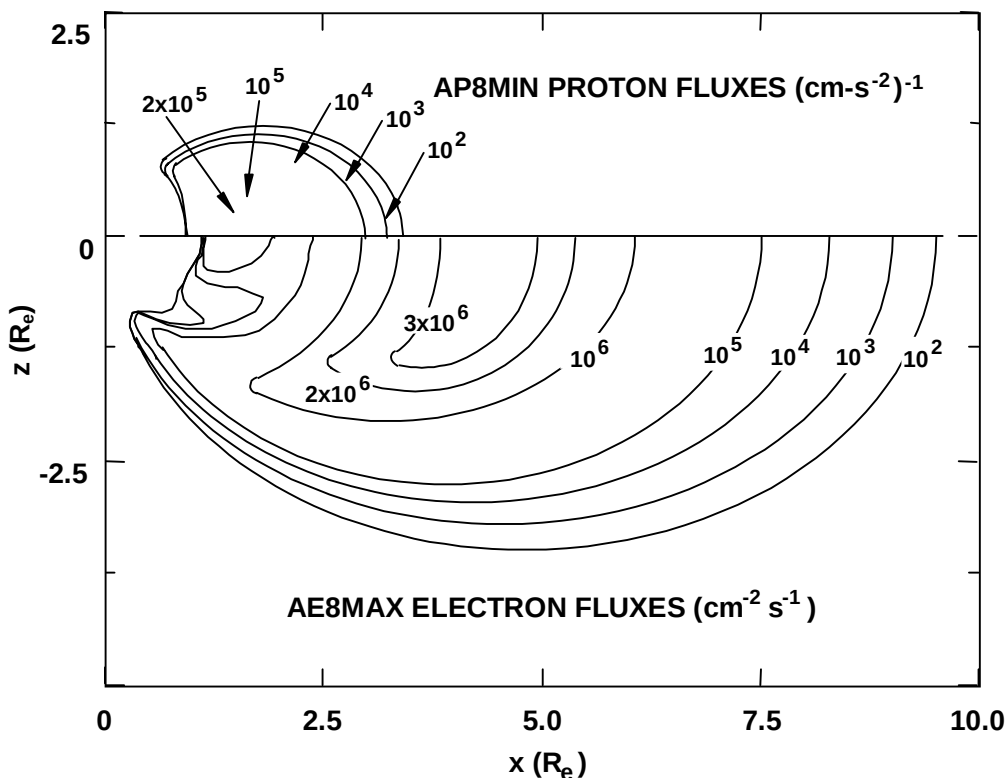


Figure 6 — The Earth's radiation belts in idealized space, according to the AP8 and AE8 models

Once captured, the motion of charged particles in the Earth's magnetic field is governed by the Lorentz force. The trapped radiation environment also exhibits large temporal variations. The inner belt zone, because of the dominance of the Earth's main field, is relatively stable. Most temporal variations in this population occur as the solar cycle proceeds and the Earth's neutral atmospheric density at a given altitude changes, causing variations in the altitude at which radiation particles can mirror without being scattered. Variations of an order of magnitude over a period of months have been observed in the electron flux intensities⁸ as a result of extended elevated geomagnetic activity. In contrast, the outer belt, which is more influenced by the Earth's highly variable geomagnetic tail, experiences greater and more rapid (less than one day) temporal fluctuations. The electron concentration in the outer zone may experience temporal fluctuations as large as a factor of 100,000.

To better understand the preceding description, we first discuss the primary characteristics of the Earth's magnetic field because it is the dominate force controlling the formation of and changes associated with the trapped radiation belts. Following this discussion, a brief review of the basic concepts of particle entrapment such as gyro radius, mirror point, and pitch angle will be presented. Although the details of these concepts are not critical to understanding the basic effects of radiation on spacecraft, they are critical to understanding how the radiation belts are modeled and how the external dose on a particular vehicle is determined. In particular, the concept of adiabatic invariants will be summarized; this concept is critical to understanding why we use B - L coordinates, the basis of almost all modern radiation models, in describing the radiation belts. Following these descriptions, several examples of the trapped radiation environment will be presented in terms of the AE8/AP8 radiation models and the predictions compared with actual data. As will become evident, given the rapid temporal variations observed in the space environment, models are only reliable in a long-term predictive sense—which is a crucial issue in establishing design margins for spacecraft.

3.2 Geomagnetic Field

Above 1000 km, the dominant geophysical environment of interest is the magnetic field of the Earth—the source of the Earth's magnetosphere. Below 1000 km, the Earth's magnetic field, primarily through the control of the ionospheric plasma, plays an important though reduced role in the dynamics of the natural environment. The Earth's geomagnetic field B is composed of three distinct components (or current systems): the core field, crustal field, and external field.⁹ The core field is a steady field due primarily to the convective motion of the conducting fluid in the Earth's internal core. The distribution of the crustal field, which is the remnant or induced magnetization of ferromagnetic materials near the Earth's surface, results in surface magnetic anomalies associated with the geologic and tectonic features of the crust. The core and crustal fields are usually combined and referred to as the internal or main field B_i . B_i varies slowly on the order of 100 years—currently $\sim 0.05\%$ per year—as illustrated in Fig. 7⁹ (be aware that although it is critical to use a magnetic field model appropriate for the year in which the magnetic field estimates are required, it is typical in radiation modeling to use the magnetic field model for the year in which the radiation belt model was derived; this has caused serious problems in the past). B_i dominates at low Earth orbit (LEO) and accounts for more than $\sim 99\%$ of B , even during extremely large geomagnetic storms (massive variations in the Earth's magnetosphere brought about by changes in the solar environment that encompasses the Earth's magnetic field). In contrast, the external field B_e , which makes up $\sim 1\%$ of the field at LEO, is due primarily to extraterrestrial sources—the Earth's ring current and the solar wind. It varies rapidly in time (from on the order of milliseconds to periods as long as the 11-year solar cycle) and is closely correlated with geomagnetic activity and solar interactions. Models of the external component of the geomagnetic field are available but are of limited importance to the trapped radiation belts.^{10–12}

Seven basic quantities, called magnetic elements, are used to specify the geomagnetic field.⁹ Their definitions and relations are given in Fig. 8. Three independent quantities [e.g., (H, D, Z) or (X, Y, Z)] are required to uniquely define B . For spacecraft operating in the radiation belts, the most convenient system to use is either the geographic (also known as geocentric) or the geomagnetic coordinate system. These systems, based on spherical coordinates, are schematically defined in Fig. 9. Geographic coordinates correspond to a geocentric longitude/latitude system based on the Greenwich Prime Meridian. Geomagnetic coordinates are similarly Earth centered but have the north pole of the system passing near the geomagnetic pole, which is offset from the geographic pole, at $\sim 78.5^\circ$ latitude and $\sim 291.1^\circ$ E. Geomagnetic longitude is measured from the great circle passing through the geographic and geomagnetic poles. Details about other coordinate systems can be found in Knecht and Shuman.⁹

Aside from the gravitational field of the Earth, the magnetic field due to the internal geomagnetic field is the most accurately known of the natural environments. It can be crudely modeled in terms of a tilted (-11° from geographic north) magnetic dipole of magnitude $M = 8 \times 10^{25} \text{ G}\cdot\text{cm}^3$ (G is the magnetic unit Gauss). Momentarily ignoring the tilt in the geomagnetic coordinate system, the magnetic field intensity induced by M at the point (r, θ, ϕ) is given by the expression:

$$B_i = -\left(\frac{M}{r^3}\right)(3\cos\theta + 1)^{1/2} \quad (8)$$

In the Gaussian unit system, r is in cm and B_i is in G. Given the above value for M , B_i is then found to have a maximum value of ~ 0.6 G near the polar cap and a minimum value of ~ 0.3 G near the equator at the Earth's surface.

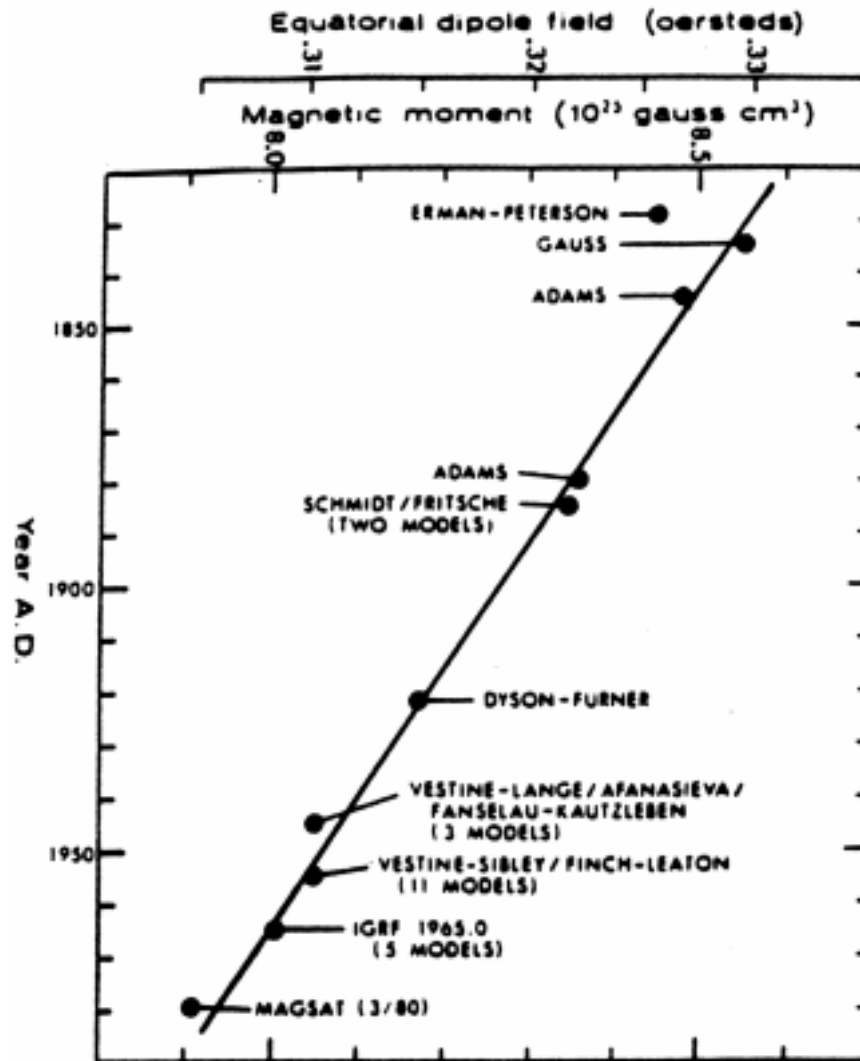


Figure 7 — Decay of the equatorial dipole field strength with time

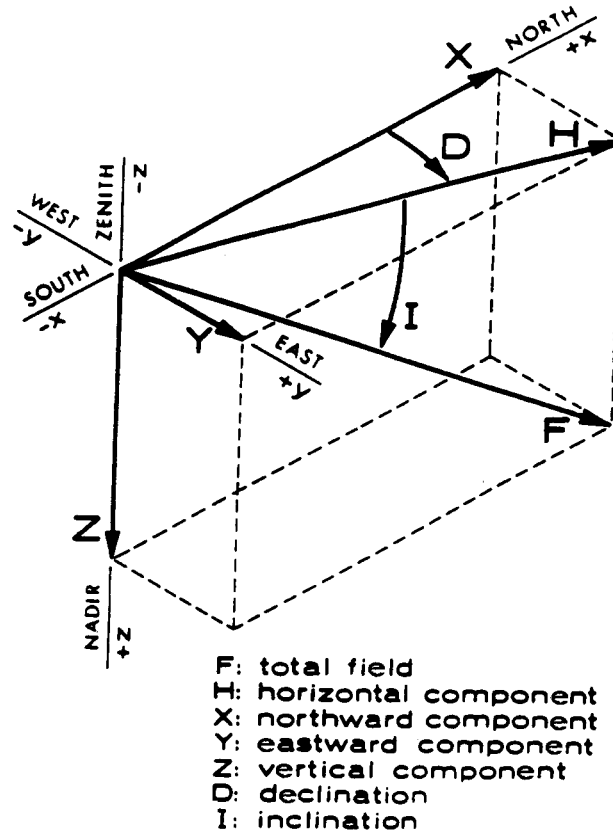


Figure 8 — Geomagnetic field magnetic elements

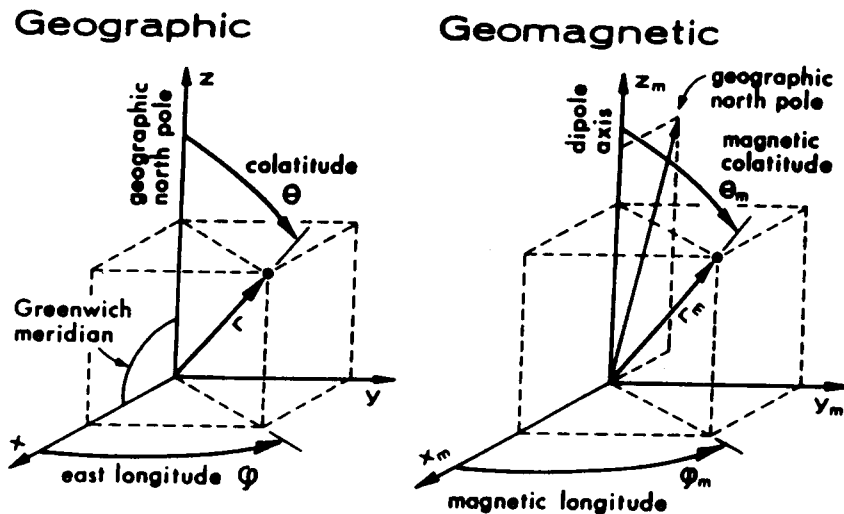


Figure 9 — Spherical coordinate system for geographic and geomagnetic coordinates

Equation (8) is valid only for an idealized configuration of a centered dipole. In reality, large scale discrepancies (as high as $\pm 25\%$) exist between the measured data and the ideal, dipole expression. Modifying the configuration from a centered dipole to an eccentric dipole reduces the discrepancies to the $\sim 10\%$ level, but this is still unacceptably large. For most purposes, the International Geomagnetic Reference Field (IGRF) series of models is the official standard. The latest version, IGRF-95,¹³ is a computer model based on the numerical fitting of measured data with a magnetic scalar potential

expanded in terms of spherical harmonics.^{13,14} The model calculates the seven magnetic elements of B_i for any given geographical location. Specifically, a scalar potential is found such that:

$$V(r, \theta, \phi) = a \sum_{n=1}^N \sum_{m=0}^n \{ [g_{n,m} \cos m\phi + h_{n,m} \sin m\phi] \left(\frac{a}{r}\right)^{n+1} + [A_{n,m} \cos m\phi + B_{n,m} \sin m\phi] \left(\frac{r}{a}\right)^n \} P_{n,m}(\cos \theta) \quad (9)$$

where

a	=	radius of the Earth
r	=	radial distance in units of a
N	=	order of expansion
θ	=	colatitude
ϕ	=	east longitude
$g_{n,m}, h_{n,m}$	=	constants for internal terms
$A_{n,m}, B_{n,m}$	=	constants for external terms

The magnetic field components are then given by:

$$\begin{aligned} B_r &= -\frac{\partial V}{\partial r} \\ B_\theta &= -\frac{1}{r} \frac{\partial V}{\partial \theta} \\ B_\phi &= -\frac{1}{r \sin \theta} \frac{\partial V}{\partial \phi} \end{aligned} \quad (10)$$

Values of g and h are presented in Table 1 for the IGRF internal model. As discussed earlier, the epoch of the magnetic model is important and should correspond either to the same date as the date of the radiation model used or to a model epoch as close to the present as possible if only the current magnetic field value is desired.