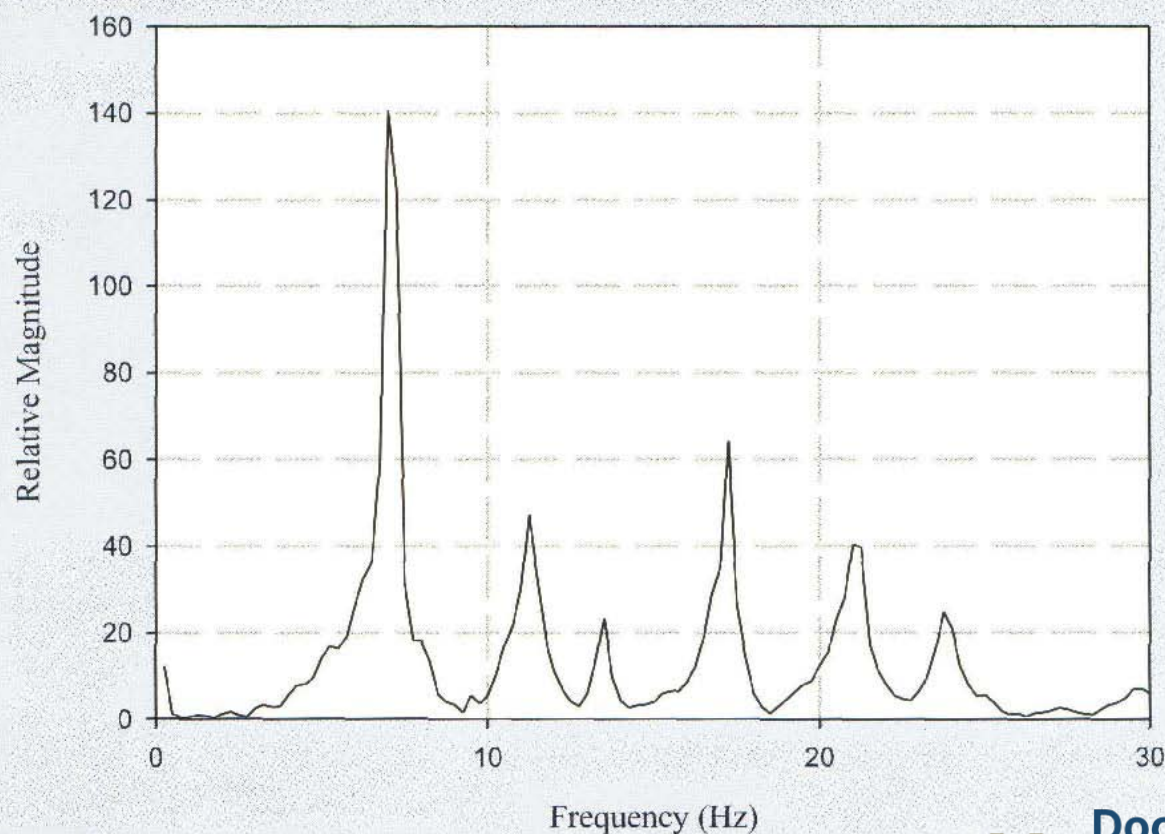




Steel Design Guide Series

Floor Vibrations Due to Human Activity



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Floor Vibrations Due to Human Activity

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Chapter 1

INTRODUCTION

1.1 Objectives of the Design Guide

The primary objective of this Design Guide is to provide basic principles and simple analytical tools to evaluate steel framed floor systems and footbridges for vibration serviceability due to human activities. Both human comfort and the need to control movement for sensitive equipment are considered. The secondary objective is to provide guidance on developing remedial measures for problem floors.

1.2 Road Map

This Design Guide is organized for the reader to move from basic principles of floor vibration and the associated terminology in Chapter 1, to serviceability criteria for evaluation and design in Chapter 2, to estimation of natural floor frequency (the most important floor vibration property) in Chapter 3, to applications of the criteria in Chapters 4, 5 and 6, and finally to possible remedial measures in Chapter 7. Chapter 4 covers walking-induced vibration, a topic of widespread importance in structural design practice. Chapter 5 concerns vibrations due to rhythmic activities such as aerobics and Chapter 6 provides guidance on the design of floor systems which support sensitive equipment, topics requiring increased specialization. Because many floor vibrations problems occur in practice, Chapter 7 provides guidance on their evaluation and the choice of remedial measures. The Appendix contains a short historical development of the various floor vibration criteria used in North America.

1.3 Background

For floor serviceability, stiffness and resonance are dominant considerations in the design of steel floor structures and footbridges. The first known stiffness criterion appeared nearly 170 years ago. Tredgold (1828) wrote that girders over long spans should be "made deep to avoid the inconvenience of not being able to move on the floor without shaking everything in the room". Traditionally, soldiers "break step" when marching across bridges to avoid large, potentially dangerous, resonant vibration.

A traditional stiffness criterion for steel floors limits the live load deflection of beams or girders supporting "plastered ceilings" to span/360. This limitation, along with restricting member span-to-depth ratios to 24 or less, have been widely applied to steel framed floor systems in an attempt to control vibrations, but with limited success.

Resonance has been ignored in the design of floors and footbridges until recently. Approximately 30 years ago, prob-

lems arose with vibrations induced by walking on steel-joint supported floors that satisfied traditional stiffness criteria. Since that time much has been learned about the loading function due to walking and the potential for resonance.

More recently, rhythmic activities, such as aerobics and high-impact dancing, have caused serious floor vibration problems due to resonance.

A number of analytical procedures have been developed which allow a structural designer to assess the floor structure for occupant comfort for a specific activity and for suitability for sensitive equipment. Generally, these analytical tools require the calculation of the first natural frequency of the floor system and the maximum amplitude of acceleration, velocity or displacement for a reference excitation. An estimate of damping in the floor is also required in some instances. A human comfort scale or sensitive equipment criterion is then used to determine whether the floor system meets serviceability requirements. Some of the analytical tools incorporate limits on acceleration into a single design formula whose parameters are estimated by the designer.

1.4 Basic Vibration Terminology

The purpose of this section is to introduce the reader to terminology and basic concepts used in this Design Guide.

Dynamic Loadings. *Dynamic loadings* can be classified as harmonic, periodic, transient, and impulsive as shown in Figure 1.1. *Harmonic or sinusoidal loads* are usually associated with rotating machinery. *Periodic loads* are caused by rhythmic human activities such as dancing and aerobics and by impactive machinery. *Transient loads* occur from the movement of people and include walking and running. Single jumps and heel-drop impacts are examples of *impulsive loads*.

Period and Frequency. *Period* is the time, usually in seconds, between successive peak excursions in repeating events. *Period* is associated with harmonic (or sinusoidal) and repetitive time functions as shown in Figure 1.1. *Frequency* is the reciprocal of period and is usually expressed in Hertz (cycles per second, Hz).

Steady State and Transient Motion. If a structural system is subjected to a continuous harmonic driving force (see Figure 1.1a), the resulting motion will have a constant frequency and constant maximum amplitude and is referred to as *steady state motion*. If a real structural system is subjected to a single impulse, damping in the system will cause the

motion to subside, as illustrated in Figure 1.2. This is one type of *transient motion*.

Natural Frequency and Free Vibration. *Natural frequency* is the frequency at which a body or structure will vibrate when displaced and then quickly released. This state of vibration is referred to as *free vibration*. All structures have a large number of natural frequencies; the lowest or "fundamental" natural frequency is of most concern.

Damping and Critical Damping. Damping refers to the loss of mechanical energy in a vibrating system. Damping is usually expressed as the percent of *critical damping* or as the ratio of actual damping (assumed to be viscous) to critical damping. Critical damping is the smallest amount of viscous damping for which a free vibrating system that is displaced from equilibrium and released comes to rest without oscillation. "Viscous" damping is associated with a retarding force that is proportional to velocity. For damping that is smaller than critical, the system oscillates freely as shown in Figure 1.2.

Until recently, damping in floor systems was generally determined from the decay of vibration following an impact (usually a heel-drop), using vibration signals from which vibration beyond 1.5 to 2 times the fundamental frequency has been removed by filtering. This technique resulted in damping ratios of 4 to 12 percent for typical office buildings. It has been found that this measurement overestimates the damping because it measures not only energy dissipation (the true damping) but also the transmission of vibrational energy to other structural components (usually referred to as geometric dispersion). To determine modal damping all modes of

vibration except one must be filtered from the record of vibration decay. Alternatively, the modal damping ratio can be determined from the Fourier spectrum of the response to impact. These techniques result in damping ratios of 3 to 5 percent for typical office buildings.

Resonance. If a frequency component of an exciting force is equal to a natural frequency of the structure, *resonance* will occur. At resonance, the amplitude of the motion tends to become large to very large, as shown in Figure 1.3.

Step Frequency. *Step frequency* is the frequency of application of a foot or feet to the floor, e.g. in walking, dancing or aerobics.

Harmonic. A *harmonic multiple* is an integer multiple of frequency of application of a repetitive force, e.g. multiple of step frequency for human activities, or multiple of rotational frequency of reciprocating machinery. (Note: Harmonics can also refer to natural frequencies, e.g. of strings or pipes.)

Mode Shape. When a floor structure vibrates freely in a particular mode, it moves up and down with a certain configuration or *mode shape*. Each natural frequency has a mode shape associated with it. Figure 1.4 shows typical mode shapes for a simple beam and for a slab/beam/girder floor system.

Modal Analysis. *Modal analysis* refers to a computational, analytical or experimental method for determining the natural frequencies and mode shapes of a structure, as well as the responses of individual modes to a given excitation. (The responses of the modes can then be superimposed to obtain a total system response.)

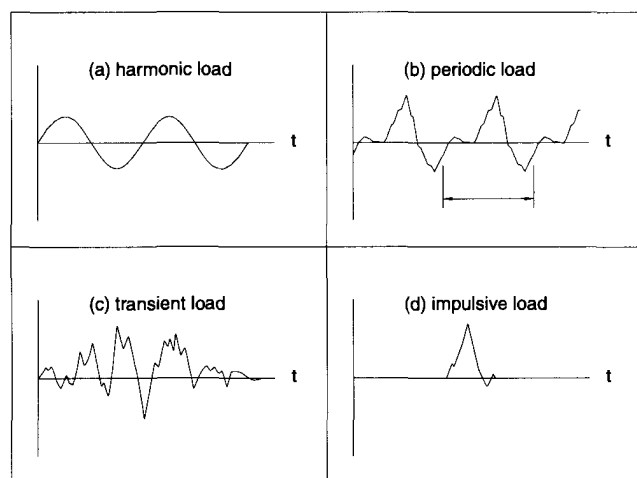


Fig. 1.1 Types of dynamic loading.

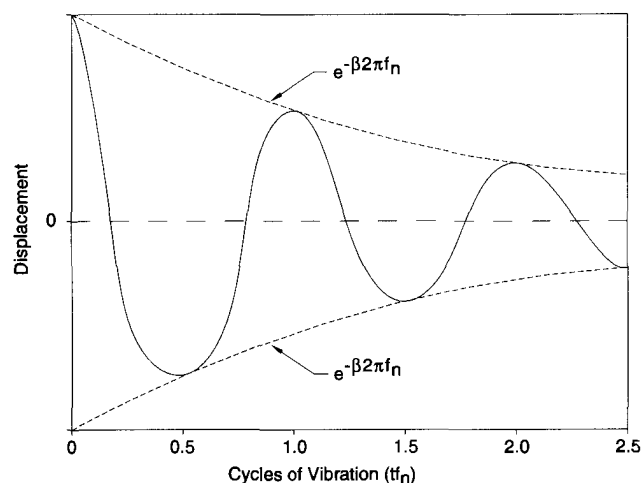


Fig. 1.2 Decaying vibration with viscous damping.

Spectrum. A *spectrum* shows the variation of relative amplitude with frequency of the vibration components that contribute to the load or motion. Figure 1.5 is an example of a frequency spectrum.

Fourier Transformation. The mathematical procedure to transform a time record into a complex frequency spectrum (Fourier spectrum) without loss of information is called a *Fourier Transformation*.

Acceleration Ratio. The acceleration of a system divided by the acceleration of gravity is referred to as the *acceleration ratio*. Usually the peak acceleration of the system is used.

Floor Panel. A rectangular plan portion of a floor encompassed by the span and an effective width is defined as a *floor panel*.

Bay. A rectangular plan portion of a floor defined by four column locations.

1.5 Floor Vibration Principles

Although human annoyance criteria for vibration have been known for many years, it has only recently become practical to apply such criteria to the design of floor structures. The reason for this is that the problem is complex—the loading is complex and the response complicated, involving a large number of modes of vibration. Experience and research have

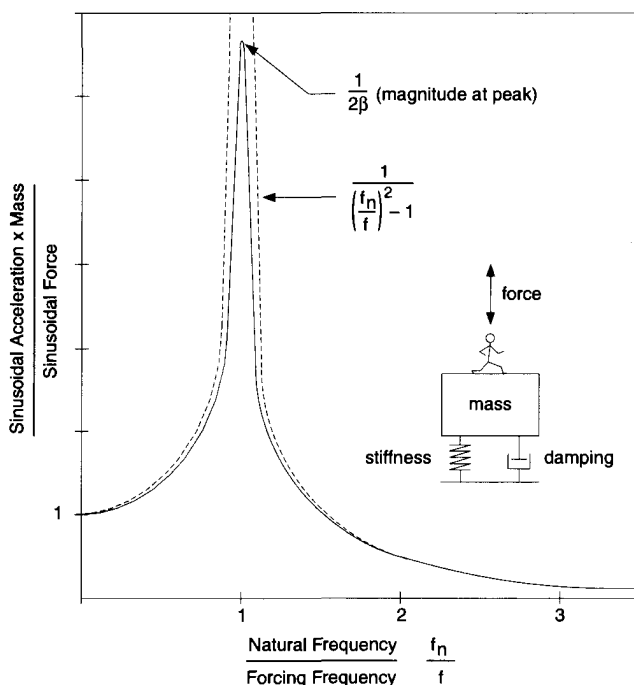


Fig. 1.3 Response to sinusoidal force.

shown, however, that the problem can be simplified sufficiently to provide practical design criteria.

Most floor vibration problems involve repeated forces caused by machinery or by human activities such as dancing, aerobics or walking, although walking is a little more complicated than the others because the forces change location with each step. In some cases, the applied force is sinusoidal or nearly so. In general, a repeated force can be represented by a combination of sinusoidal forces whose frequencies, f , are multiples or harmonics of the basic frequency of the force repetition, e.g. step frequency, f_{step} , for human activities. The time-dependent repeated force can be represented by the Fourier series

$$F = P [1 + \sum \alpha_i \cos(2\pi i f_{step} t + \phi_i)] \quad (1.1)$$

where

P = person's weight

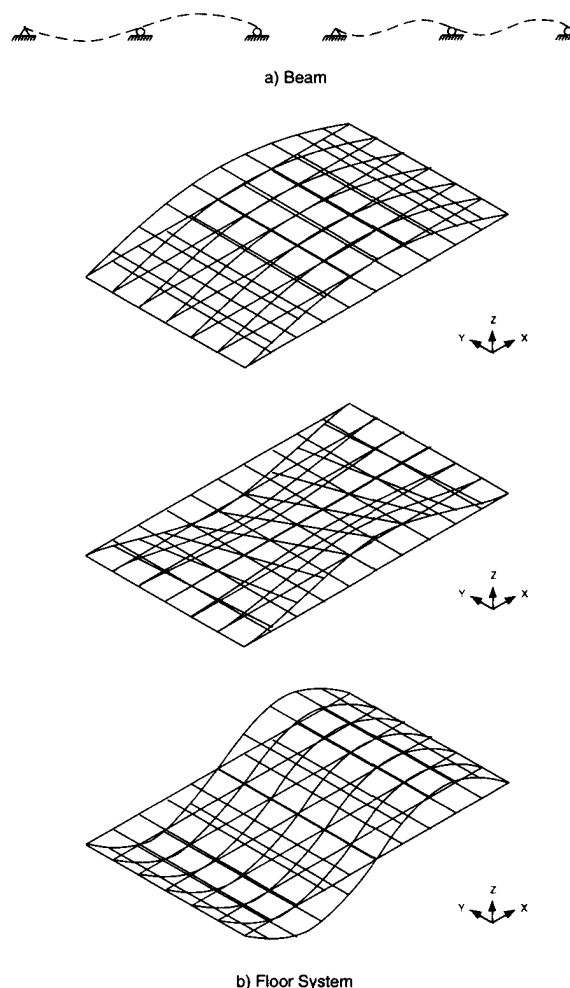


Fig. 1.4 Typical beam and floor system mode shapes.

- α_i = dynamic coefficient for the harmonic force
- i = harmonic multiple (1, 2, 3,...)
- f_{step} = step frequency of the activity
- t = time
- ϕ_i = phase angle for the harmonic

As a general rule, the magnitude of the dynamic coefficient decreases with increasing harmonic, for instance, the dynamic coefficients associated with the first four harmonics of walking are 0.5, 0.2, 0.1 and 0.05, respectively. In theory, if any frequency associated with the sinusoidal forces matches the natural frequency of a vibration mode, then resonance will occur, causing severe vibration amplification.

The effect of resonance is shown in Figure 1.3. For this figure, the floor structure is modeled as a simple mass connected to the ground by a spring and viscous damper. A person or machine exerts a vertical sinusoidal force on the mass. Because the natural frequency of almost all concrete slab-structural steel supported floors can be close to or can match a harmonic forcing frequency of human activities, resonance amplification is associated with most of the vibration problems that occur in buildings using structural steel.

Figure 1.3 shows sinusoidal response if there is only one mode of vibration. In fact, there may be many in a floor system. Each mode of vibration has its own displacement configuration or "mode shape" and associated natural frequency. A typical mode shape may be visualized by considering the floor as divided into an array of panels, with adjacent panels moving in opposite directions. Typical mode shapes for a bay are shown in Figure 1.4(b). The panels are large for low-frequency modes (panel length usually corresponding to

a floor span) and small for high frequency modes. In practice, the vibrational motion of building floors are localized to one or two panels, because of the constraining effect of multiple column/wall supports and non-structural components, such as partitions.

For vibration caused by machinery, any mode of vibration must be considered, high frequency, as well as, low frequency. For vibration due to human activities such as dancing or aerobics, a higher mode is more difficult to excite because people are spread out over a relatively large area and tend to force all panels in the same direction simultaneously, whereas adjacent panels must move in opposite directions for higher modal response. Walking generates a concentrated force and therefore may excite a higher mode. Higher modes, however, are generally excited only by relatively small harmonic walking force components as compared to those associated with the lowest modes of vibration. Thus, in practice it is usually only the lowest modes of vibration that are of concern for human activities.

The basic model of Figure 1.3 may be represented by:

$$\text{Sinusoidal Acceleration} = \left(\frac{\text{Force}}{\text{Mass}} \right) \times \text{Response Factor} \quad (1.2)$$

where the response factor depends strongly on the ratio of natural frequency to forcing frequency (f_n/f) and, for vibration at or close to resonance, on the damping ratio (β). It is these parameters that control the vibration serviceability design of most steel floor structures.

It is possible to control the acceleration at resonance by increasing damping or mass since acceleration = force divided by damping times mass (see Figure 1.3). The control is most effective where the sinusoidal forces are small, as they are for walking. Natural frequency also always plays a role, because sinusoidal forces generally decrease with increasing harmonic—the higher the natural frequency, the lower the force. The design criterion for walking vibration in Chapter 4 is based on these principles.

Where the dynamic forces are large, as they are for aerobics, resonant vibration is generally too great to be controlled practically by increasing damping or mass. In this case, the natural frequency of any vibration mode significantly affected by the dynamic force (i.e. a low frequency mode) must be kept away from the forcing frequency. This generally means that the fundamental natural frequency must be made greater than the forcing frequency of the highest harmonic force that can cause large resonant vibration. For aerobics or dancing, attention should be paid to the possibility of transmission of vibrations to sensitive occupancies in other parts of the floor and other parts of the building. This requires the consideration of vibration transfer through supports, such as columns, particularly to parts of the building which may be in resonance with the harmonic force. The design criterion for rhythmic activities in Chapter 5 takes this into account.

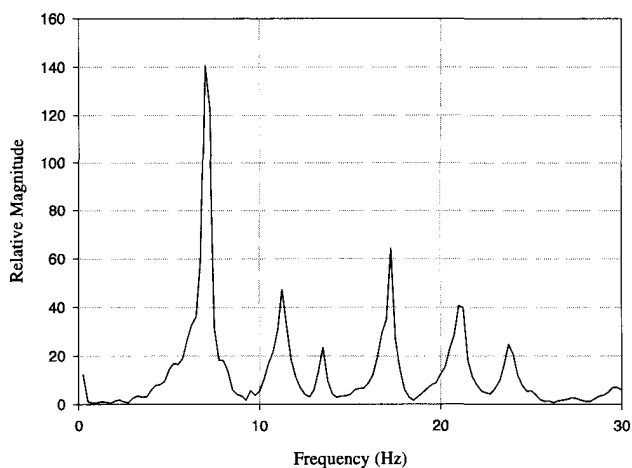


Fig. 1.5 Frequency spectrum.

Where the natural frequency of the floor exceeds 9-10 Hz or where the floors are light, as for example wood deck on light metal joists, resonance becomes less important for human induced vibration, and other criteria related to the response of the floor to footstep forces should be used. When floors are very light, response includes time variation of static

deflection due to a moving repeated load (see Figure 1.6), as well as decaying natural vibrations due to footstep impulses (see Figure 1.7). A point load stiffness criterion is appropriate for the static deflection component and a criterion based on footstep impulse vibration is appropriate for the footstep impulses.

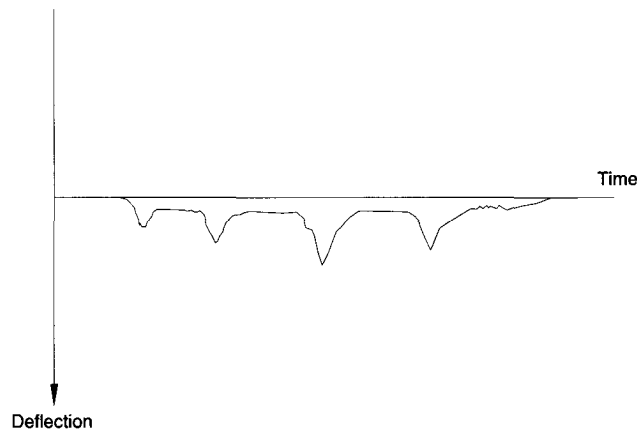


Fig. 1.6 Quasi-static deflection of a point on a floor due to a person walking across the floor.

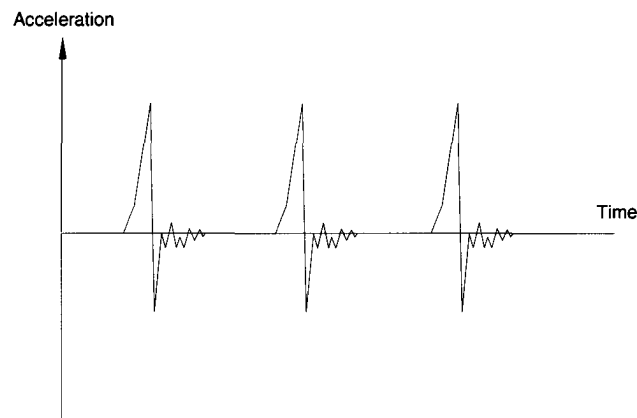


Fig. 1.7 Floor vibration due to footstep impulses during walking.

Chapter 2

ACCEPTANCE CRITERIA FOR HUMAN COMFORT

2.1 Human Response to Floor Motion

Human response to floor motion is a very complex phenomenon, involving the magnitude of the motion, the environment surrounding the sensor, and the human sensor. A continuous motion (steady-state) can be more annoying than motion caused by an infrequent impact (transient). The threshold of perception of floor motion in a busy workplace can be higher than in a quiet apartment. The reaction of a senior citizen living on the fiftieth floor can be considerably different from that of a young adult living on the second floor of an apartment complex, if both are subjected to the same motion.

The reaction of people who feel vibration depends very strongly on what they are doing. People in offices or residences do not like "distinctly perceptible" vibration (peak acceleration of about 0.5 percent of the acceleration of gravity, g), whereas people taking part in an activity will accept vibrations approximately 10 times greater (5 percent g or more). People dining beside a dance floor, lifting weights beside an aerobics gym, or standing in a shopping mall, will accept something in between (about 1.5 percent g). Sensitivity within each occupancy also varies with duration of vibration and remoteness of source. The above limits are for vibration frequencies between 4 Hz and 8 Hz. Outside this frequency range, people accept higher vibration accelerations as shown in Figure 2.1.

2.2 Recommended Criteria for Structural Design

Many criteria for human comfort have been proposed over the years. The Appendix includes a short historical development of criteria used in North American and Europe. Following are recommended design criteria for walking and rhythmic excitations. The recommended walking excitation criterion, methods for estimating the required floor properties, and design procedures were first proposed by Allen and Murray (1993). The criterion differs considerably from previous "heel-drop" based approaches. Although the proposed criterion for walking excitation is somewhat more complex than previous criteria, it has a wider range of applicability and results in more economical, but acceptable, floor systems.

2.2.1 Walking Excitation

As part of the effort to develop this Design Guide, a new criterion for vibrations caused by walking was developed with broader applicability than the criteria currently used in North America. The criterion is based on the dynamic response of steel beam- or joist-supported floor systems to

walking forces, and can be used to evaluate structural systems supporting offices, shopping malls, footbridges, and similar occupancies (Allen and Murray 1993). Its development is explained in the following paragraphs and its application is shown in Chapter 4.

The criterion was developed using the following:

- Acceleration limits as recommended by the International Standards Organization (International Standard ISO 2631-2, 1989), adjusted for intended occupancy. The ISO Standard suggests limits in terms of rms acceleration as a multiple of the baseline line curve shown in Figure 2.1. The multipliers for the proposed criterion, which is expressed in terms of peak acceleration, are 10 for offices, 30 for shopping malls and indoor footbridges, and 100 for outdoor footbridges. For design purposes, the limits can be assumed to range between 0.8 and 1.5 times the recommended values depending on

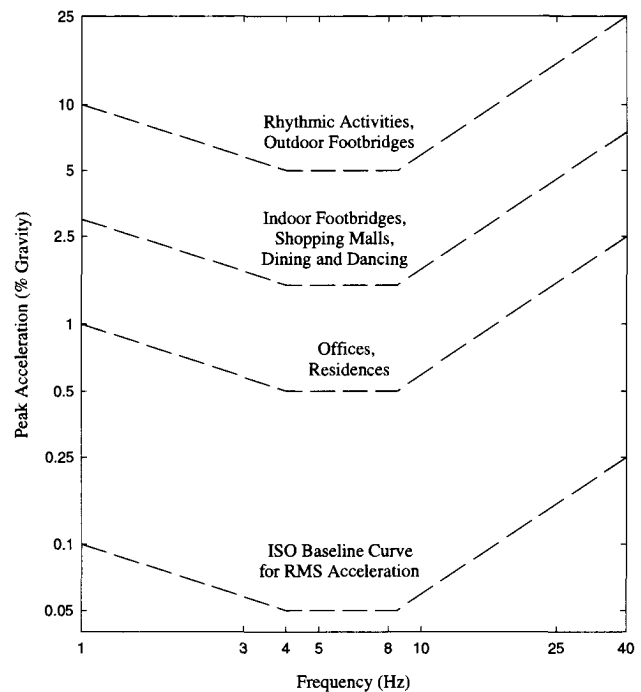


Fig. 2.1 Recommended peak acceleration for human comfort for vibrations due to human activities (Allen and Murray, 1993; ISO 2631-2: 1989).

the duration of vibration and the frequency of vibration events.

- A time dependent harmonic force component which matches the fundamental frequency of the floor:

$$F_i = P\alpha_i \cos(2\pi f_{step} t) \quad (2.1)$$

where

P = person's weight, taken as 0.7 kN (157 pounds) for design

α_i = dynamic coefficient for the i th harmonic force component

i = harmonic multiple of the step frequency

f_{step} = step frequency

Recommended values for α_i are given in Table 2.1. (Only one harmonic component of Equation (1.1) is used since all other harmonic vibrations are small in comparison to the harmonic associated with resonance.)

- A resonance response function of the form:

$$\frac{a}{g} = \frac{R\alpha_i P}{\beta W} \cdot \cos(2\pi f_{step} t) \quad (2.2)$$

where

a/g = ratio of the floor acceleration to the acceleration of gravity

R = reduction factor

β = modal damping ratio

W = effective weight of the floor

The reduction factor R takes into account the fact that full steady-state resonant motion is not achieved for walking and that the walking person and the person annoyed are not simultaneously at the location of maximum modal displacement. It is recommended that R be

taken as 0.7 for footbridges and 0.5 for floor structures with two-way mode shape configurations.

For evaluation, the peak acceleration due to walking can be estimated from Equation (2.2) by selecting the lowest harmonic, i , for which the forcing frequency, $f = i \cdot f_{step}$, can match a natural frequency of the floor structure. The peak acceleration is then compared with the appropriate limit in Figure 2.1. For design, Equation (2.2) can be simplified by approximating the step relationship between the dynamic coefficient, α_i , and frequency, f , shown in Figure 2.2 by the formula $\alpha = 0.83 \exp(-0.35f)$. With this substitution, the following simplified design criterion is obtained:

$$\frac{a_p}{g} = \frac{P_o \exp(-0.35f_n)}{\beta W} \leq \frac{a_o}{g} \quad (2.3)$$

where

a_p/g = estimated peak acceleration (in units of g)

a_o/g = acceleration limit from Figure 2.1

f_n = natural frequency of floor structure

P_o = constant force equal to 0.29 kN (65 lb.) for floors and 0.41 kN (92 lb.) for footbridges

The numerator $P_o \exp(-0.35f_n)$ in Inequality (2.3) represents an effective harmonic force due to walking which results in resonance response at the natural floor frequency f_n . Inequality (2.3) is the same design criterion as that proposed by Allen and Murray (1993); only the format is different.

Motion due to quasi-static deflection (Figure 1.6) and footstep impulse vibration (Figure 1.7) can become more critical than resonance if the fundamental frequency of a floor is greater than about 8 Hz. To account approximately for footstep impulse vibration, the acceleration limit a_o/g is not increased with frequency above 8 Hz, as it would be if

Table 2.1
Common Forcing Frequencies (f) and
Dynamic Coefficients* (α_i)

Harmonic i	Person Walking		Aerobics Class		Group Dancing	
	f , Hz	α_i	f , Hz	α_i	f , Hz	α_i
1	1.6–2.2	0.5	2–2.75	1.5	1.5–3	0.5
2	3.2–4.4	0.2	4–5.5	0.6	—	—
3	4.8–6.6	0.1	6–8.25	0.1	—	—
4	6.4–8.8	0.05	—	—	—	—

*dynamic coefficient = peak sinusoidal force/weight of person(s).

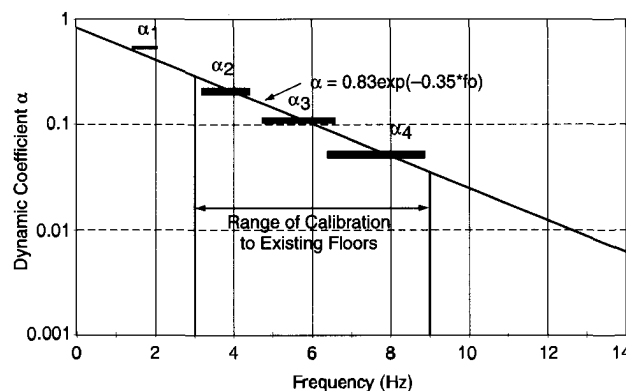


Fig. 2.2 Dynamic coefficient, α , versus frequency.

Figure 2.1 were used. That is, the horizontal portion of the curves between 4 Hz and 8 Hz in Figure 2.1 are extended to the right beyond 8 Hz. To account for motion due to varying static deflection, a minimum static stiffness of 1 kN/mm (5.7 kips/inch) under concentrated load is introduced as an additional check if the natural frequency is greater than 9-10 Hz. More severe criteria for static stiffness under concentrated load are used for sensitive equipment as described in Chapter 6.

Guidelines for the estimation of the parameters used in the above design criterion for walking vibration and application examples are found in Chapter 4.

2.2.2 Rhythmic Excitation

Criteria have recently been developed for the design of floor structures for rhythmic exercises (Allen 1990, 1990a; NBC 1990). The criteria are based on the dynamic response of structural systems to rhythmic exercise forces distributed over all or part of the floor. The criteria can be used to evaluate structural systems supporting aerobics, dancing, audience participation and similar events, provided the loading function is known. As an example, Figure 2.3 shows a time record of the dynamic loading function and an associated spectrum for eight people jumping at 2.1 Hz. Table 2.1 gives common forcing frequencies and dynamic coefficients for rhythmic activities.

The peak acceleration of the floor due to a harmonic rhythmic force is obtained from the classical solution by assuming that the floor structure has only one mode of vibration (Allen 1990):

$$\frac{a_p}{g} = \frac{1.3\alpha_i w_p / w_t}{\sqrt{\left[\left(\frac{f_n}{f}\right)^2 - 1\right]^2 + \left[\frac{2\beta f_n}{f}\right]^2}} \quad (2.4)$$

where

- a_p / g = peak acceleration as a fraction of the acceleration due to gravity
- α_i = dynamic coefficient (see Table 2.1)
- w_p = effective weight per unit area of participants distributed over floor panel
- w_t = effective distributed weight per unit area of floor panel, including occupants
- f_n = natural frequency of floor structure
- f = forcing frequency
- $= i \cdot f_{step}$ where f_{step} is the step frequency
- β = damping ratio

Equation (2.4) can be simplified as follows:

At resonance ($f_n = f$):

$$\frac{a_p}{g} = \frac{1.3}{2\beta} \cdot \frac{\alpha_i w_p}{w_t} \quad (2.5a)$$

Above resonance ($f_n > 1.2f$):

$$\frac{a_p}{g} = \frac{1.3}{(f_n/f)^2 - 1} \cdot \frac{\alpha_i w_p}{w_t} \quad (2.5b)$$

Most problems occur if a harmonic forcing frequency, $f = i \cdot f_{step}$, is equal to or close to the natural frequency, f_n , for which case the acceleration is determined from Equation (2.5a). Vibration from lower harmonics (first or second), however, may still be substantial, and the acceleration for a lower harmonic is determined from Equation (2.5b). The effective maximum acceleration, accounting for all harmonics, can then be estimated from the combination rule (Allen 1990a):

$$a_m = [\sum a_i^{1.5}]^{1/1.5} \quad (2.6)$$

where

a_i = peak acceleration for the i 'th harmonic.

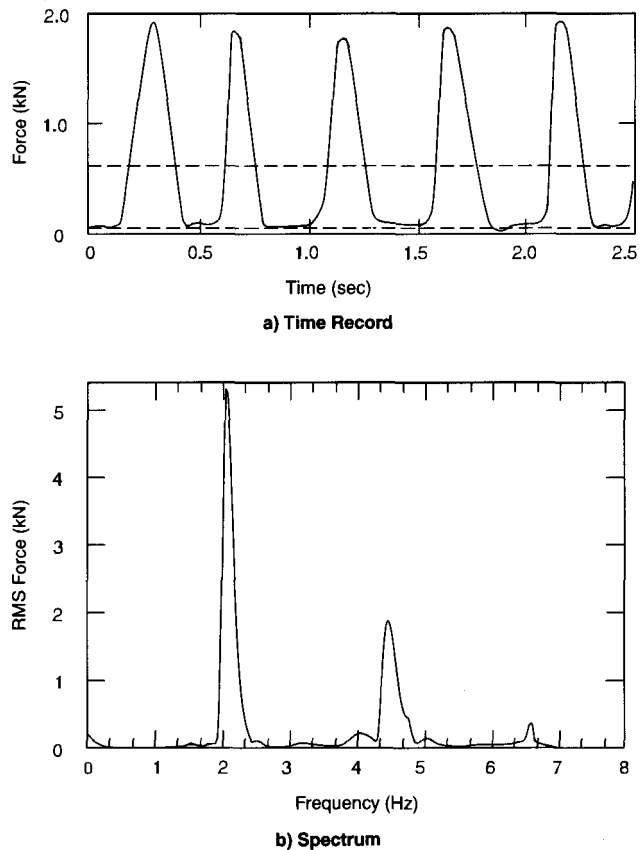


Fig. 2.3 Example loading function and spectrum from rhythmic activity.

The effective maximum acceleration determined from Equation (2.6) can then be compared to the acceleration limit for people participating in the rhythmic activity (approximately 5 percent gravity from Figure 2.1). Experience shows, however, that many problems with building vibrations due to rhythmic exercises concern more sensitive occupancies in the building, especially for those located near the exercising area. For these other occupancies, the effective maximum acceleration, a_m , calculated for the exercise floor should be reduced in accordance with the vibration mode shape for the structural system, before comparing it to the acceleration limit for the sensitive occupancy from Figure 2.1.

The dynamic forces for rhythmic activities tend to be large and resonant vibration is generally too great to be reduced practically by increasing the damping and or mass. This means that for design purposes, the natural frequency of the structural system, f_n , must be made greater than the forcing frequency, f , of the highest harmonic that can cause large resonant vibration. Equation (2.5b) can be inverted to provide the following design criterion (Allen 1990a):

$$f_n \geq f \sqrt{1 + \frac{k}{(a_o/g)} \frac{\alpha_i w_p}{w_t}} \quad (2.7)$$

where

k = constant (1.3 for dancing, 1.7 for lively concert or sports event, 2.0 for aerobics)

a_o/g = acceleration limit (0.05, or less, if sensitive occupancies are affected)

and the other parameters are defined above. Careful analysis by use of Equations (2.5) and (2.6) can provide better guidance than Equation (2.7), as for example if resonance with the highest harmonic is acceptable because of sufficient mass or partial loading of the floor panel.

Guidance on the estimation of parameters, including building vibration mode shape, and examples of the application of Equations (2.5) to (2.7) are given in Chapter 5.

Chapter 3

NATURAL FREQUENCY OF STEEL FRAMED FLOOR SYSTEMS

The most important parameter for the vibration serviceability design and evaluation of floor framing systems is natural frequency. This chapter gives guidance for estimating the natural frequency of steel beam and steel joist supported floor systems, including the effects of continuity.

3.1. Fundamental Relationships

Steel framed floors generally are two-way systems which may have several vibration modes with closely spaced frequencies. The natural frequency of a critical mode in resonance with a harmonic of step frequency may therefore be difficult to assess. Modal analysis of the floor structure can be used to determine the critical modal properties, but there are factors that are difficult to incorporate into the structural model (composite action, boundary and discontinuity conditions, partitions, other non-structural components, etc). An unfinished floor with uniform bays can have a variety of modal pattern configurations extending over the whole floor area, but partitions and other non-structural components tend to constrain significant dynamic motions to local areas in such a way that the floor vibrates locally like a single two-way panel. The following simplified procedures for determining the first natural frequency of vertical vibration are recommended.

The floor is assumed to consist of a concrete slab (or deck) supported on steel beams or joists which are supported on walls or steel girders between columns. The natural frequency, f_n , of a critical mode is estimated by first considering a "beam or joist panel" mode and a "girder panel" mode separately and then combining them. Alternatively, the natural frequency can be estimated by finite element analyses.

Beam or joist and girder panel mode natural frequencies can be estimated from the fundamental natural frequency equation of a uniformly loaded, simply-supported, beam:

$$f_n = \frac{\pi}{2} \left[\frac{gE_s I_t}{wL^4} \right]^{1/2} \quad (3.1)$$

where

- f_n = fundamental natural frequency, Hz
- g = acceleration of gravity, 9.86 m/s² or 386 in/sec²
- E_s = modulus of elasticity of steel
- I_t = transformed moment of inertia; effective transformed moment of inertia, if shear deformations are included
- w = uniformly distributed weight per unit length (actual,

not design, live and dead loads) supported by the member

L = member span

The combined mode or system frequency, can be estimated using the Dunkerley relationship:

$$\frac{1}{f_n^2} = \frac{1}{f_j^2} + \frac{1}{f_g^2} \quad (3.2)$$

where

- f_j = beam or joist panel mode frequency
- f_g = girder panel mode frequency

Equation (3.1) can be rewritten as

$$f_n = 0.18 \sqrt{\frac{g}{\Delta}} \quad (3.3)$$

where

Δ = midspan deflection of the member relative to its supports due to the weight supported (i.e. $\Delta = 5wL^4 / (384E_s I_t)$)

Sometimes, as described later in this Design Guide, shear deformations must also be included in determining Δ .

For the combined mode, if both the beam or joist and girder are assumed simply supported, the Dunkerley relationship can be rewritten as

$$f_n = 0.18 \sqrt{\frac{g}{(\Delta_j + \Delta_g)}} \quad (3.4)$$

where

Δ_j and Δ_g = beam or joist and girder deflections due to the weight supported, respectively.

Tall buildings can have vertical column frequencies low enough to create serious resonance problems with rhythmic activity. For these cases, Equation (3.4) is modified to include the column effect:

$$f_n = 0.18 \sqrt{\frac{g}{(\Delta_j + \Delta_g + \Delta_c)}} \quad (3.5)$$

where

Δ_c = axial shortening of the column due to the weight supported

Further guidance on the estimation of deflection of joists, beams and girders due to flexural and shear deformation is found in the following sections.

3.2 Composite Action

In calculating the fundamental natural frequency using the relationships in Section 3.1, the transformed moment of inertia is to be used if the slab (or deck) is attached to the supporting member. This assumption is to be applied even if structural shear connectors are not used, because the shear forces at the slab/member interface are resisted by deck-to-

member spot welds or by friction between the concrete and metal surfaces.

If the supporting member is separated from the slab (for example, the case of overhanging beams which pass over a supporting girder) composite behavior should not be assumed. For such cases, the fundamental natural frequency of the girder can be increased by providing shear connection between the slab and girder flange (see Section 7.2).

To take account of the greater stiffness of concrete on metal deck under dynamic as compared to static loading, it is recommended that the concrete modulus of elasticity be taken equal to 1.35 times that specified in current structural standards for calculation of the transformed moment of inertia. Also for determining the transformed moment of inertia of typical beams or joists and girders, it is recommended that the effective width of the concrete slab be taken as the member spacing, but not more than 0.4 times the member span. For edge or spandrel members, the effective slab width is to be taken as one-half the member spacing but not more than 0.2 times the member span plus the projection of the free edge of the slab beyond the member Centerline. If the concrete side of the member is in compression, the concrete can be assumed to be solid, uncracked.

See Section 3.5 and for special considerations needed for trusses and open web joist framing.

3.3 Distributed Weight

The supported weight, w , used in the above equations must be estimated carefully. The actual dead and live loads, not the design dead and live loads, should be used in the calculations. For office floors, it is suggested that the live load be taken as 0.5 kN/m^2 (11 psf). This suggested live load is for typical office areas with desks, file cabinets, bookcases, etc. A lower value should be used if these items are not present. For residential floors, it is suggested that the live load be taken as 0.25 kN/m^2 (6 psf). For footbridges, and gymnasium and shopping center floors, it is suggested that the live load be taken as zero, or at least nearly so.

Equations (3.1) and (3.3) are based on the assumption of a simply-supported beam, uniformly loaded. Joists, beams or girders usually are uniformly loaded, or nearly so, with the exception of girders that support joists or beams at mid-span only, in which case the calculated deflection should be multiplied by $4/\pi$ (≈ 1.3) to take into account the difference between the frequency for a simply-supported beam with distributed mass and that with a concentrated mass at mid-span.

3.4 Deflection Due to Flexure: Continuity

Continuous Joists, Beams or Girders

Equations (3.3) through (3.5) also apply approximately for continuous beams over supports (such as beams shear-connected through girders or joists connected through girders at

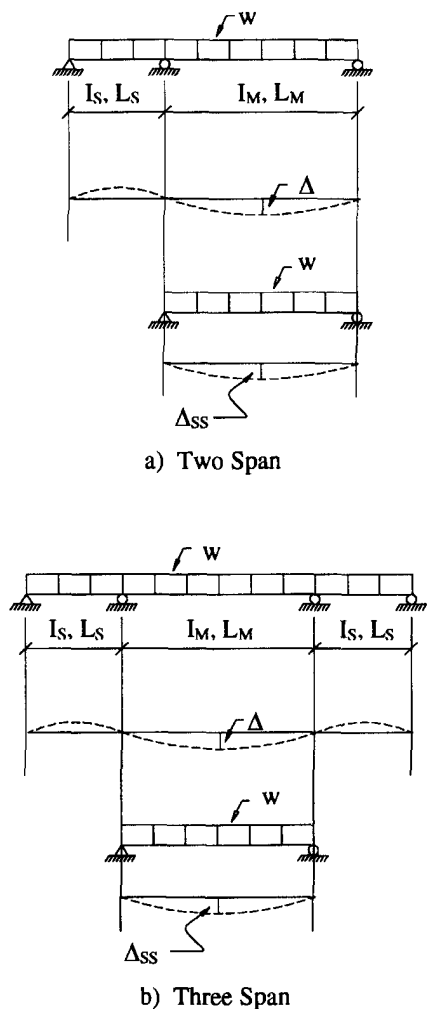


Fig. 3.1 Modal flexural deflections, Δ , for beams continuous over supports.

top and bottom chords) for the situation where the distributed weight acts in the direction of modal displacement, i.e. down where the modal displacement is down, and up where it is up (opposite to gravity). Adjacent spans displace in opposite directions and, therefore, for a continuous beam with equal spans, the fundamental frequency is equal to the natural frequency of a single simply-supported span.

Where the spans are not equal, the following relations can be used for estimating the flexural deflection of a continuous member from the simply supported flexural deflection, Δ_{ss} , of the main (larger) span, L_m due to the weight supported. For two continuous spans:

$$\Delta = \left[\frac{0.4 + \frac{k_m}{k_s} \left(1 + 0.6 \frac{L_s^2}{L_m^2} \right)}{1 + \frac{k_m}{k_s}} \right] \Delta_{ss} \quad (3.6)$$

For three continuous spans

$$\Delta = \left[\frac{0.6 + 2 \frac{k_m}{k_s} \left(1 + 1.2 \frac{L_s^2}{L_m^2} \right)}{3 + 2 \frac{k_m}{k_s}} \right] \Delta_{ss} \quad (3.7)$$

where

$$\begin{aligned} k_m &= I_M / L_M \\ k_s &= I_S / L_S \\ I &= \text{moment of inertia} \\ L_M \text{ and } L_S &\text{ are defined in Figure 3.1.} \end{aligned}$$

Members Continuous with Columns

The natural frequency of a girder or beam moment-connected to columns is increased because of the flexural restraint of the

columns. This is important for tall buildings with large columns. The following relationship can be used for estimating the flexural deflection of a girder or beam moment connected to columns in the configuration shown in Figure 3.2.

$$\Delta = \left[\frac{0.6 + 2 \frac{k_m}{k_s} (1 + 1.2\lambda) + \frac{1.2 n_c k_c}{k_s}}{3 + 2 \frac{k_m}{k_s} + \frac{1.2 n_c k_c}{k_s}} \right] \Delta_{ss} \quad (3.8)$$

where

$$\begin{aligned} k_m &= I_M / L_M \\ k_s &= I_S / L_S \\ k_c &= I_C / L_C \\ L_M, L_S \text{ and } L_C &\text{ are defined in Figure 3.2} \\ \Delta_{ss} &= \text{flexural deflection of the simply supported main (longer) beam or girder of span } L_M \text{ due to the weight supported} \\ \lambda &= (L_S / L_M)^2 \end{aligned}$$

Cantilevers

The natural frequency of a fixed cantilever can be estimated using Equation (3.3) through (3.5), with the following used to calculate Δ . For uniformly distributed mass

$$\Delta_f = \frac{wL^4}{8EI} \quad (3.9)$$

and for a mass concentrated at the tip

$$\Delta_f = \frac{WL^3}{3EI} \quad (3.10)$$

Cantilevers, however, are rarely fully fixed at their supports. The following equations can be used to estimate the flexural deflection of a cantilever/backspan/column condition shown in Figure 3.3. If the cantilever deflection, Δ_T , exceeds the deflection of the backspan, Δ_B , then

$$\Delta = \Delta_T = C_m \left[1 + \frac{4}{3} \frac{L_B}{L_t} \cdot \frac{1 + 0.25 L_B^2 / L_t^2}{1 + n_c k_c / k_b} \right] \Delta_F \quad (3.11)$$

If the opposite is true, then

$$\Delta = \Delta_B = \left[1 + 2.4 \left(\frac{L_t^2 / L_B^2 - 0.5 k_c / k_b}{1 + n_c k_c / k_b} \right) \right] \Delta_{ss} \quad (3.12)$$

where L_C , L_B and L_T are defined in Figure 3.3 and

$$\begin{aligned} k_b &= I_B / L_B \\ k_c &= I_C / L_C \\ C_m &= 0.81 \text{ for distributed mass and } 1.06 \text{ for mass concentrated at the tip} \\ n_c &= 2 \text{ if columns occur above and below, } 1 \text{ if only above or below} \\ \Delta_f &= \text{flexural deflection of a fixed cantilever, due to the weight supported} \end{aligned}$$

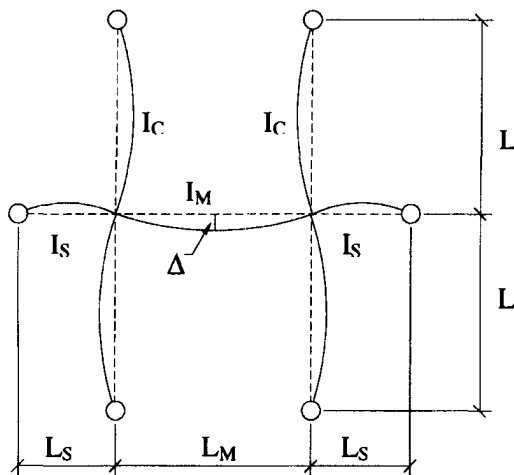


Fig. 3.2 Modal flexural deflections, Δ , for beams or girders continuous with columns.

Δ_{ss} = flexural deflection of backspan, assumed simply supported

If the cantilever/backspan beam is supported by a girder, $k_c = 0$ in Equations (3.11) and (3.12).

3.5 Deflection Due to Shear in Beams and Trusses

Sometimes shear may contribute substantially to the deflection of the member. Two types of shear may occur:

- Direct shear due to shear strain in the web of a beam or girder, or due to length changes of the web members of a truss;
- Indirect shear in trusses as a result of eccentricity of member forces through joints.

For wide flange members, the shear deflection is simply equal to the accumulated shear strain in the web from the

support to mid-span. For rolled shapes, shear deflection is usually small relative to flexural deflection and can be neglected.

For simply supported trusses, the shear deformation effect can usually be taken into account using:

$$I_{eff} = \frac{I_{comp}}{1 + 0.15 I_{comp} / I_{chords}} \quad (3.13)$$

where

I_{eff} = the "effective" transformed moment of inertia which accounts for shear deformation

I_{comp} = the fully composite moment inertia

I_{chords} = the moment of inertia of the joist chords alone

Equation (3.13) is applicable only to simply supported trusses with span-to-depth ratios greater than approximately 12.

For deep long-span trusses the shear strain can be considerable, substantially reducing the estimated natural frequency from that based on flexural deflection (Allen 1990a). The following method may be used for estimating such shear deflection assuming no eccentricity at the joints:

1. Determine web member forces, due to the weight supported.
2. Determine web member length changes $\delta_i = F_i L_i / A_i E_s$, where for the i th member, F_i is the axial force due to the real loads, L_i is the length, and A_i is the cross-section area.
3. Determine shear increments, $v_i = \delta_i / \cos \gamma_i$, where γ_i is the angle of the web member to vertical.
4. Sum the shear increments for each web member from the support to mid-span.

The total deflection, Δ , is then the sum of flexural and shear deflections, generally at mid-span.

3.6 Special Considerations for Open Web Joists and Joist Girders

The effects of joist seats, web shear deformation, and eccentricity of joints must be considered in calculating the natural frequency of open web joist and hot-rolled girder or joist-girder framed floor systems.

For the case of a girder or joist girder supporting standard open web joists, it has been found that the joist seats are not sufficiently stiff to justify the full transformed moment of inertia assumption for the girder or joist girder. It is recommended that the effective moment of inertia of girders supporting joist seats be determined from

$$I_g = I_{nc} + (I_c - I_{nc}) / 4 \quad (3.14)$$

where

I_{nc} and I_c = non-composite and fully composite moments of inertia, respectively.

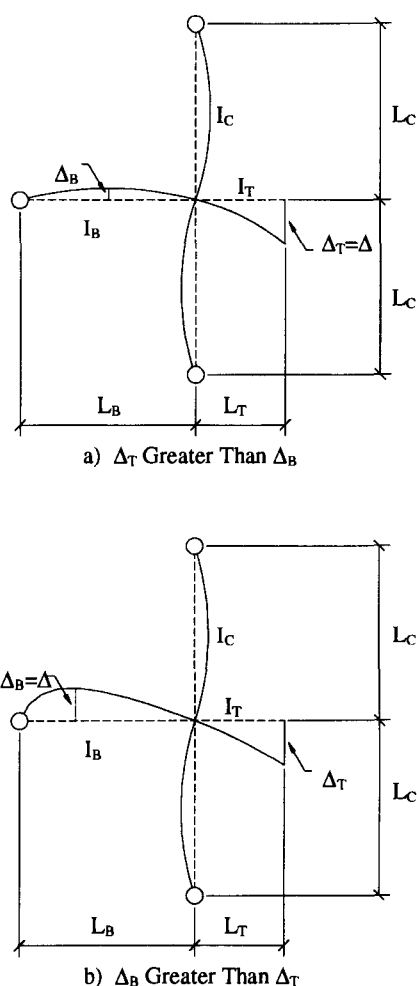


Fig. 3.3 Modal flexural deflections, Δ , for cantilever/backspan/columns.

The effective moment of inertia of joists and joist girders that is used to calculate the limiting span/360 load in Steel Joist Institute (SJI) load tables is 0.85 times the moment of inertia of the chord members. This factor accounts for web shear deformation. It has recently been reported (Band and Murray 1996) that the 0.85 coefficient can be increased to 0.90 if the span-to-depth ratio of the joist or joist-girder is not less than about 20. For smaller span-to-depth ratios, the effective moment of inertia of the joist or joist-girder can be as low as 0.50 times the moment of inertia of the chords. Barry and Murray (1996) proposed the following method to estimate the effective moment of inertia of joists or joist girders:

$$I_{mod} = C_r I_{chords} \quad (3.15)$$

where, for joists or joist girders with single or double angle web members,

$$C_r = 0.90(1 - e^{-0.28(L/D)})^{2.8} \quad (3.16)$$

for $6 \leq L/D \leq 24$, L = span length, and D = nominal depth of

the joist and for joists with continuous round rod web members

$$C_r = 0.721 + 0.00725(L/D) \quad (3.17)$$

for $10 \leq L/D \leq 24$.

The effective transformed moment of inertia of joist supported tee-beams can then be calculated using

$$I_{eff} = \frac{1}{\frac{\gamma}{I_{chords}} + \frac{1}{I_{comp}}} \quad (3.18)$$

where

$$\gamma = \frac{1}{C_r} - 1 \quad (3.19)$$

and

I_{comp} = the transformed moment of inertia using the actual chord areas. (See Examples 4.5 and 4.6 in Section 4.4.2).

Chapter 4

DESIGN FOR WALKING EXCITATION

4.1 Recommended Criterion

Existing North American floor vibration design criteria are generally based on a reference impact such as a heel-drop and were calibrated using floors constructed 20-30 years ago. Annoying floors of this vintage generally had natural frequencies between 5 and 8 Hz because of traditional design rules, such as live load deflection less than span/360, and common construction practice. With the advent of limit states design and the more common use of lightweight concrete, floor systems have become lighter, resulting in higher natural frequencies for the same structural steel layout. However, beam and girder spans have increased, sometimes resulting in frequencies lower than 5 Hz. Most existing design criteria do not properly evaluate systems with frequencies below 5 Hz and above 8 Hz.

The design criterion for walking excitations recommended in Section 2.2.1 has broader applications than commonly used criteria. The recommended criterion is based on the dynamic response of steel beam and joist supported floor systems to walking forces. The criterion can be used to evaluate concrete/steel framed structural systems supporting footbridges, residences, offices, and shopping malls.

The criterion states that the floor system is satisfactory if the peak acceleration, a_p , due to walking excitation as a fraction of the acceleration of gravity, g , determined from

$$\frac{a_p}{g} = \frac{P_o \exp(-0.35f_n)}{\beta W} \quad (4.1)$$

does not exceed the acceleration limit, a_o/g , for the appropriate occupancy. In Equation (4.1),

- P_o = a constant force representing the excitation,
- f_n = fundamental natural frequency of a beam or joist panel, a girder panel, or a combined panel, as applicable,
- β = modal damping ratio, and
- W = effective weight supported by the beam or joist panel, girder panel or combined panel, as applicable.

Recommended values of P_o and β , as well as a_o/g limits for several occupancies, are given in Table 4.1. Figure 2.1 can also be used to evaluate a floor system if the original ISO plateau between 4 Hz and approximately 8 Hz is extended from 3 Hz to 20 Hz as discussed in Section 2.2.1.

If the natural frequency of a floor is greater than 9-10 Hz, significant resonance with walking harmonics does not occur, but walking vibration can still be annoying. Experience indi-

cates that a minimum stiffness of the floor to a concentrated load of 1 kN per mm (5.7 kips per in.) is required for office and residential occupancies. To ensure satisfactory performance of office or residential floors with frequencies greater than 9-10 Hz, this stiffness criterion should be used in addition to the walking excitation criterion, Equation (4.1) or Figure 2.1. Floor systems with fundamental frequencies less than 3 Hz should generally be avoided, because they are liable to be subjected to "rogue jumping" (see Chapter 5).

The following section, based on Allen and Murray (1993), provides guidance for estimating the required floor properties for application of the recommended criterion.

4.2 Estimation Of Required Parameters

The parameters in Equation (4.1) are obtained or estimated from Table 4.1 (P_o , β , and a_o/g) and Chapter 3 (f_n). For simply supported footbridges f_n is estimated using Equation (3.1) or (3.3) and W is equal to the weight of the footbridge. For floors, the fundamental natural frequency, f_n , and effective panel weight, W , for a critical mode are estimated by first considering the 'beam or joist panel' and 'girder panel' modes separately and then combining them as explained in Chapter 3.

Effective Panel Weight, W

The effective panel weights for the beam or joist and girder panel modes are estimated from

$$W = wBL \quad (4.2)$$

where

- w = supported weight per unit area
- L = member span
- B = effective width

For the beam or joist panel mode, the effective width is

$$B_j = C_j(D_s/D_j)^{1/4}L_j \quad (4.3a)$$

but not greater than $2/3 \times$ floor width

where

- C_j = 2.0 for joists or beams in most areas
- = 1.0 for joists or beams parallel to an interior edge
- D_s = transformed slab moment of inertia per unit width
- = $d_c^3 / (12n)$ mm³ or $12d_c^3 / (12n)$ in⁴/ft
- d_c = effective depth of the concrete slab, usually taken as

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Table 4.1 Recommended Values of Parameters in Equation (4.1) and a_o / g Limits			
	Constant Force P_o	Damping Ratio β	Acceleration Limit $a_o / g \times 100\%$
Offices, Residences, Churches	0.29 kN (65 lb)	0.02–0.05*	0.5%
Shopping Malls	0.29 kN (65 lb)	0.02	1.5%
Footbridges—Indoor	0.41 kN (92 lb)	0.01	1.5%
Footbridges—Outdoor	0.41 kN (92 lb)	0.01	5.0%
* 0.02 for floors with few non-structural components (ceilings, ducts, partitions, etc.) as can occur in open work areas and churches, 0.03 for floors with non-structural components and furnishings, but with only small demountable partitions, typical of many modular office areas, 0.05 for full height partitions between floors.			

the depth of the concrete above the form deck plus one-half the depth of the form deck

n = dynamic modular ratio = $E_s / 1.35E_c$

E_s = modulus of elasticity of steel

E_c = modulus of elasticity of concrete

D_j = joist or beam transformed moment of inertia per unit width

I_t / S , mm³ (in.⁴/ft)

I_t = effective moment of inertia of the tee-beam

S = joist or beam spacing

L_j = joist or beam span.

For the girder panel mode, the effective width is

$$B_g = C_g(D_j / D_g)^{1/4} L_g \quad (4.3b)$$

but not greater than $2/3 \times$ floor length

where

C_g = 1.6 for girders supporting joists connected to the girder flange (e.g. joist seats)

= 1.8 for girders supporting beams connected to the girder web

D_g = girder transformed moment of inertia per unit width

= I_g / L_j for all but edge girders

= $[2I_g / L_j]$ for edge girders

L_g = girder span.

Where beams, joists or girders are continuous over their supports and an adjacent span is greater than 0.7 times the span under consideration, the effective panel weight, W_j or W_g , can be increased by 50 percent. This liberalization also applies to rolled sections shear-connected to girder webs, but not to joists connected only at their top chord. Since continuity effects are not generally realized when girders frame directly into columns, this liberalization does not apply to such girders.

For the combined mode, the equivalent panel weight is approximated using

$$W = \frac{\Delta_j}{\Delta_j + \Delta_g} W_j + \frac{\Delta_g}{\Delta_j + \Delta_g} W_g \quad (4.4)$$

where

Δ_j and Δ_g = maximum deflections of the beam or joist and girder, respectively, due to the weight supported by the member

W_j and W_g = effective panel weights from Equation (4.2) for the beam or joist and girder panels, respectively

Composite action with the concrete deck is normally assumed when calculating Δ_j and Δ_g , provided there is sufficient shear connection between the slab/deck and the member. See Sections 3.2, 3.4 and 3.5 for more details.

If the girder span, L_g , is less than the joist panel width, B_j , the combined mode is restricted and the system is effectively stiffened. This can be accounted for by reducing the deflection, Δ_g , used in Equation (4.4) to

$$\Delta_g' = \frac{L_g}{B_j} (\Delta_g) \quad (4-5)$$

where L_g / B_j is taken as not less than 0.5 nor greater than 1.0 for calculation purposes, i.e. $0.5 \leq L_g / B_j \leq 1.0$.

If the beam or joist span is less than one-half the girder span, the beam or joist panel mode and the combined mode should be checked separately.

Damping

The damping associated with floor systems depends primarily on non-structural components, furnishings, and occupants. Table 4.1 recommends values of the modal damping ratio, β . Recommended modal damping ratios range from 0.01 to 0.05. The value 0.01 is suitable for footbridges or floors with

no non-structural components or furnishings and few occupants. The value 0.02 is suitable for floors with very few non-structural components or furnishings, such as floors found in shopping malls, open work areas or churches. The value 0.03 is suitable for floors with non-structural components and furnishings, but with only small demountable partitions, typical of many modular office areas. The value 0.05 is suitable for offices and residences with full-height room partitions between floors. These recommended modal damping ratios are approximately half the damping ratios recommended in previous criteria (Murray 1991, CSA S16.1-M89) because modal damping excludes vibration transmission, whereas dispersion effects, due to vibration transmission are included in earlier heel drop test data.

Floor Stiffness

For floor systems having a natural frequency greater than 9-10 Hz., the floor should have a minimum stiffness under a concentrated force of 1 kN per mm (5.7 kips per in.). The following procedure is recommended for calculating the stiffness of a floor. The deflection of the joist panel under concentrated force, Δ_{jp} , is first estimated using

$$\Delta_{jp} = \frac{\Delta_{oj}}{N_{eff}} \quad (4.6)$$

where

Δ_{oj} = the static deflection of a single, simply supported, tee-beam due to a 1 kN (0.225 kips) concentrated force calculated using the same effective moment of inertia as was used for the frequency calculation

N_{eff} = number of effective beams or joists. The concentrated load is to be placed so as to produce the maximum possible deflection of the tee-beam. The effective number of tee-beams can be estimated from

$$N_{eff} = 0.49 + 34.2 \frac{d_e}{S} + (9.0 \times 10^{-9}) \frac{L_j^4}{I_t} - 0.00059 \left(\frac{L_j}{S} \right)^2 \geq 1.0 \quad (4.7)$$

for

$$0.018 \leq \frac{d_e}{S} \leq 0.208$$

$$4.5 \times 10^6 \leq \frac{L_j^4}{I_t} \leq 257 \times 10^6$$

$$2 \leq \frac{L_j}{S} \leq 30$$

where

d_e = effective slab depth,
 S = joist or beam spacing,
 L_j = joist or beam span, and
 I_t = transformed moment of inertia of the tee-beam.

Equation (4.7) was developed by Kittennan and Murray (1994) and replaces two traditionally used equations, one developed for open web joist supported floor systems and the other for hot-rolled beam supported floor systems; see Murray (1991).

The total floor deflection, Δ_P , is then estimated using

$$\Delta_P = \Delta_{jp} + \Delta_{gp} / 2 \quad (4.8)$$

where

Δ_{gp} = maximum deflection of the more flexible girder due to a 1 kN (0.225 kips) concentrated load, P_1 , using the same effective moment of inertia as used in the frequency calculation.

The deflections Δ_{oj} and Δ_{gp} are usually estimated using

$$\Delta = \frac{P_1 L^3}{48 E I_t} \quad (4.9)$$

which assumes simple span conditions. To account for rotational restraint provided by beam and girder web framing connections, the coefficient 1/48 may be reduced to 1/96, which is the geometric mean of 1/48 (for simple span beams) and 1/192 (for beams with built-in ends). This reduction is commonly used when evaluating floors for sensitive equipment use, but is not generally used when evaluating floors for human comfort.

4.3 Application Of Criterion

General

Application of the criterion requires careful consideration by the structural engineer. For example, the acceleration limit for outdoor footbridges is meant for traffic and not for quiet areas like crossovers in hotel or office building atria.

Designers of footbridges are cautioned to pay particular attention to the location of the concrete slab relative to the beam height. The concrete slab may be located between the beams (because of clearance considerations); then the footbridge will vibrate at a much lower frequency and at a larger amplitude because of the reduced transformed moment of inertia.

As shown in Figure 4.1, an open web joist is typically supported at the ends by a seat on the girder flange and the bottom chord is not connected to the girders. This support detail provides much less flexural continuity than shear connected beams, reducing both the lateral stiffness of the girder panel and the participation of the mass of adjacent bays in resisting walker-induced vibration. These effects are accounted for as follows:

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1. The reduced lateral stiffness requires that the coefficient 1.8 in Equation (4.3b) be reduced to 1.6 when joist seats are present.
2. The non-participation of mass in adjacent bays means that an increase in effective joist panel weight should not be considered, that is, the 50 percent increase in panel weight, as recommended for shear-connected beam-to-girder or column connections should not be used.

Also, the separation of the girder from the concrete slab results in partial composite action and the moment of inertia of girders supporting joist seats should therefore be determined using the procedure in Section 3.6.

Unequal Joist Spans

For the common situation where the girder stiffnesses or effective girder panel weights in a bay are different, the following modifications to the basic design procedure are necessary.

1. The combined mode frequency should be determined using the more flexible girder, i.e. the girder with the greater value of Δ_g or lowest f_g .
2. The effective girder panel width should be determined using the average span length of the joists supported by the more flexible girder, i.e., the average joist span length is substituted for L_j when determining D_g .
3. In some instances, calculations may be required for both girders to determine the critical case.

Interior Floor Edges

Interior floor edges, as in mezzanine areas or atria, require special consideration because of the reduced effective mass due to the free edge. Where the edge member is a joist or beam, a practical solution is to stiffen the edge by adding another joist or beam, or by choosing an edge beam with

moment of inertia 50 percent greater than for the interior beams. If the edge joist or beam is not stiffened, the estimation of natural frequency, f_n , and effective panel weight, W , should be based on the general procedure with the coefficient C_j in Equation (4.3a) taken as 1.0. Where the edge member is a girder, the estimation of natural frequency, f_n , and effective panel weight, W , should be based on the general procedure, except that the girder panel width, B_g , should be taken as $\frac{2}{3}$ of the supported beam or joist span. See Examples 4.9 and 4.10.

Experience so far has shown that exterior floor edges of buildings do not require special consideration as do interior floor edges. Reasons for this include stiffening due to exterior cladding and walkways generally not being adjacent to exterior walls. If these conditions do not exist, the exterior floor edges should be given special consideration.

Vibration Transmission

Occasionally, a floor system will be judged particularly annoying because of vibration transmission transverse to the supporting joists. In these situations, when the floor is impacted at one location there is a perception that a "wave" moves from the impact location in a direction transverse to the supporting joists. The phenomenon is described in more detail in Section 7.2. The recommended criterion does not address this phenomenon, but a small change in the structural system will eliminate the problem. If one beam or joist stiffness or spacing is changed periodically, say by 50 percent in every third bay, the "wave" is interrupted at that location and floor motion is much less annoying. Fixed partitions, of course, achieve the same result.

Summary

Figure 4.2 is a summary of the procedure for assessing typical building floors for walking vibrations.

4.4 Example Calculations

The following examples are presented first in the SI system of units and then repeated in the US Customary (USC) system of units. Table 4.2 identifies the intent of each example.

4.4.1 Footbridge Examples

Example 4.1—SI Units

An outdoor footbridge of span 12m with pinned supports and the cross-section shown is to be evaluated for walking vibration.

Deck Properties

$$\begin{aligned}\text{Concrete: } m_c &= 2400 \text{ kg/m}^3 \\ f'_c &= 30 \text{ MPa} \\ E_c &= 24,000 \text{ MPa} \\ \text{Slab + deck weight} &= 3.6 \text{ kPa}\end{aligned}$$

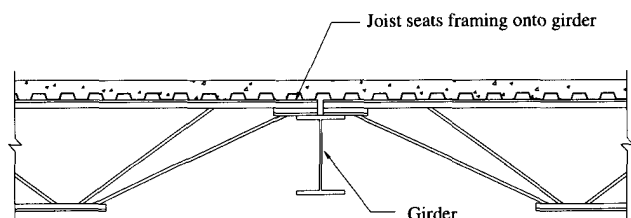


Fig. 4.1 Typical joist support.