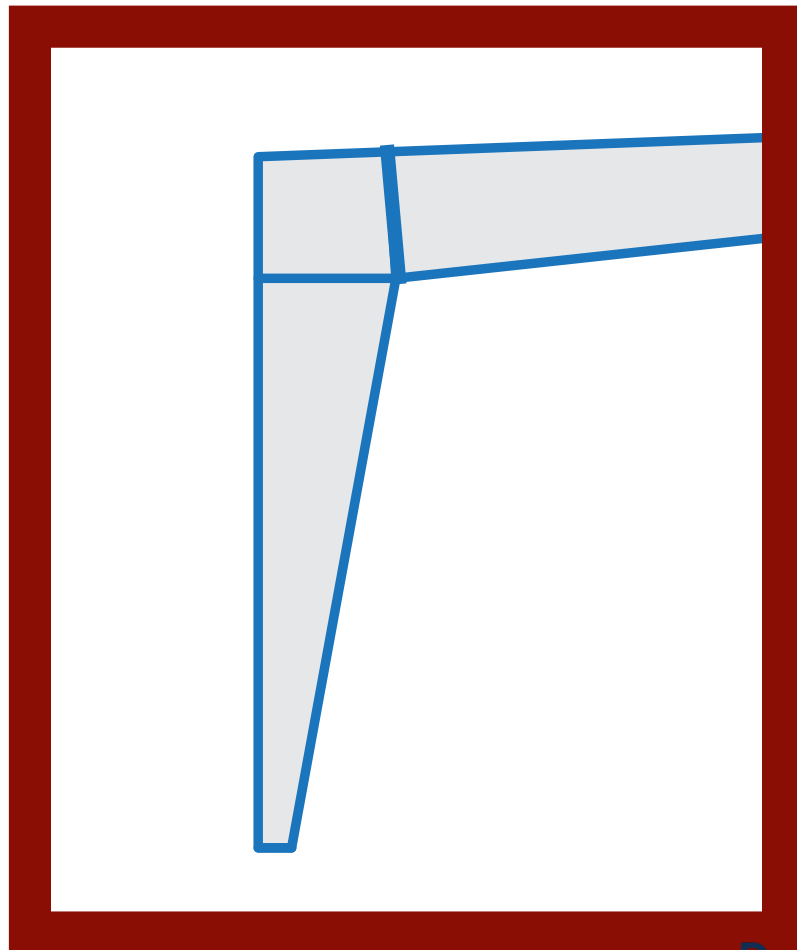




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Steel Design Guide

Frame Design Using Web-Tapered Members



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Preface

This design guide is based on the 2005 AISC *Specification for Structural Steel Buildings*. It provides guidance in the application of the provisions of the *Specification* to the design of web-tapered members and frames composed of web-tapered members. The recommendations of this document apply equally to the 2010 AISC *Specification for Structural Steel Buildings*, although some section and equation numbers have changed in the 2010 *Specification*.

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Chapter 1

Introduction

This document provides suggested methods for the design of web-tapered I-shaped beams and columns, as well as frames that incorporate web-tapered I-shaped beams and/or columns. Both the requirements for analysis and rules for proportioning of web-tapered framing members are addressed. The emphasis is on members and frames with proportions and bracing details commonly used in metal building systems. However, this information is equally applicable to similar tapered members used in conventional steel construction.

The methods contained herein are primarily interpretations of, and extensions to, the provisions of the 2005 *AISC Specification for Structural Steel Buildings* (AISC, 2005), hereafter referred to as the *AISC Specification*. The recommendations of this document apply equally to the 2010 *AISC Specification for Structural Steel Buildings*, although some section and equation numbers have changed in the 2010 *AISC Specification*. These recommendations are not intended to apply to structures designed using earlier editions of the *AISC Specification*.

The 2005 *AISC Specification* is a significant departure from past *AISC Specifications*, particularly the ASD Specifications, with which almost all metal buildings have been designed in the United States. Engineers and other users familiar with the previous ASD editions will find significant changes in the presentation of the *AISC Specification*, the member design provisions, and the requirements for analysis. The *AISC Specification* contains no provisions specific to tapered members.

The methods presented in this document comply with the 2005 *AISC Specification* and provide additional information needed to apply the *Specification* to tapered members. In some instances, procedures are provided for situations not addressed by the *AISC Specification*. These are noted where they occur.

The publication of the recommendations in this document is not intended to preclude the use of other methods that comply with the *AISC Specification*.

1.1 BASIS FOR RECOMMENDATIONS

The following sources were used extensively in the preparation of this document, are referenced extensively herein, and should be used in conjunction with this publication for a fuller understanding of its recommendations:

1. ANSI/AISC 360-05, *Specification for Structural Steel Buildings* (AISC, 2005) and its commentary
2. “A Prototype Application of the AISC (2005) Stability

Analysis and Design Provisions to Metal Building Structural Systems” (White and Kim, 2006)

The References and Annotated Bibliography sections of this document provide references to other publications relevant to the design of tapered members and frames composed of tapered members. Additional requirements for seismic design and detailing can be found in the ANSI/AISC 341-05, *Seismic Provisions for Structural Steel Buildings* (AISC, 2005a).

A significant research program was conducted as part of the development of this Design Guide. This research was conducted by White, Kim and others at the Georgia Institute of Technology. The focus of this work was the verification and adaptation of the *AISC Specification* provisions for tapered members and frames composed of tapered members. The researched topics included studies on the following:

1. Beam lateral-torsional buckling (LTB)
2. Column in-plane and out-of-plane flexural buckling
3. Column torsional and flexural-torsional buckling
4. Influence of local buckling on member resistances
5. Combined influence of local buckling and member yielding on overall structure stiffness and strength
6. Synthesis of approaches for calculation of second-order forces and moments in general framing systems
7. Benchmarking of second-order elastic analysis software
8. Consideration of rotational restraint at nominally simply supported column bases
9. Consideration of general end restraint effects on the LTB resistance of web-tapered members

The reader is referred to Kim and White (2006a, 2006b, 2007a, 2007b); Kim (2010); Ozgur et al. (2007); and Guney and White (2007) for a detailed presentation of research results for these topics.

1.2 LIMITATIONS

Except where otherwise noted in the text, these recommendations apply to members satisfying the following limits:

1. Specified minimum yield strength, $F_y \leq 55$ ksi.
2. Homogeneous members only (hybrid members are not

considered); i.e., $F_{yf} = F_{yw}$, where F_{yf} and F_{yw} are the flange and web minimum specified yield strengths.

3. Web taper is linear or piecewise linear.
4. Web taper angle is between 0° and 15° .
5. Thickness of each flange is greater than or equal to the web thickness.
6. Flange slenderness ratio is such that

$$\frac{b_f}{2t_f} \leq 18$$

where

b_f = flange width, in.

t_f = flange thickness, in.

7. Flange width is such that

$$b_f \geq \frac{h}{7}$$

throughout each unbraced length, L_b . Exception: if $L_b \leq 1.1 r_t \sqrt{E/F_y}$

$$b_f \geq \frac{h}{9}$$

throughout the unbraced length. In the foregoing equations,

h = web height, in.

r_t = radius of gyration of the flange in flexural compression plus one third of the web area in compression due to the application of major axis bending moment alone, calculated using the largest section depth within the length under consideration, in.

8. Web slenderness (without transverse stiffeners or with stiffeners at $a/h > 1.5$) is such that

$$\frac{h}{t_w} \leq \frac{0.40E}{F_y} \leq 260$$

where

E = modulus of elasticity, ksi

t_w = web thickness, in.

9. Web slenderness (with transverse stiffeners at $a/h \leq 1.5$) is such that

$$\frac{h}{t_w} \leq 12 \sqrt{\frac{E}{F_y}}$$

It is expected that these recommendations can be extended to homogeneous members with larger yield strengths. However, the background research for these recommendations

was focused on $F_y = 55$ ksi, because the use of larger yield strengths is not common in current practice.

In addition, it is expected that the recommendations can be extended to hybrid members. The background research for the recommendations in this Design Guide was focused on homogeneous members and the AISC *Specification* does not address hybrid members. Comprehensive provisions for flexural design of hybrid members are provided in the American Association of State Highway and Transportation Officials (AASHTO) *LRFD Bridge Design Specifications* (AASHTO, 2004, 2007).

Furthermore, it is expected that the recommendations can be applied to members with parabolic or other tapered web geometries. However, calculation of the elastic buckling resistances of these types of members is beyond the scope of this document. The general approach provided in this document also accommodates members with steps in the cross-section geometry at field splices or transitions in cross-section plate dimensions. However, the primary focus of this document is on members with linear or piecewise linear web taper.

1.3 BENEFITS OF WEB-TAPERED MEMBERS

Web-tapered members have been utilized extensively in buildings and bridges for more than 50 years.

Design Optimization—Web-tapered members can be shaped to provide maximum strength and stiffness with minimum weight. Web depths are made larger in areas with high moments, and thicker webs are used in areas of high shear. Areas with less required moment and shear strength can be made shallower and with thinner webs, respectively, saving significant amounts of material when compared with rolled shapes.

Fabrication Flexibility—Fabricators equipped to produce web-tapered members can create a wide range of optimized members from a minimal stock of different plates and coil. This can result in time and cost savings compared with the alternative of ordering or stocking an array of rolled shapes. In many cases, the savings in material can offset the increased labor involved in fabricating web-tapered members.

1.4 FABRICATION OF WEB-TAPERED MEMBERS

Web-tapered I-shaped members are fabricated by welding the inside and outside flange plates to a tapered web plate. In the metal building industry, this welding is generally performed by automated welding machines. One typical process is as follows:

1. Flanges and webs are cut to size or selected from plate, coil, or bar stock, and spliced as required to length.
2. Flanges and webs are punched as required for attachments (bracing, purlin and girt bolts, etc.).

3. Flanges are tack-welded to the web, with the web in a horizontal position.
4. With the web in the horizontal position, both flanges are simultaneously welded to the webs from the top side only, using an automated process that proceeds along the length of the member from one end to the other. Exception: welding on both sides of the web at member ends may be required for intermediate moment frames (IMF) and special moment frames (SMF) used in seismic applications.
5. End plates and stiffeners, if required, are manually welded to complete the member.

Although the thicknesses of the two flanges at any given cross section generally need not be the same, the constraints of most automated welding equipment require that the flanges be of the same width along the full length of a fabricated member. Consequently, web-tapered members in metal building construction usually have the same flange widths on the inside and outside of the members. Other welding systems, such as vertical pull-through welders and horizontal welders with blocking, permit the automated welding of cross sections with different flange widths but are not as common. The production of members with unequal flange widths therefore is usually avoided. I-shaped members with unequal flange sizes (thickness and/or width) are categorized as singly symmetric in the AISC *Specification*.

The automated equipment used by metal building manufacturers to join the flanges with the web is typically capable of welding from one side only. These flange-to-web welds must be capable of transferring the local shear flow (VQ/I) as

well as any localized concentrated loads between the webs and flanges, where V is the required shear strength, Q is the static moment of area of the flange taken about the neutral axis, and I is the moment of inertia of the full cross section. In most cases, the calculated strength requirements can be met easily with one-sided welds. In special cases, such as for IMF and SMF seismic applications, additional strength is provided where required by reinforcing the automated weld with additional manual welding on one or both sides of the web-to-flange junction.

The one-sided automated welds used in tapered member production in the metal building industry have a long history of satisfactory performance. Two-sided welds are not required unless the calculated required weld strength exceeds the strength of a one-sided weld. Research by Chen et al. (2001) shows that one-sided welds are acceptable to transfer shear loads.

1.5 GENERAL NOTES ON DOCUMENT

- (1) Unless otherwise noted, references to a section or chapter are references to the sections and chapters of this Design Guide.
- (2) Extensive references to prior research and development efforts are provided in the Annotated Bibliography (Chapter 7). The Annotated Bibliography is organized chronologically under several topic areas. References cited within the other chapters of this Design Guide may be found in the Annotated Bibliography but are also included in the main reference list for the convenience of the reader.

Chapter 2

Web-Tapered Member Behavior and Design Approaches

The behavior of web-tapered members is not qualitatively different from that of prismatic members. Tapered members are subject to the same limit states as prismatic members, but adjustments in the calculation of the strengths are required for some limit states due to the continuously varying geometry.

Strength limit states involving “local” member behavior do not differ from those of prismatic members. These include the limit states of:

1. Tension yielding
2. Compression yielding
3. Tension rupture
4. Shear yielding
5. Shear rupture
6. Local buckling
7. Shear buckling of unstiffened web panels

Local member strengths for these limit states can be calculated by directly applying the provisions of the AISC *Specification* using the section properties at the point of interest on the member.

The calculation of strengths involving the overall member behavior requires adjustments to the procedures given in the AISC *Specification*. These include the limit states of:

1. In-plane buckling (strong-axis flexural column buckling)
2. Out-of-plane buckling (weak-axis flexural, torsional or flexural-torsional column buckling, as well as lateral-torsional beam buckling)
3. Strength under combined axial load and bending, where in-plane or out-of plane buckling is a controlling limit state
4. Shear buckling strength or shear tension-field strength of stiffened web panels

Strength calculations in the AISC *Specification* for these limit states are based on the assumption of constant section properties over the member unsupported lengths. When designing web-tapered members, adjustments to the procedures are needed to account for the varying section properties along the unsupported lengths. These adjustments are detailed in Chapters 4 and 5 of this Design Guide.

2.1 PREVIOUS RESEARCH

Research on stability of members of varying cross sections can be traced back to the work of Euler (Ostwald, 1910), who derived the differential equation of the deflection curve and discussed columns of various shapes, including a truncated cone or pyramid. Lagrange (1770–1773) discussed the stability of bars bounded by a surface of revolution of the second degree. Timoshenko (1936) summarized various analytical and energy method solutions for the elastic buckling of nonprismatic columns, and cited related work as early as Bairstow and Stedman (1914) and Dinnik (1914, 1916, 1929, 1932). He also discussed a powerful procedure called the method of successive approximations, which makes it possible to estimate buckling loads along with upper and lower bounds for any variation of the geometry and/or axial loading along a member length. Timoshenko demonstrated a graphical application of the method of successive approximations to a simply supported column with a stepped cross section subjected to a constant axial load.

Bleich (1952) provided analytical solutions for the elastic buckling of simply supported columns with linear and parabolically varying depths between their “chords.” Furthermore, he provided an overview of the method of successive approximations in his Sections 27 and 28 (Bleich, 1952, pp. 81–91), including a proof of its convergence. In addition, Bleich provided detailed discussions of numerical solution procedures utilized with the method of successive approximations for column flexural buckling and thin-walled open section beam lateral-torsional buckling problems. These developments were based largely on the research by Newmark (1943) as well as by Salvadori (1951).

Timoshenko and Gere (1961) retained the solutions presented in Timoshenko’s earlier work (Timoshenko, 1936) and added a numerical solution for Timoshenko’s original stepped column demonstration of the method of successive approximations (see Timoshenko, 1936, pp. 116–125). Timoshenko and Gere attributed the specific numerical implementation details they presented to Newmark (1943), and referenced Newmark for more extensive discussions and additional applications. More recent discussions of the method of successive approximations are provided by Chen and Lui (1987) in their Section 6.7, and by Bazant and Cedolin (1991) in their Section 5.8. Timoshenko and Gere (1961) also discussed the calculation of inelastic strengths of bars with variable cross section using column curves based on the tangent modulus, E_t , at the cross section with the maximum compressive stress.

In 1966, the Column Research Council (CRC) and the Welding Research Council (WRC) initiated the first concerted effort to address the complete strength behavior of metal building frames composed of tapered I-shaped members. Prior experimental studies by Butler and Anderson (1963) and Butler (1966) had addressed the elastic stability behavior of I-shaped beams tapered in both the flanges and webs, and tested as cantilevered beam-columns. Starting in 1966, researchers at the State University of New York at Buffalo worked on numerous aspects of the problem. This research concluded with the development of the provisions in AISC (1978), as well as a synthesis of these provisions, plus additional design procedures and recommendations by Lee et al. (1981).

The first set of experimental tests aimed at understanding the inelastic stability behavior of tapered I-shaped beam-columns was conducted under the technical guidance of the CRC-WRC joint task committee, and was documented by Prawel et al. (1974). These tests and other analytical studies provided the basis for an overall design approach summarized by Lee et al. (1972). These developments targeted members with linearly tapered web depths. A key characteristic of the resulting design calculations was the use of member length modification factors. The modification factors mapped the physical linearly tapered member to an equivalent prismatic member composed of the cross section at its shallower end. The modified length for the equivalent prismatic member was selected such that this hypothetical member would buckle elastically at the same applied load as the physical linearly tapered member. Length modification factors were developed by curve fitting to representative results from members with five different cross sections. For in-plane flexural buckling under constant axial load, the modification factor was denoted by the symbol, g . For out-of-plane lateral-torsional buckling (LTB) under approximately constant compression flange stress, two length modification factors were developed that paralleled the idealizations used in the AISC *Specification* two-equation approach. One modification factor, h_s , was based on considering only the St. Venant torsional stiffness, while the other, h_w , was based on considering only the warping torsion stiffness.

The equivalent column length, gL , only addressed the in-plane flexural buckling of columns with simply supported end conditions. Therefore, a second length modification factor was applied to this length to account for the rotational restraint provided at the column ends by adjacent members. Idealized rectangular frame models similar to those employed in the development of the AISC alignment charts were used to derive design charts for the corresponding effective length factors, K_γ . Both of the ideal rectangular frame alignment chart cases—sidesway inhibited and sidesway uninhibited—were addressed. The total equivalent prismatic column length was therefore taken as the product of g and

K_γ with the resulting physical tapered member length, $K_\gamma gL$. Actually, the g parameter was absorbed into the charts provided for determination of K_γ , but the two factors are shown separately here to emphasize the concepts.

Once the equivalent prismatic column length, $K_\gamma gL$, was determined, the AISC ASD equations were used to determine the column elastic or inelastic design strengths (LRFD). It is important to note that all the preceding steps were simply a means of estimating the maximum axial stress along the length of the column at incipient elastic buckling. This was followed by the mapping of this elastic buckling stress to the elastic or inelastic design stress. This last step used the same mapping of the theoretical to the design buckling resistance employed for prismatic members.

The preceding calculations only addressed the in-plane flexural buckling column resistance of linearly tapered web I-shaped members. The out-of-plane flexural buckling resistance was addressed in exactly the same way as for prismatic members, because the weak-axis moment of inertia, I_y , is nearly constant along the length for members with prismatic flanges.

The calculation of the LTB strength involved the combination of the square root of the sum of the squares of the two elastic LTB contributions (one corresponding to the St. Venant torsional resistance and one corresponding to the warping torsional resistance) to determine an estimate of the theoretical total elastic LTB stress under uniform bending and simply supported end conditions. This stress was then multiplied by an additional parameter, labeled B in AISC (1978), which increased the calculated elastic buckling stress accounting for an estimate of end restraint from adjacent unbraced segments and/or the effects of a flexural stress gradient along the tapered member length. The B parameter equations were developed by Lee et al. (1972), Morrell and Lee (1974), and Lee and Morrell (1975). The base elastic LTB stress modified by B was taken as the estimated maximum flexural stress at incipient elastic LTB of the tapered member. Similar to the column strength determination, this elastic stress was used with the AISC ASD prismatic member mapping from the theoretical elastic buckling resistance to the design LTB resistance (LRFD).

Lee et al. (1972) recommended interaction equations for checking of linearly tapered web I-shaped members for combined axial and flexural loadings that paralleled the AISC ASD beam-column strength interaction equations for prismatic I-shaped members. The only change in the interaction equations implemented in AISC (1978) was a simplification in the C_m parameter, referred to as C'_m in the AISC tapered member provisions. Lee et al. (1972) developed a relatively general C_m equation to approximate the second-order elastic amplification of the maximum major-axis bending stress in linearly tapered members at load levels corresponding to the nominal first-yield condition. The general equation accounts

for the influence of linear web taper and a linear variation of the bending moment between the member ends. The AISC (1978) C'_m equations are identical to the general C_m equation but correspond to the specific cases of single-curvature bending with equal maximum flexural stress at both ends of the member and single-curvature bending with zero moment (or flexural stress) at the smaller end.

The preceding procedures formed the primary basis for the AISC design provisions in Appendix D of the *ASD Specification for Design, Fabrication and Erection of Structural Steel for Buildings* (AISC, 1978), Appendix F, Section F4 of the *Load and Resistance Factor Design Specification for Structural Steel Buildings* (AISC, 1986), Appendix F, Section F7 of the *Specification for Structural Steel Buildings—Allowable Stress and Plastic Design* (AISC, 1989), and Appendix F, Section F3 of the *Load and Resistance Factor Design Specification for Structural Steel Buildings* (AISC, 1993, 1999).

These approaches did not account for torsional or flexural-torsional buckling limit states in tapered columns and beam-columns. The flexural-torsional buckling limit state can be of particular importance for tapered members with unequal flange areas. Lee and Hsu (1981) addressed this design requirement by providing an alternative beam-column strength interaction equation that estimated the flexural-torsional buckling resistance of tapered members subjected to combined bending and axial compression, and charts that provided a coefficient required in the alternative beam-column strength interaction equation. These charts were included in Lee et al. (1981) but were never formally adopted within any of the AISC *Specification* provisions.

Furthermore, these approaches did not address the in-plane stability design of I-shaped members consisting of two or more linearly tapered segments. These types of members are used commonly for roof girders or rafters in metal building frames. Lee et al. (1979) developed another extensive set of design charts that permitted the calculation of (1) the equivalent pinned-end prismatic column length for doubly symmetric, doubly tapered I-shaped members (analogous to the length gL), and (2) the *effective* equivalent prismatic column length accounting for the influence of end rotational end restraints for these members (analogous to the length $K_y gL$). The second of these calculations was based again on idealized rectangular frame models similar to those associated with the AISC alignment charts. The authors provided charts and procedures for calculation of the equivalent rotational stiffness provided by adjacent tapered members again using the concept of the equivalent length of an alternative prismatic member composed of the shallowest cross-section along the tapered member length. These charts were included in Lee et al. (1981) but were never formally adopted within any of the AISC *Specification* provisions.

The provisions within the AISC *Specifications* from AISC (1978) through AISC (1999) were limited only to I-shaped members with equal-size flanges and linearly varying web depths. This, combined with the unpopularity of design charts without underlying equations for calculation of the corresponding parameters, led to limited use of these provisions. Instead, metal building manufacturers have tended to develop their own specific mappings of the AISC prismatic member equations for design of the wide range of general nonprismatic member geometries encountered in practice, often based upon research to validate their design approaches. As a result, the AISC Committee on Specifications decided to remove the explicit consideration of nonprismatic I-shaped members entirely from the AISC *Specification* in favor of subsequent development of separate updated guidelines for these member types. It was anticipated that the subsequent developments could take significant advantage of the many advances that have been implemented for member and frame stability design in the time since the seminal work by Lee et al. (1981).

Since the culmination of the work by Lee et al. (1981), numerous other studies have been conducted to investigate various attributes of the behavior of nonprismatic I-shaped members and frames composed of these member types. Salter et al. (1980); Shiomi et al. (1983); and Shiomi and Kurata (1984) have reported on additional experimental tests of isolated doubly symmetric beam-columns with linearly tapered webs. However, these tests focused only on members with compact webs and flanges.

Practical web-tapered members produced by American manufacturers often have noncompact or slender webs and flanges. Forest and Murray (1982) tested eight full-scale clear-span gable frames with proportions representative of American design practices under the sponsorship of Star Building Systems. They provided an early assessment of the Star Building Systems design rules in place at that time, as well as the procedures recommended by Lee et al. (1981). Forest and Murray concluded, “No consistent set of design rules adequately predicted the frame strengths for all the loading combinations.” However, the Star Building Systems design rules were judged to be safe.

Jenner et al. (1985a, 1985b) tested four clear-span frames. These tests demonstrated the importance of providing sufficient panel zone thickness to maintain the stiffness of the knee joint area. Davis (1996) conducted comparisons of AISC load and resistance factor design (LRFD) (AISC, 1993) calculation procedures to the results from two other full-scale, clear-span gable frame tests conducted at Virginia Tech. Local buckling of the rafter flanges governed the design resistances as well as the experimental failure modes. The predictions of the experimental resistances were consistently conservative by a small margin.

Watwood (1985) discussed the calculation of the appropriate effective length of the rafters in an example gable frame, accounting for the rafter axial compression and its effect on the sidesway stability of the overall structure. Watwood also investigated the sensitivity of his example frame design to foundation boundary conditions and unbalanced gravity loads. He suggested an approach for design of the rafters that in essence equates the buckling load of these members to their axial force at incipient sidesway buckling of the full structure. This typically results in an effective length factor for the rafters significantly larger than one. Numerous other researchers have considered the influence of axial compression in the rafters of gable clear-span frames in the calculation of the overall sidesway buckling loads and in the design of the gable frame columns, [e.g., Lu (1965), Davies (1990), Silvestre and Camotim (2002), and White and Kim (2006)].

These results highlight an anomaly of the effective length method (ELM) for structural stability design. Members that have small axial stress at incipient buckling of the frame generally have large effective length factors (K). In some cases, these K factors are justified, while in other cases they are not. If the member is indeed participating in the governing buckling mode, a large K value is justified. If the member is largely undergoing rigid-body motion in the governing buckling mode, or if it has a relatively light axial load and is predominantly serving to restrain the buckling of other members, a large K value is sometimes not justified. The distinction between these two situations requires engineering judgment (White and Kim, 2006). In any case, the ELM procedures recommended by Lee et al. (1981) rely on the first-order elastic stiffness of the adjacent members in determining the K_γ values. Unfortunately, if the adjacent members are also subjected to significant axial compression, their effective stiffnesses can be reduced substantially. In these cases, the Lee et al. (1981) K_γ procedures in essence rely on one member to restrain the buckling of its neighbor, then turn around and rely on the neighbor to restrain the buckling of the member. Watwood (1985) shows a clear example illustrating the fallacy of this approach.

Cary and Murray (1997) developed a significant improvement upon the traditional calculation of alignment chart frame effective length factors for sway frames. Their approach built upon Lui's (1992) development of a story-stiffness-based method for prismatic member frameworks. A common useful attribute of story-stiffness-based methods is that they use the results of a first-order elastic drift analysis (usually conducted for service design lateral loadings) to quantify the overall story buckling resistance. In addition, one of the most significant attributes of these methods is the fact that they account for the influence of leaning (gravity-only) columns on the frame sidesway buckling resistance. Conversely, the traditional AISC alignment chart and the Lee et al. (1981) effective length factor methods do not

account for such influences. This attribute can be a very important factor in the proper stability design of wide modular frames having multiple bays and a large number of leaning columns. Cary and Murray (1997) did not address the potential significant degradation in the story buckling resistance due to axial compression in the beams or rafters of metal building structures. This axial compression is often negligible for modular building frames, but it can be quite significant in some clear-span gable frames, such as the frame considered by Watwood (1985). Also, these investigators did not account for the influence of different height columns. This characteristic generally needs to be addressed in modular building frames as well as in monoslope roof clear-span frames. White and Kim (2006) explain how the story-stiffness equations from the Commentary on the AISC *Specification* (AISC, 2005a) can be extended to account both for the influence of axial compression in the roof girders as well as variable column heights. EuroCode3 (CEN, 2005) provides guidance on when these approximations are appropriate for gable frames, although the origins and basis for the EuroCode3 guidelines are unknown.

White and Kim (2006) explain that all of the preceding sidesway buckling analysis developments focus on the wrong parts of the stability design problem, because the behavior of metal building frames is almost always a moment amplification (load-deflection) problem rather than a sidesway buckling (bifurcation) problem. The behavior of metal building frames is typically dominated by the moment terms. Therefore, calculation of the appropriate amplified moment from a load-deflection analysis of the structure is key, not the determination of a buckling load that is typically many times larger than the ultimate strength of the structure. The Direct Analysis Method in the AISC *Specification* allows the engineer to focus more appropriately on the most important part of the metal building frame design problem, i.e., the calculation of the amplified internal moments (or bending stresses) under relatively small axial loads (or axial stresses), and the corresponding proportioning of the structural system to resist these actions.

Metal building frame members are usually proportioned such that they encounter some yielding prior to reaching their maximum resistance. Subsequent to Lee et al. (1981), a number of other research studies have focused on evaluation of inelastic beam and beam-column resistances and frame design. Jimenez (1998, 2005, 2006) and Jimenez and Galambos (2001) conducted numerous inelastic stability studies of linearly tapered I-shaped members accounting for a nominal initial out-of-straightness, the nominal Lehigh (Galambos and Ketter, 1959) residual stress pattern commonly used in the literature for rolled wide-flange members, and assuming compact cross-section behavior (i.e., no consideration of web or flange plate slenderness effects). Jimenez showed that the AISC (1999) provisions predicted

the column inelastic buckling resistance with some minor conservatism for these types of members. Also, he observed that the inelastic LTB curve for these types of members, predicted from inelastic buckling analyses, exhibited more of a pinched or concave up shape [rather than the linear transition curve assumed for the inelastic LTB range in AISC (1999)]. In addition, he observed that very short unbraced lengths were necessary for the compact I-shaped members considered in his study to reach their plastic moment capacity. Nevertheless, it is important to note that this type of behavior has been observed as well in some inelastic buckling studies of prismatic I-shaped members. White and Jung (2008) and White and Kim (2008) show that the linear transition curve for inelastic LTB in AISC (2005) is a reasonable fit to the mean resistances from experimental test data for all types of prismatic I-shaped members and justify the AISC (2005) resistance factor $\phi_b = 0.90$.

Other researchers have suggested simpler and more intuitive ways of determining the elastic buckling resistance of I-shaped members than the equivalent prismatic member (with a modified length) approach. Polyzois and Raftoyiannis (1998) reexamined the B factor equations from AISC (1978, 1986, 1989, 1993 and 1999) and suggested changes that covered a wider range of geometry and loading cases. They questioned the use of the single modification factor, B , to account for both the stress gradient effects and the influence of LTB end restraint from adjacent segments, and they developed separate modification factors for each of these contributions to the elastic LTB resistance. In other developments, Yura and Helwig (1996) suggested a method of determining the elastic LTB resistance of linearly tapered I-shaped members based on (1) the use of the AISC (2005) C_b equations but written in terms of the compression flange stresses rather than the member moments, and (2) the use of the tapered member cross section at the middle of the segment unbraced length. Kim and White (2007a) have validated the Yura and Helwig (1996) approach and have generalized this approach to other elastic member buckling calculations.

Numerous researchers have worked on refined calculations of elastic LTB resistances for tapered I-shaped members in recent years. Andrade et al. (2005) and Boissonnade and Maquoi (2005) show that the use of prismatic beam elements for the analysis of tapered beams (i.e., subdivision of the member into a number of small prismatic element lengths) can lead to significant errors when the behavior involves torsion. Kim and White (2007a) use a three-dimensional beam finite element formulation similar to the formulations by Andrade et al. (2005) and Boissonnade and Maquoi (2005) for their elastic buckling studies. More recently, Andrade et al. (2007) provide further validations of their one-dimensional beam model for capturing elastic LTB of web-tapered cantilevers and simply-supported beams.

Kim (2010) demonstrates that the procedures presented in this design guide for calculating the LTB resistances may be applied equivalently to both tapered and prismatic I-section members. That is, given the calculation of an elastic buckling resistance and the moment gradient parameter, C_b , the physical flexural strength is effectively the same at the most highly stressed section regardless of whether the member is tapered or prismatic. Kim (2010) also addresses the fact that virtual test simulation studies by refined full-nonlinear finite element analysis typically lead to smaller nominal strength estimates than obtained by analysis of experimental test data. These differences appear to be largely due to the geometric imperfections and internal residual stresses being smaller on average in the physical tests compared to common deterministic values assumed in virtual simulation studies. The nominal flexural strengths calculated using the AISC *Specification* and this Design Guide essentially give the mean of the resistances from experimental tests (White and Jung, 2008; White and Kim, 2008; Kim, 2010)

Davies and Brown (1996), King (2001a, 2001b), and Silvestre and Camotim (2002) have presented substantial information about the overall design of gable frame systems, including clear-span frames and multiple-span gable frames with moment continuity throughout and lightweight interior columns. Much of their discussions are oriented toward European practices and design standards, including plastic analysis and design of single-story gable frames using compact rolled I-shaped members with haunches at the frame knees. However, these studies also provide useful insights that are of value to American practices, which typically involve welded I-shapes with thinner web and flange plates.

There are numerous other prior efforts that deserve mention, but due to the abbreviated scope of this section are not referenced herein. See Chapter 7 for an extensive annotated bibliography on the stability design of frames composed of tapered and general nonprismatic I-shaped members.

2.2 RELATIONSHIP TO PRIOR AISC PROVISIONS FOR WEB-TAPERED MEMBERS

The member resistance provisions provided in this Design Guide differ somewhat from the Appendix F provisions of AISC (1989). Nevertheless, the fundamental concepts are largely the same. The primary differences between the current provisions and those in AISC (1989) are as follows:

1. The prior AISC (1989) provisions required the flanges to be of equal and constant area. The recommended provisions apply generally to cases such as singly symmetric members and unbraced segments having cross-section transitions.
2. The prior AISC (1989) provisions required the depth to vary linearly between the ends of the unbraced lengths. The recommended provisions apply to all

cases within the scope of this document, including unbraced lengths having cross-section transitions and/or multiple tapered segments.

3. The recommended resistance provisions define a mapping of the beam and column resistances from a theoretical elastic buckling value to an elastic or inelastic resistance using the AISC (2005) beam and column resistance equations as a base. The Appendix F provisions of AISC (1989) define a similar mapping to the design resistances, but use the AISC (1989) beam and column equations. The AISC (2005) design resistance equations provide improved simplicity and accuracy for the base prismatic member cases compared to the AISC (1989) equations (White and Chang, 2007).
4. The prior AISC (1989) column resistance equations for tapered members were based on the calculation of an equivalent elastic effective length factor, $K_y g$. The effective length, $K_y g L$, was the length at which an equivalent prismatic member composed of the smallest cross section would buckle elastically at the same constant axial load as in the actual tapered column of length L . As noted in Section 2.1, the separate g parameter, which gives the equivalent length for simply supported end conditions, was actually absorbed into charts for determination of the rotational end restraint effects. Therefore, AISC (1989) shows just one factor, labeled as K_y [i.e., K_y in AISC (1989) is the same as $K_y g$ in this discussion]. The length $K_y g L$ was used in the AISC (1989) equations to accomplish the preceding mapping from the theoretical elastic buckling stress to the column buckling resistance, expressed in terms of the allowable axial stress. The AISC (1989) column buckling resistance corresponded specifically to the axial stress state at the smallest cross section.

The recommended provisions focus directly on the calculation of the controlling elastic buckling load (or stress) ratio,

$$\gamma_e = \frac{P_e}{P_r} = \frac{F_e}{f_r} \quad (2.2-1)$$

where

F_e = corresponding axial stress at the most highly stressed cross section (the smallest cross section if the axial force is constant along the member length), ksi

P_e = smallest member axial force at flexural buckling about the major- or minor-axis of bending, torsional buckling, or flexural-torsional buckling, kips

P_r = member required axial load resistance, kips

$f_r = P_r/A_g$ at the most highly stressed cross section, ksi

A_g = gross area of member, in.²

The calculation of γ_e , which is the same for all cross sections along the member length (because it is an overall member buckling load ratio), is more easily generalized to address all potential column buckling limit states for all types of member geometries than the equivalent length procedures of AISC (1989). Also, it accommodates all three of the overall stability analysis-and-design approaches in AISC (2005), i.e., the Direct Analysis Method, the Effective Length Method and the First-Order Analysis Method. Simplified procedures are provided in this design guide for calculation of γ_e . Furthermore, the ratio $\gamma_e = P_e/P_r = F_e/f_r$ can be obtained directly from general buckling analysis methods. Nevertheless, both the prior calculation of $K_y g L$ and the current calculation of γ_e focus on the same fundamental question: what is the elastic buckling load (or stress) for the unsupported length under consideration?

5. The prior AISC (1989) flexural resistance equations also focused on a modification of the tapered member length, L . The basic concept was to replace the tapered beam by an “equivalent” prismatic beam with a different length, and with a cross section identical to the one at the smaller end of the tapered beam. The equivalency condition was that both the actual tapered member and the equivalent prismatic member buckle elastically at the same flexural stress if the compression flange is subjected to uniform flexural compression. This led to two different length modifiers, labeled h_s and h_w , which were used with the ASD two-equation lateral-torsional buckling (LTB) resistance equations depending on whether the LTB resistance was dominated by the St. Venant torsion stiffness or the warping torsion stiffness. Rather than taking the elastic buckling stress as the larger of these two estimates, F_{sy} and F_{wy} , as in the AISC (1989) prismatic member provisions, AISC (1989) Appendix F used the more refined estimate of $(F_{sy}^2 + F_{wy}^2)^{0.5}$ to determine the base elastic LTB stress. A separate modifier, labeled B , was applied to this elastic buckling estimate to account for moment gradient effects and lateral restraint offered by adjacent unbraced segments. Finally, for $B(F_{sy}^2 + F_{wy}^2)^{0.5} > F_y/3$, the AISC (1989) flexural resistance equations mapped the above elastic buckling stress estimate, $B(F_{sy}^2 + F_{wy}^2)^{0.5}$, to an inelastic LTB design resistance using the prismatic member equations [for $B(F_{sy}^2 + F_{wy}^2)^{0.5} \leq F_y/3$, the design LTB resistance was taken the same as the theoretical elastic LTB resistance]. The maximum flexural stress within

the unbraced segment was then compared against this design LTB resistance.

In contrast, the recommended LTB resistance provisions focus on the calculation of (1) the buckling load ratio $(\gamma_{e.LTB})_{C_b=1} = (M_{e.LTB})_{C_b=1} / M_r$ and the moment gradient modifier, C_b , or more generally the buckling load ratio, $\gamma_{e.LTB} = M_{e.LTB} / M_r$, including the moment gradient effects for the unbraced length under consideration, where $M_{e.LTB}$ is the elastic lateral-torsional buckling strength and M_r is the required flexural strength (ASD or LRFD), and (2) the calculated flexural stress state, f_r/F_y , at key locations along the length. Simplified procedures are provided for the calculation of C_b and $(\gamma_{e.LTB})_{C_b=1}$ for linearly tapered members. The parameters C_b , $(\gamma_{e.LTB})_{C_b=1}$ and f_r/F_y are then used with a form of the base AISC (2005) flexural resistance equations to accomplish a general mapping from the theoretical elastic LTB resistance to the elastic or inelastic design LTB resistance.

6. Both the prior AISC (1989) provisions as well as the recommended provisions address compression flange local buckling (FLB) on a cross section by cross section basis using the base prismatic member equations. The AISC (2005) FLB equations, on which the recommended provisions are based, give a simpler and more accurate characterization of the FLB resistance of I-shaped members (White and Chang, 2007) than the prior AISC (1989) provisions.
7. The AISC (1989) provisions restrict both the tension and the compression flange to the same allowable LTB stress. The recommended provisions specify a more rational tension flange yielding (TFY) limit for singly symmetric I-shaped members with a smaller tension flange and a larger depth of the web in flexural tension.

8. The AISC (1989) Appendix F provisions applied the base ASD prismatic beam-column strength interaction equations to assess the resistance of members subjected to combined flexure and axial force. A modified factor, labeled C'_m , was defined for two specific cases: (1) single curvature bending and approximately equal computed bending stresses at the ends; and (2) computed bending stress at the smaller end equal to zero. The recommended provisions utilize the base AISC (2005) prismatic beam-column strength interaction equations. These equations are applied to define the strength interaction for all types of beam-column geometries and all combinations of column and beam resistance limit states.
9. The prior AISC (1989) Appendix F provisions required extensive use of charts for the calculation of the in-plane column buckling resistances (i.e., for the determination of $K_y g$). The current provisions do not require the use of any charts.

The prior AISC LRFD provisions (AISC, 1999) for web-tapered members were patterned largely after AISC ASD provisions (AISC, 1989). The flexural resistance provisions were essentially identical to the latter. The column resistance provisions utilized the same $K_y g$ as in the AISC ASD provisions (AISC, 1989) but applied these parameters with the AISC LRFD column curve [which is retained as the AISC (2005) column curve]. Furthermore, the beam-column resistance was checked using the AISC LRFD (AISC, 1999) bilinear interaction curve, but with the C'_m from the AISC ASD provisions (AISC, 1989). The AISC LRFD (AISC, 1999) bilinear equations are retained as the base beam-column strength curve in AISC (2005).

The recommended provisions represent a natural progression in terms of simplification, improvement in accuracy, and improvement in breadth of applicability from the AISC ASD (AISC, 1989) and the AISC LRFD (AISC, 1999) provisions.

Chapter 3

Design Basis

The primary basis for the following design recommendations is the *AISC Specification*. In cases where supplemental recommendations are given to account for the unique nature of web-tapered members, these procedures conform to the intent of the *AISC Specification*. Users are cautioned against selecting individual provisions and incorporating them into their current design methods based on earlier *AISC Specifications*.

Structures may be designed using the *AISC Specification* using either allowable strength design (ASD) or load and resistance factor design (LRFD). The *Specification* voices no preference, so the choice can be made by the designer on the basis of personal preference. Designs produced by ASD and LRFD may differ slightly, but both are acceptable according to the *AISC Specification* and the building codes that reference the *AISC Specification*.

The LRFD procedure is intended to provide a mathematically predictable level of reliability, i.e., a known probability that the strength of the structure will exceed the demands imposed upon it over its lifetime. The safety factors used in ASD have been derived from LRFD to provide a similar level of safety and reliability.

3.1 KEY TERMINOLOGY

The five following terms are used throughout the *AISC Specification* and this document:

1. *Required strength* is the member (or component) force or moment that must be resisted. This usually comes from a structural analysis. The required strength for any given load combination is calculated using the appropriate ASD or LRFD load combinations. In this document, required strength is represented by the following symbols:

R_r = Generalized required strength, which applies to both ASD and LRFD. R_r is a generic term that can refer to forces or moments. The specific required forces and moments are designated by:

P_r = required axial strength using LRFD or ASD load combinations, kips

V_r = required shear strength using LRFD or ASD load combinations, kips

M_r = required flexural strength using LRFD or ASD load combinations, kip-in.

R_a = ASD required strength calculated using ASD load combinations. R_a is a generic term that can refer to forces or moments. The specific required ASD forces and moments are designated by:

P_a = required axial strength using ASD load combinations, kips

V_a = required shear strength using ASD load combinations, kips

M_a = required flexural strength using ASD load combinations, kip-in.

R_u = LRFD required strength calculated using LRFD load combinations. R_u is a generic term that can refer to forces or moments. The specific required LRFD forces and moments are designated by:

P_u = required axial strength using LRFD load combinations, kips

V_u = required shear strength using LRFD load combinations, kips

M_u = required flexural strength using LRFD load combinations, kip-in.

2. *Nominal strength* is the calculated strength without reduction by safety factors (ASD) or resistance factors (LRFD). Nominal strength is represented by the following symbols:

R_n = Generalized nominal strength. Specific nominal axial forces, shear forces and moments are designated by:

P_n = nominal axial strength, kips

V_n = nominal shear strength, kips

M_n = nominal flexural strength, kip-in.

3. *Available strength* is the generalized term for calculated strength including reductions by safety factors (ASD) or resistance factors (LRFD). Available strength refers inclusively to both allowable strength and design strength.

P_c = available axial strength (allowable strength in ASD or design strength in LRFD), kips

M_c = available flexural strength (allowable strength in ASD or design strength in LRFD), kip-in.

4. *Allowable strength* is the nominal strength divided by the safety factor (ASD),

$$\text{Allowable strength} = \frac{R_n}{\Omega}$$

5. *Design strength* is the nominal strength multiplied by the resistance factor (LRFD),

$$\text{Design strength} = \phi R_n$$

3.2 LIMIT STATES DESIGN

Although the AISC *Specification* permits design by either the ASD or LRFD methods, all designs produced using the provisions of the AISC *Specification* are limit states based. In both ASD and LRFD, required strengths are compared against available strengths calculated for each of the limit states by which the member can be governed.

The roots of the AISC *Specification* are primarily the provisions from the 1999 LRFD *Specification*, enhanced with numerous changes based on more recent research and aspects of ASD that were preferable or better for practice. Safety factors have been provided for use in ASD. The safety factors are calibrated to give essentially identical results to LRFD for each limit state when the ratio of live load to dead load is 3.0.

When the live load to dead load ratio is higher than 3.0, ASD will tend to produce a somewhat lighter design. When the live load to dead load ratio is less than 3.0, LRFD will tend to produce a lighter design. The differences between designs produced using the two methods are rather small, even when the ratio of live-to-dead load becomes extreme. A similar result occurs for other load combinations. For structures with large second-order effects, the ASD second-order analysis requirements (i.e., the second-order effects must be considered at an ultimate strength load level taken as 1.6 times the load combinations in ASD) tend to reduce or eliminate the apparent economic advantage ASD has for structures with high live load to dead load ratios.

Although the 1.6 factor used to increase ASD loads to ultimate levels is usually more conservative than the load factors used for LRFD, this value is lower than that used in previous editions of the ASD *Specifications*. In the 1989 and earlier editions, second-order amplification was handled by the term [see AISC (1989) Equation H1-1],

$$\frac{C_m}{1 - \frac{f_a}{F'_e}}$$

where

F'_e = Euler stress for a prismatic member divided by a safety factor, ksi

f_a = computed axial stress, ksi

The safety factor of 23/12 = 1.92 in the term F'_e effectively resulted in second-order amplification occurring at 1.92 times the ASD load levels.

Other than the load combinations, the safety and resistance factors, and a few details of second-order analysis, there are no significant differences between the ASD and LRFD design procedures in the AISC *Specification*.

3.2.1 LRFD Design Basis

The design basis for LRFD is formally expressed as:

$$R_u \leq \phi R_n \quad (3.2-1, \text{Spec. Eq. B3-1})$$

where

R_u = required strength computed using LRFD load combinations, kips

R_n = nominal strength of the applicable limit state, kips

ϕ = LRFD resistance factor corresponding to the limit state

Stated simply, the required strength, R_u , must be less than or equal to the design strength, ϕR_n .

3.2.2 ASD Design Basis

There is an important difference between ASD as defined in the AISC *Specification* and ASD as has been customarily practiced in the United States. In prior ASD *Specifications*, ASD was an acronym for *allowable stress design*. In past editions, the *Specification* provided maximum allowable stresses that were compared with calculated working load stresses in the member. In the AISC *Specification*, ASD is an acronym for *allowable strength design*. The *Specification* now provides maximum allowable forces and moments that are compared with required forces and moments in the member. This is the same format that has been used in the *Specification for Cold-Formed Structural Steel Members* (AISI, 1996, 2001, 2007) since 1996.

The design basis for ASD is formally expressed as:

$$R_u \leq \frac{R_n}{\Omega} \quad (3.2-2, \text{Spec. Eq. B3-2})$$

where

R_a = required strength computed using ASD load combinations, kips

R_n = nominal strength of the applicable limit state, kips

Ω = ASD safety factor corresponding to the limit state

Stated simply, the required strength, R_a , must be less than or equal to the allowable strength, R_n/Ω .

3.2.3 Allowable Stress Design

Although the AISC *Specification* provides ASD strengths in terms of forces and moments, it is possible to convert these strengths to a stress-based format for the convenience of users accustomed to working with stresses. Stress-based design holds several advantages over load-based design. These include the ability of the engineer to more readily assess the reasonableness of the allowable strengths, in most cases, and the potential for greater compatibility with the existing ASD software base. This technique has been presented in an article by Fisher (2005) and in literature distributed by AISC on the AISC website at www.aisc.org and at seminars. Although this procedure is not explicitly endorsed in the AISC *Specification*, it produces mathematically identical results to load-based ASD designs produced in accordance with the *Specification* when properly used.

Required strengths are converted to required stresses by dividing the required strength by the appropriate section property [gross area (A), section modulus (S), area of web, etc.] in the usual way. *Allowable strengths* are converted to allowable stresses by dividing the allowable strength by the same section property used to calculate the corresponding required stress. Thus, the design basis becomes:

$$\text{Required stress} \leq \text{Allowable stress} \quad (3.2-3)$$

$$\text{For axial compression force, } \frac{P_a}{A} \leq \frac{P_n}{A\Omega_c} \quad (3.2-4)$$

$$\text{For flexure, } \frac{M_a}{S} \leq \frac{M_n}{S\Omega_b} \quad (3.2-5)$$

Allowable flexural stresses computed in this manner can exceed $0.66F_y$ in cases where the nominal flexural strength approaches the plastic moment. This is particularly the case for highly singly symmetric sections, which can have a shape factor, M_p/M_y , significantly larger than 1.1, where M_p is the plastic bending moment and M_y is the yield moment.

The design calculations are mathematically equivalent to those produced by the allowable strength design procedure if the details of these conversions are handled consistently. This stress-based procedure should not be used to produce predicted strengths in excess of those calculated using forces and moments.

Chapter 4

Stability Design Requirements

The most significant and possibly the most challenging changes in the AISC *Specification* are in the area of stability design, that is, the analysis of framing systems and the application of rules for proportioning of the frame components accounting for stability effects. With a few exceptions, designers using the 1989 AISC *Specification for Structural Steel Buildings—Allowable Stress and Plastic Design* (AISC, 1989) have conducted linearly elastic structural analysis without any explicit consideration of second-order effects, geometric imperfections, residual stresses, or other nonideal conditions. Changes in the AISC *Specification* make explicit consideration of some, or all, of these factors mandatory in the analysis phase.

4.1 KEY TERMINOLOGY

The following key terms are used in the AISC *Specification* and this document.

P-Δ effect. Additional force or moment (couple) due to axial force acting through the relative transverse displacement of the member (or member segment) ends (see Figure 4-1).

P-δ effect. Additional bending moment due to axial force acting through the transverse displacement of the cross-section centroid relative to a chord between the member (or member segment) ends (see Figure 4-2). In singly symmetric web-tapered I-shaped members, and in members with steps in the cross-section geometry along their length, this transverse displacement includes both the deflections relative to the chord between the member or element ends, due to applied loads, as well as the offset of the (nonstraight)

cross-section centroidal axis from the chord. When members are subdivided into shorter-length elements in a second-order matrix analysis, the *P-δ* effects at the member level are captured partly by *P-Δ* effects at the individual member or segment level (see Figure 4-3).

Second-order analysis. Structural analysis in which the equilibrium conditions are formulated on the deformed structure. Second-order effects (both *P-δ* and *P-Δ*, unless specified otherwise) are included. First-order elastic analysis with appropriate usage of amplification factors is a second-order analysis. Other methods of second-order elastic analysis include matrix formulations based on the deformed geometry and *P-Δ* analysis procedures applied with a sufficient number of elements per member. See Chapter 6, Section 6.2, for a brief summary and assessment of different methods of second-order analysis. See Chapter 6, Section 6.2.1, for a discussion of the required number of elements per member for various types of second-order matrix analysis.

Second-order effect. Effect of loads acting on the deformed configuration of a structure; includes *P-δ* effect and *P-Δ* effect.

4.2 ASCE 7 AND IBC SEISMIC STABILITY REQUIREMENTS

Requirements for consideration of second-order effects under some conditions were introduced into the seismic provisions of the American Society of Civil Engineers' standard, ASCE 7, beginning in 1998 (ASCE, 1998) and the International Building Code (IBC) beginning in 2000 (IBC,

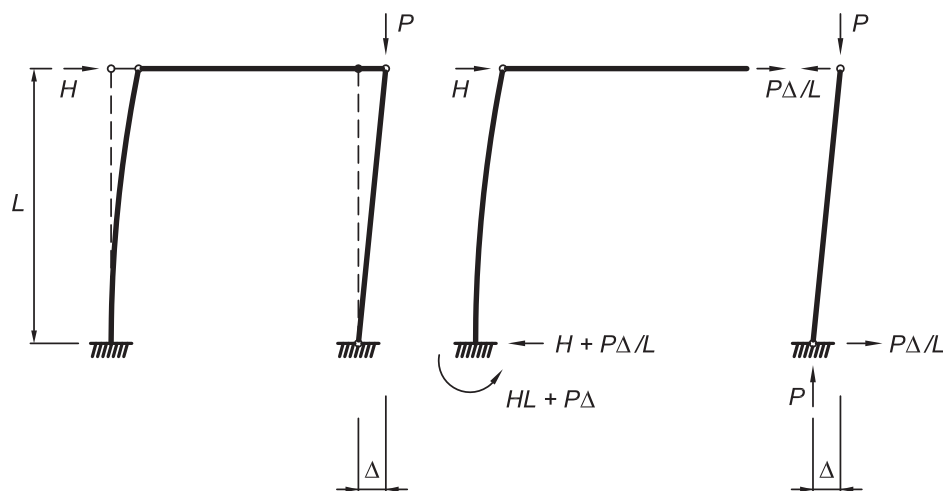


Fig. 4-1. Illustration of *P-Δ* effect.

2000). These provisions established limits on the maximum P - Δ effects and imposed second-order analysis requirements in some cases. The current provisions, summarized from ASCE/SEI 7-05 (ASCE, 2005), are as follows:

Section 12.8.7 requires the calculation of a *seismic stability coefficient*, θ , for each seismic load combination:

$$\theta = \frac{P_x \Delta}{V_x h_{sx} C_d} \quad (4.2-1, \text{ASCE/SEI 7 12.8-16})$$

where

P_x = gravity load in the combination (with a maximum load factor of 1.0), kips

$\frac{\Delta}{V_x C_d}$ = elastic sidesway flexibility of the structure under a lateral load, V_x , calculated using the nominal elastic (unreduced) structural stiffness, in./kip

h_{sx} = story height at the level being considered, in.

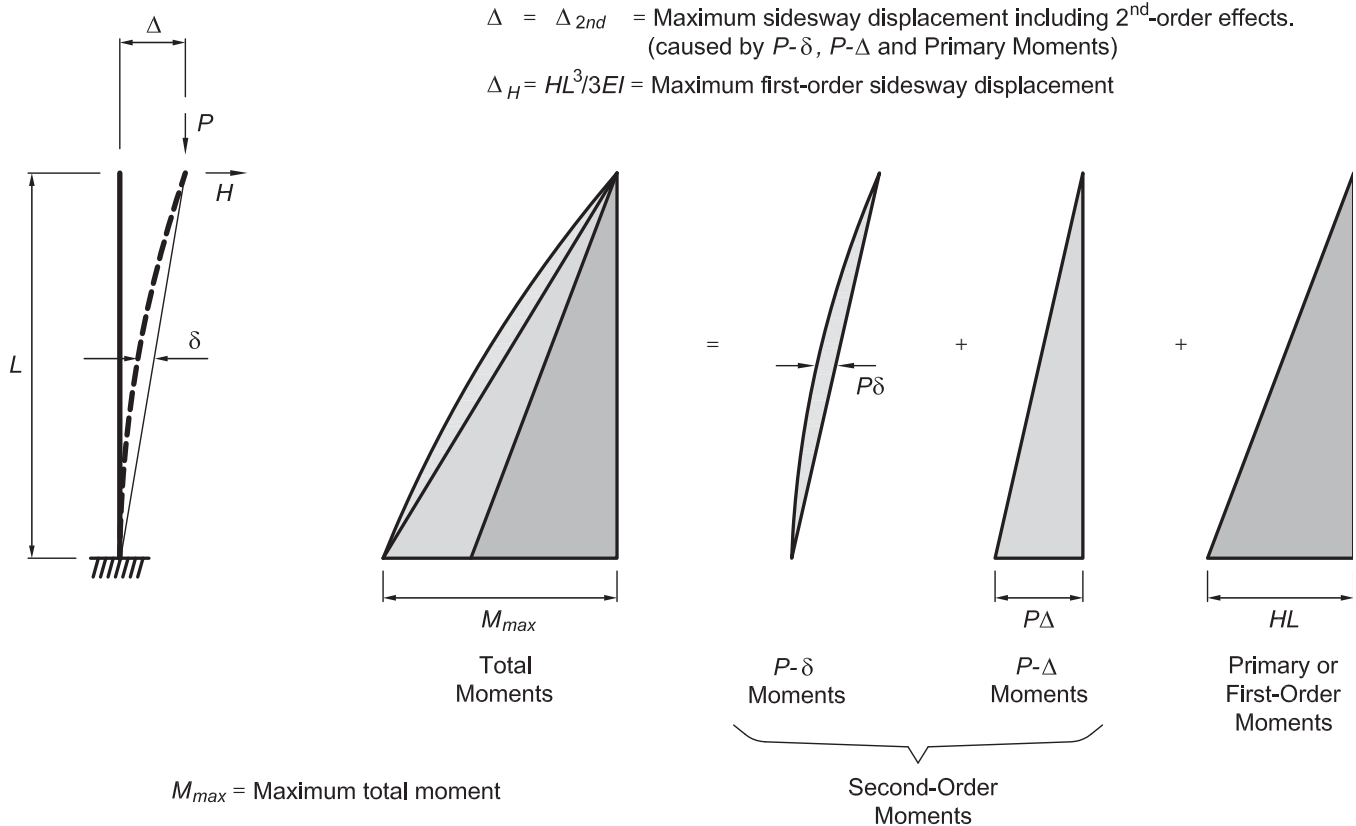


Fig. 4-2. Illustration of combined P - δ and P - Δ effects on sidesway moments and displacements.

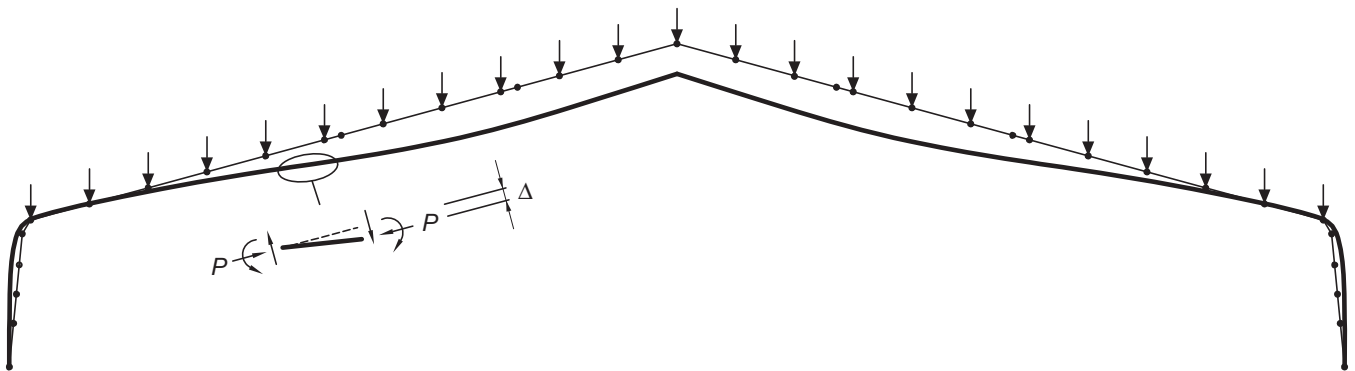


Fig. 4-3. Capture of member P - δ effects by subdivision into shorter-length elements.

Here, θ is an estimate of the ratio of the gravity load to the elastic sidesway buckling strength of the frame and is an indicator of the magnitude of the expected P - Δ effects. Structures with θ less than or equal to 0.10 have small P - Δ effects and are exempt from any ASCE 7 second-order analysis requirement. Structures with θ between 0.10 and an upper limit that can range as high as 0.25 are permitted, but must be designed using an analysis that includes P - Δ effects. Structures with θ above the upper limit of 0.25, corresponding to a P - Δ amplification of the sidesway deflections and moments of $1/(1 - 0.25) = 1.33$, are not permitted.

These provisions have been interpreted to apply only to seismic load combinations. Bachman et al. (2004) indicate that the calculation of θ need never include the roof live load or snow load except in the case of flat roof snow loads of greater than 30 psf, where 20% of the snow load is to be included unless otherwise required by the authority having jurisdiction. This usually limits P_x to a fairly small percentage of the full gravity design load. As a result, for single-story metal building frames, θ seldom exceeds the upper limit. Wide modular frames can have θ exceeding 0.10, but θ can usually be brought down to 0.10 or less by increasing the frame lateral stiffness slightly.

These provisions require consideration of significant P - Δ effects under seismic loading but do not provide any assurance of adequate second-order response under other load combinations that have much higher gravity loading. These conditions are addressed in Section 4.3.

4.3 AISC STABILITY REQUIREMENTS

Section C1 of the AISC *Specification* requires that “Stability shall be provided for the structure as a whole and for each of its elements.” Stability for the individual members of the structure is provided by compliance with the design provisions of Chapters E, F, G, H and I along with the member bracing requirements of Appendix 6. Overall stability of the structure is provided by selecting an appropriate analysis approach combined with a corresponding set of member (or component) design constraints.

Any method of design that considers the following effects is permitted by the AISC *Specification*.

1. Second-order effects
 - P - Δ effects
 - P - δ effects
2. Geometric imperfections
 - System out-of-plumbness
 - Member out-of-straightness
3. Member stiffness reductions due to residual stress
4. Member flexural, shear and axial deformations
5. Connection flexibility

The second-order effects required for the design calculations are those from the geometric nonlinearity of the elastic structure. In essence, this means that equilibrium must be considered in the deflected elastic configuration of the structure rather than in the initial geometry, as is the case for first-order elastic analysis. A wide variety of approaches for handling elastic geometric nonlinearity are available in commercial and in-house software, some of which are discussed in Chapter 6. Various approximate hand methods are also available and are satisfactory in certain cases.

Overall geometric imperfections in a frame can be handled in the preceding elastic analysis in two ways. The most intuitively obvious approach is to incorporate the maximum expected or permitted out-of-plumbness of the structure in the initial modeling of the geometry of the structure. An alternative approach is to include notional loads, which are lateral loads calibrated to produce the same sidesway as the expected out-of-plumbness. Member out-of-straightness has traditionally been handled in the column strength curves but can alternatively be handled by explicit modeling of out-of-straightness between member ends, if preferred. For members and frames subjected predominantly to in-plane bending, the geometric imperfections represented by explicit modeling or notional loads are those in the plane of the member and/or frame.

The effect of member stiffness reduction due to residual stress has traditionally been incorporated in the column strength equations in conjunction with the use of member effective lengths, rather than being considered directly in the analysis. This approach is still permitted in the Effective Length Method. However, it is now possible to consider this effect directly in the analysis. This is the approach taken in the direct analysis method and the first-order analysis method procedures outlined later, which do not require calculation of effective length factors.

The calculation of axial and flexural deformations is a basic component of the direct stiffness approach used in most modern elastic frame analysis software. Shear deformations are not often included in the analysis because their influence on the results is usually small, and therefore, the extra required calculations are not justified. For cases in which shear deformations are significant, they are an option to include in most general analysis programs and can be incorporated into in-house software.

Connection flexibility is routinely handled in elastic analysis software for cases in which the connections are fully restrained (FR) moment connections or simple shear connections by specifying ideally rigid or ideally pinned connections, respectively. For prismatic members, the AISC *Specification* Commentary (AISC, 2005) suggests that a connection with a rotational secant spring stiffness of at least $20EI/L$ at full-service loads can be considered rigid and one with a stiffness below $2EI/L$ can be considered pinned with

respect to stiffness. However, connection strength must also be considered when evaluating whether connections may be considered as ideally rigid or ideally pinned.

Bjorhovde, Colson and Brozzetti (1990) propose a connection classification system that may be interpreted as follows: Connections may be considered rigid when they have a secant rotational stiffness greater than $0.5EI/d$ at $0.7M_p$ of the connecting member, where d is the member depth. Connections with a secant rotational stiffness less than $0.1EI/d$ at $0.2M_p$ of the connecting member should be considered as pinned. Although the Bjorhovde et al. (1990) system was originally developed with prismatic members in mind, it may be applied as an approximate classification approach in frames composed of web-tapered members, using d as the depth of the member at the connection. Connections with stiffnesses between these limits are classified as partially restrained (PR). Inclusion of PR connection stiffness and strength in the analysis is required by Section B3.6b of the *AISC Specification*. Including PR spring stiffnesses in off-the-shelf or in-house software is technically straightforward, but is complicated by shakedown behavior in PR connections and the fact that connections cannot be designed until after the members are selected. A number of commercial software programs currently allow the engineer to also address connection strength by defining the connection's full moment-rotation response. Section B3.6 of the *Specification* also requires that the ductility of simple and PR connections be checked.

4.4 STABILITY DESIGN METHODS

The *AISC Specification* provides three stability design methods that account for items 1 through 3 in Section 4.3. In the following discussions these methods are referred to as:

1. The Effective Length Method (ELM), referred to as “design by second-order analysis” in Section C2.2a of the *AISC Specification*.
2. The Direct Analysis Method (DM) in Appendix 7 of the *AISC Specification*.
3. The First-Order Analysis Method (FOM), referred to as “design by first-order analysis” in Section C2.2b of the *AISC Specification*.

Each of the methods holds certain advantages. The *AISC Specification* also permits the use of any other design method that accounts for all of the elements listed in Section 4.3; however, selecting from the three methods included in the *AISC Specification* is the most practical approach for most engineers.

The primary advantage of the ELM is that experienced steel engineers will already be familiar with many of its

elements. The DM holds the advantages that (1) it may be used for all structures and load combinations, (2) it provides the most accurate assessment of internal forces and moments, and (3) columns may be designed without calculation of K factors. The virtues of the FOM are that (1) it permits design without a second-order analysis (an assumed second-order amplification is implicit in this method), and (2) it permits the design of columns without the calculation of K factors.

Theoretical details of the differences between the methods are covered in the *AISC Specification* Commentary and numerous research papers (Maleck and White, 2003; Deierlein, 2003, 2004; Kuchenbecker et al., 2004; Surovek-Maleck and White, 2004a, 2004b; Nair, 2005; Martinez-Garcia and Ziemian, 2006; White et al., 2007a; and White et al., 2007b) and are not discussed in this document. From an implementation viewpoint, the differences between the methods are in the areas of:

1. Limits on structural characteristics that establish the applicability of the methods
2. The type of structural analysis to be employed (first-order or second-order)
3. The method of accounting for nominal out-of-straightness and out-of-plumbness (use of notional loads or explicit modeling of imperfections in the analysis, or implicit inclusion in column strength equations via effective lengths)
4. The method for considering stiffness reduction from residual stress effects (directly in the analysis or implicitly in column strength equations via effective lengths)
5. Corresponding design constraints

The three methods differ (in analysis details, notional loads, and stiffness reductions, for example) and result in somewhat different required strengths for use in design. In general, for structures with significant second-order effects, the DM and the FOM will generate larger and more realistic sidesway moments than those determined using the ELM. On the other hand, the calculated in-plane column available strengths are larger and more easily determined using the DM and FOM. This is because these methods increase the member required flexural strengths, M_r , rather than reduce the member required axial strengths, P_c , to account for sidesway instability effects. Conversely, the ELM accounts for sidesway stability effects by reducing P_c via the use of $K > 1$ in moment frames, where K is the effective length factor, or by explicit use of buckling analyses to determine the theoretical column buckling loads.

The following sections provide an overview of the major implementation differences between the three methods.

4.4.1 Limits of Applicability

The DM is permitted for all structures and all load combinations. The usage of the other two methods is restricted to load combinations in which $\Delta_{2nd}/\Delta_{1st} \leq 1.5$, where Δ_{2nd} is the second-order drift and Δ_{1st} is the first-order drift for the strength combination being considered (the ASD load combinations multiplied by 1.6 or the LRFD load combinations). The limit of $\Delta_{2nd}/\Delta_{1st} \leq 1.5$ is applicable to an analysis conducted using unreduced stiffness, EA and EI . If reduced member stiffness is used for the analysis, as discussed in Section 4.4.4, this limit is $\Delta_{2nd}/\Delta_{1st} \leq 1.71$. Clear span frames often meet this restriction for all load combinations, but wide modular frames will often exceed this limit under the maximum gravity load combinations. As a result, the DM is the only method suitable for some load combinations of many metal building frames unless the designer is willing to limit $\Delta_{2nd}/\Delta_{1st}$ to no more than 1.5 for every load combination.

4.4.2 Type of Analysis

Both the ELM and the DM require that a second-order analysis be performed. As the name implies, the FOM does not require a second-order analysis. It provides a second-order amplification indirectly through the use of larger notional loads. The detailed requirements for the second-order analysis calculations required by the ELM and DM differ somewhat and are covered later in the sections that describe each method in detail.

4.4.3 Out-of-Plumbness

Each of the three methods requires the application of notional loads, or explicit modeling of the out-of-plumbness on which the notional loads are based, for at least some load combinations in the structural analysis. Notional loads are artificial lateral loads applied to the structure to account for geometric imperfections and other nonideal conditions that can induce or increase the sway of a structure.

Notional Loads

In all the methods of the AISC *Specification*, the equations for calculating notional loads are based on an assumed uniform out-of-plumbness of $L/500$. However, Appendix 7, Section 7.3(2), of the *Specification* states that this can be adjusted by the ratio of the expected out-of-plumbness to $L/500$ to account for a lesser assumed out-of-plumbness. Prior to the 2007 edition, Section 6.8 of Common Industry Standards in the *Metal Building Systems* (MBMA, 2002) specified an out-of-plumbness erection tolerance of $L/300$. Structures to be built to this standard should have their notional loads or nominal out-of-plumbness increased by the factor of $(L/300)/(L/500) = 1.67$ relative to the specified AISC value to account for this more liberal tolerance. The latest edition

of the *Metal Building Systems Manual* (MBMA, 2007) has eliminated this exception; therefore, structures to be built to the 2007 MBMA standard should be designed using the notional loads specified in the AISC *Specification*.

Notional loads are calculated for each load combination as a percentage of the vertical load acting at each level for that load combination. Although the text of the AISC *Specification* defines notional loads as a percentage of the *gravity* load, the notional loads are more properly defined as a percentage of the *vertical* load, regardless of the source. This is apparent based on the direct correspondence between notional loads and the alternative explicit modeling of out-of-plumbness.

The physical out-of-plumb imperfections in the structure may be in either sidesway direction. However, the direction of application of the notional loads for each load combination is selected to increase the overall destabilizing effect for that combination. For gravity-only load combinations that cause a net (i.e., weighted average) sidesway either due to nonsymmetry of loads or geometry, the notional loads should be applied in the direction that increases the net sidesway. For structures with multiple stories or levels, and in which the sidesway deformations are in different directions in different stories or levels, it is necessary to include a pair of load combinations, separately considering the notional loads associated with a uniform out-of-plumbness in each direction. For load combinations involving lateral loads, the notional loads should be applied only in the direction that adds to the effect of the lateral loads. One need not apply notional loads in a direction opposite from the total lateral loads to minimize the reduction in internal forces in certain components due to the lateral load. For gravity load combinations with no sidesway, it is necessary to include a pair of load combinations, separately considering notional loads in each direction, unless symmetry of the frame is enforced by other means.

Separate notional loads should be applied at the top of each of the columns in proportion to the vertical load transferred to each column. In columns with axial forces applied at an intermediate location along the length, a proportional notional load should be placed at that location on the column. For any instances in which questions arise about the calculation and application of notional loads, the question may be answered by determining the lateral forces that are equivalent to the effect of the intended uniform nominal out-of-plumbness.

For ASD designs, notional loads calculated based on ASD load combination factors must be increased by a factor of 1.6 in all three methods. The 1.6 factor overestimates the second-order effects somewhat in ASD designs relative to LRFD, particularly where second-order effects are significant.

Explicit Modeling of Out-of-Plumbness

The AISC *Specification* permits explicit modeling of out-of-plumbness in the structural analysis in lieu of the use of notional loads for the DM. This avoids the need to determine how to apply notional loads in buildings with sloping roofs or floors, where the building geometry is nonrectangular or nonregular, or in structures where axial loads are applied at intermediate positions along the length of a member. This approach is easy to automate in computer-based design, and it allows the designer to better understand the true nature of internal forces set up in the structure from out-of-plumbness effects. However, unless automated methods of specifying out-of-plumbness are available in analysis software, it is often easier to apply notional loads along with the other applied loads on the structure rather than to modify the structure geometry.

The modeled out-of-plumbness should be consistent with the erection tolerances specified for the structure. Therefore, if the erection tolerances are smaller than $L/500$, a reduced uniform out-of-plumbness equal to the specified erection tolerance may be employed. Also, where a larger erection tolerance is permitted, this larger tolerance should be used as the modeled uniform out-of-plumbness in the structural analysis.

The physical out-of-plumb imperfections in the structure may be in either sidesway direction. However, the direction of the modeled uniform out-of-plumbness for each load combination is selected to increase the overall destabilizing effect for that combination. For gravity-only load combinations that induce a net sidesway, the modeled out-of-plumbness should be in the direction of the net sidesway. For structures with multiple stories or levels and in which the sidesway deformations are in different directions in different stories or levels, two different uniform out-of-plumb geometries are required to capture the potential overall destabilizing effects in both directions. For load combinations involving lateral loads, the out-of-plumbness should be in the direction of the lateral loads. For a gravity load combination with no sidesway, it is necessary to consider a uniform out-of-plumbness in both directions unless any symmetry of the design is enforced by other means.

Only two different out-of-plumb geometries are typically required to cover the overall destabilizing effects for all load combinations. In contrast, the corresponding notional loads discussed in the previous section are, in general, different for each load combination (although the notional loads may be taken conservatively as the maximum values from all the load combinations).

For the ELM in all cases, and the DM in some cases, the notional loads are specified as minimum lateral loads in the gravity-only load combinations. That is, they are not used in combination with other lateral loads. Consequently,

out-of-plumbness need not be included in the model for these load combinations. Modeling out-of-plumbness for all load combinations is permitted but will result in conservative results for lateral load combinations in which notional loads are not required. In cases where the notional loads are specified as additive rather than minimum, the model must include out-of-plumbness (or out-of-plumbness effects via notional loads) regardless of the magnitude of the lateral loads.

4.4.4 Stiffness Reduction

Only the DM requires explicit consideration of member stiffness reduction due to the combined effects of residual stresses and distributed yielding with member axial forces and moments in the structural analysis. This is handled by reducing the flexural stiffness in moment frames and the axial stiffness in braced frames to 80% of their nominal elastic values in the second-order structural analysis. Where the axial compression load in a flexural member contributing to lateral stability exceeds 50% of the yield load, the flexural stiffness is further reduced. This is discussed in detail later. The other two methods consider member stiffness reductions only implicitly, either via the calculation of the column strengths using effective lengths or by the calculation of a larger notional lateral load.

4.4.5 Design Constraints

The DM and the FOM permit the design of columns for in-plane buckling using a length equal to the actual unsupported length ($K = 1$), or a smaller length in some cases. For moment frames, the ELM requires the calculation and use of elastic buckling load values (or the corresponding K factors) determined using a sidesway buckling analysis of the structure unless $\Delta_{2nd} / \Delta_{1st} \leq 1.1$. If $\Delta_{2nd} / \Delta_{1st} \leq 1.1$, the in-plane flexural buckling load may be calculated based on the actual member length and idealized pinned-pinned end conditions (i.e., $K = 1$).

4.5 COMMON ANALYSIS PARAMETERS

Several parameters are used throughout the text of the stability requirements in the AISC *Specification* to establish limits of applicability of various provisions and in other calculations. These are defined as follows:

4.5.1 αP_r

αP_r is the required axial compressive strength, P_r , multiplied by the factor α where $\alpha = 1.6$ for ASD and $\alpha = 1.0$ for LRFD.

For LRFD, αP_r is simply the required axial compressive strength determined using the LRFD load combination factors for the load combination under consideration. For ASD, αP_r is the required axial compression strength determined

using ASD load combination factors multiplied by 1.6, to give an approximation of the required strength under ultimate conditions.

αP_r is used in the determination of whether the FOM may be used. It is also used in the DM to determine whether P - δ effects can be neglected in the calculation of the sidesway displacements, and to calculate the required stiffness reduction. αP_r is also used in approximate second-order analysis techniques such as the B_1 - B_2 method.

4.5.2 P_{eL} or $\gamma_{eL}P_r$

P_{eL} is the nominal in-plane elastic flexural buckling strength of a member subjected to axial compression force and having assumed ideally pinned-pinned end conditions. This parameter is used extensively in the provisions for both the ELM and the DM as well as in the member design provisions. Consideration of the actual member rotational and sidesway restraint end conditions is handled subsequently within the AISC design procedures, via the calculation of other buckling loads or the corresponding effective length factors in the ELM, and via the modeling of the structure for the structural analysis.

In many cases it is more convenient to work with the equivalent parameter $\gamma_{eL}P_r$, which is the required strength, P_r , multiplied by the elastic buckling load ratio, $\gamma_{eL} = P_{eL}/P_r$. That is,

$$P_{eL} = \gamma_{eL}P_r \quad (4.5-1)$$

where

P_{eL} = Euler buckling load, evaluated in the plane of bending, kips. This is the internal axial force at elastic buckling of the member, assuming simply supported end conditions.

P_r = required axial strength for the column, kips

γ_{eL} = a scalar ratio

Regardless of the complexity of the loadings (e.g., stepped loading or distributed axial loading) or the member geometry (e.g., tapered and/or stepped geometry), there is only one γ_{eL} corresponding to the member elastic flexural buckling strength. However, for stepped or distributed axial loading, P_r and P_{eL} vary along the member length.

The elastic flexural buckling strength can also be expressed as:

$$F_{eL} = \gamma_{eL}F_r \quad (4.5-2)$$

where

F_{eL} = axial stress at elastic buckling of the member, assuming simply supported end conditions, ksi

F_r = required axial stress for the column, ksi

For a straight, geometrically perfect prismatic column with a constant axial loading,

$$P_{eL} = \frac{\pi^2 EI}{L^2} \quad (4.5-3)$$

For a tapered I-shaped member, there is no exact closed-form solution for P_{eL} . However, several approaches to a solution are available:

1. P_{eL} can be determined by an elastic eigenvalue buckling analysis. Many advanced finite element and/or frame analysis programs can be used to calculate elastic buckling multipliers, γ_{eL} , corresponding to a given required axial strength using numerical eigenvalue solution techniques. P_{eL} is then determined as the required axial compression strength, used in the analysis, multiplied by γ_{eL} . The quality of such solutions depends on the accuracy of tapered member modeling, element choice, and meshing. The engineer should run the benchmark problems provided in Appendix C to establish the appropriateness of the computer program and modeling techniques prior to use in design. Although this approach has the advantage of handling essentially any imaginable geometry and loading, it may not be practical in a production environment unless the finite element modeling is automated and integrated into analysis-design software.
2. P_{eL} can be determined by the method of successive approximations (Timoshenko and Gere, 1961). This technique uses an iterative beam analysis to find the axial load, $\gamma_{eL}P_r$, at which the beam deflections resulting from applied P - δ moments are a uniform multiple of the deflections assumed to calculate the P - δ moments. This is an iterative process in which (1) a load, P_r , and a deflected buckling mode shape are assumed; (2) the P - δ moments from the assumed deflections times the assumed axial load are calculated; (3) the calculated P - δ moments are applied in a beam analysis of the member to compute a new deflected shape; and (4) the new deflected shape is substituted as a new approximation for the buckled geometry. The process is continued iteratively until the calculated deflections everywhere along the beam are a uniform multiple, γ_{eL} , of the assumed deflections. P_{eL} is then determined as the assumed axial load, P_r , multiplied by γ_{eL} .

The method of successive approximations requires relatively few calculations compared with eigenvalue solution techniques, is easily programmed, and is adaptable to handle various tapers and steps in the member loading and geometry. The method is illustrated in Timoshenko and Gere (1961) with an example in a format easily adapted to a spreadsheet or procedural computer program. See Appendix C of this Guide for benchmark examples of web-tapered members.

3. P_{eL} can be approximated with good accuracy for a single linearly tapered member with simply supported conditions and supporting a constant internal axial force, and having no plate or taper changes, as:

$$P_{eL} = \frac{\pi^2 EI'}{L^2} \quad (4.5-4)$$

where

I' = moment of inertia calculated using the depth at $0.5L$ ($I_{\text{small}}/I_{\text{large}})^{0.0732}$ from the small end, in.⁴

This empirically derived expression gives results accurate to within several percentage points for the range of members addressed in this document. The preceding approximation should not be used for any buckling solution in which anything other than simply supported end conditions are assumed. That is, the preceding expression for I' is valid solely for idealized simply supported end conditions.

4. For linearly tapered members subjected to nonuniform axial compression, γ_{eL} can be calculated conservatively as $P_{eL}/(P_r)_{\text{max}}$, where P_{eL} is calculated using Equation 4.5-4 and $(P_r)_{\text{max}}$ is the largest internal axial compression along the member length.

4.5.3 $\Delta_{2nd}/\Delta_{1st}$

$\Delta_{2nd}/\Delta_{1st}$ is the ratio of story drifts calculated from a second-order and first-order analysis, respectively. This ratio is used to establish the applicability of the approved design methods, to establish the applicability of the $K = 1$ provisions of the ELM [2005 AISC *Specification* Section C2.2a(4)], to determine whether notional loads are additive to lateral loads in the DM, and also in the B_1 - B_2 method.

Unless otherwise noted, this ratio is calculated from analyses using unreduced member stiffnesses. For information on calculating $\Delta_{2nd}/\Delta_{1st}$ for gable frames, see Section 6.3.3.

$\Delta_{2nd}/\Delta_{1st}$ is calculated separately for each load combination. This parameter gives an indication of the significance of the second-order effects in a load combination. No maximum limit on $\Delta_{2nd}/\Delta_{1st}$ is established by the AISC *Specification* (AISC, 2005). Values below 1.1 are considered insignificant. Values above 1.5 are considered large second-order effects. As such, the design must be conducted using the DM when this threshold is exceeded. Values between 1.1 and 1.5 are considered moderate second-order effects. The design may be conducted either by the DM or by the ELM in these cases.

The reader is cautioned against using the ratio M_{2nd}/M_{1st} as a substitute for $\Delta_{2nd}/\Delta_{1st}$. The moments usually include significant first-order gravity components that will obscure the magnitude of the second-order effects.

4.6 DETAILED REQUIREMENTS OF THE STABILITY DESIGN METHODS

The following sections summarize the detailed requirements for each of the three stability design methods discussed earlier. Additional information on first-order and second-order frame analysis is given in Chapter 6.

4.6.1 The Effective Length Method (ELM)

1. The ELM is only permitted for load combinations where $\Delta_{2nd}/\Delta_{1st} \leq 1.5$.
2. A second-order analysis, considering both P - Δ and P - δ effects as detailed below, is required:
 - (a) The P - Δ effects on the nodal displacements must be considered. The P - δ effects on the nodal displacements may be neglected in the calculation of required strengths because the ELM beam-column unity checks are insensitive to these effects.
 - (b) The P - δ effects on the internal element moments (between the nodes) may be neglected in individual elements in load combinations when $\alpha P_r \leq 0.02P_{e\ell}$ for that element,

where

$P_{e\ell}$ = flexural column buckling load based on the cross-section geometry and the element length between the nodal locations with idealized simply supported nodal end conditions, kips

Otherwise, they must be considered.

- (c) Internal P - δ moments may be included by performing a second-order analysis to determine the nodal displacements, forces and moments, and then calculating the second-order internal moments in each element as follows (Guney and White, 2007):
 - (i) Calculate δ_{1st} , the first-order displacement perpendicular to the element chord caused by the second-order nodal forces and any applied loads within the element length, at any locations of interest.
 - (ii) Calculate the second-order displacement at each of the preceding locations as

$$\delta_{2nd} = \frac{\delta_{1st}}{1 - \alpha P_r / P_{e\ell}} \quad (4.6-1)$$

where $P_{e\ell}$ may be estimated for linearly tapered segments using P_{eL} from Equation 4.5-4, but applied to the element length, ℓ .

- (iii) Calculate the required internal second-order moment at each of the above locations as

$$M_r = M_{1st} + \alpha P_r \delta_{2nd} \quad (4.6-2)$$

where

M_{1st} = first-order moment at a given position along the element length, caused by the second-order nodal forces and any applied loads within the element length, kip-in.

This procedure provides good accuracy for general cases involving prismatic or nonprismatic member geometry for values of $\alpha P_r / P_{el} \leq 0.7$. This limit is satisfied in all cases when (1) a P - Δ only analysis or a second-order analysis using an element geometric stiffness based on element cubic transverse displacements is used, and (2) the number of elements per member is greater than or equal to that specified by the guidelines discussed subsequently in Section 6.2. Alternatively, the nonsway amplifier

$$B_1 = \frac{C_m}{1 - \alpha P_r / P_{el}} \geq 1.0 \quad (4.6-3a)$$

may be applied to all the moments M_{1st} throughout the length of a given element, except those at the ends. Equation 4.6-3a is useful for elements in linearly tapered members that do not have any applied transverse loads. In this case, the equivalent uniform moment factor, C_m , may be expressed approximately as

$$C_m = 0.6 + 0.4(f_1/f_2) \quad (4.6-3b)$$

where

f_2 = the absolute value of the largest compressive flexural stress at either element end node, ksi

$$f_1 = 2f_{mid} - f_2, \text{ ksi} \quad (4.6-3c)$$

f_{mid} = flexural stress due to M_{1st} at the mid-length of the element in the flange containing the stress f_2 , taken as positive for compression and negative for tension, ksi

Equation 4.6-3b accounts for the fact that a linear variation in M_{1st} produces a nonlinear variation in the corresponding flexural stress along the length of a tapered member. The value f_1 is the flange stress obtained by extending a line through f_2 and f_{mid} to the opposite element end node.

In many cases, Equation 4.6-3a gives $B_1 = 1.0$, indicating that the second-order amplification of the internal moments may be neglected. Equations 4.6-1 and 4.6-2 generally provide better accuracy for both prismatic and nonprismatic members compared to the amplified moments determined using Equation 4.6-3a. This is particularly true for elements with transverse applied loads, where the AISC *Specification* gives a conservative value of $C_m = 1.0$ and Table C-C2.1 in the AISC *Specification* Commentary gives refined equations for C_m that are applicable only for prismatic members with ideally pinned or ideally fixed end conditions. The use of $C_m = 1.0$ is recommended for general cases with transverse applied loads.

- (d) P - δ effects must be included in the calculation of elastic column buckling strengths, P_{el} , when using either an eigenvalue or the successive approximation approach. See Appendix B1.2 for guidance on subdividing the members into a sufficient number of smaller-length elements for matrix eigenvalue analyses.
- (e) The accuracy of any second-order analysis program used should be tested using appropriate benchmark problems such as those provided in Appendix C. Particular care should be taken to establish whether P - δ effects are correctly included in the analysis. Sections 6.2.1 and 6.2.2 provide guidelines for subdivision of members to ensure sufficient accuracy with respect to these effects.

The *amplified first-order elastic analysis approach* (e.g., the B_1 - B_2 approach) is an acceptable second-order analysis method. If this approach is employed, Equation C2-6b in the AISC *Specification* (or the more refined Commentary Equation C-C2-6 not including the limit $1.7HL/\Delta_H$) is recommended for the definition of ΣP_{e2} in Equation C2-3.

Implementation of the B_1 - B_2 may involve more work compared to other alternative approaches [e.g., a general P - Δ analysis as discussed earlier, or the alternate amplifier-based method discussed by White et al. (2007a, 2007b)]. The amplifier-based methods are particularly difficult to implement and lose accuracy for gable frames, where the sidesway column displacements are generally not the same, and for frames with unequal height columns, where the methods must be modified to account for the different column heights (White and Kim, 2006).

The reader should note that the term *amplified first-order elastic analysis* is typically used to refer to the specific B_1 - B_2 method of calculating the second-order forces and moments. It is important to distinguish this term from the terms used for the different design methods, i.e., the ELM, the DM and the FOM. The B_1 - B_2 second-order analysis method is one of many methods of second-order analysis that may be used for the calculation of the forces and moments in either of the design methods that require a second-order analysis (the ELM and the DM).

3. Given the satisfaction of the preceding requirements for the second-order elastic analysis calculations, the ELM structural analysis model must include the following attributes:

- (a) The analysis is conducted with nominal elastic stiffnesses, i.e., no member stiffness reductions.
- (b) Minimum lateral loads of 0.002 times the vertical load, Y_i , applied at each level are required for all gravity-only load combinations. For gable frames and for frames with stories having unequal height columns, it is recommended that individual notional lateral loads equal to $0.002y_i$ should be applied at the top of each column, where y_i is the vertical load transferred to the column at its top. Also, for columns with intermediate vertical loads along their length, a notional lateral load of $0.002y_i$ should be applied at the location of the intermediate vertical loads, where y_i is the intermediate vertical load applied to the column. This is necessary to capture the geometric imperfection effects on different height columns, as well as to capture the behavior in cases where the lateral displacements are generally different at the different column locations.

In lieu of applying the notional lateral loads, one can impose an out-of-plumbness of $0.002H$ on the structure for analysis of the gravity-only load combinations, where H is the vertical height above the base, or in general, the node(s) having the minimum vertical coordinate. This may be implemented by shifting all the nodes of the analysis model horizontally by $0.002H$ relative to the node(s) at the base of the structure. For cases in which questions arise about the appropriate application of the notional lateral loads, one should always return to the model where the uniform out-of-plumbness is represented explicitly in the structural model. The appropriate notional loads are the ones that are equivalent to the effect of this out-of-plumbness.

For both notional loads and explicit modeling of out-of-plumbness, the factor 0.002 is based on an assumed erection tolerance of $L/500$. For adjustments to this factor to account for structures built to different tolerances, see Section 4.4.3.

- (c) For ASD, the analysis is conducted using loads of 1.6 times those from ASD load combinations. The resulting member forces and moments are divided by 1.6 for the member design calculations. The 1.6 multiplier also applies to any notional loads added to satisfy item 3(b).
4. The in-plane flexural buckling strength of columns and beam-columns, P_{ni} , is determined as follows:
 - (a) For members in load combinations where $\Delta_{2nd}/\Delta_{1st} \leq 1.1$, calculate P_{ni} based on the actual unbraced length with $K = 1.0$, i.e., assuming idealized pinned-pinned end conditions on the actual unbraced length.
 - (b) For other cases, P_{ni} must be calculated using an effective length factor, K , or the corresponding column buckling stress, F_e , determined from a sidesway buckling analysis of the structure. Because member taper violates one of the inherent assumptions of the traditional alignment charts, more advanced methods of determining K or F_e are normally required. See Appendix B for further information on the calculation of elastic buckling strengths of tapered columns and frames.

4.6.2 The Direct Analysis Method (DM)

1. The DM is permitted for all structures and load combinations.
2. A second-order analysis with characteristics similar to those discussed in item 2 of Section 4.6.1 is required. However, there are a few important differences. The following discussion repeats much of the discussion in item 2 of Section 4.6.1 with an emphasis on the specific requirements in the context of the DM.
 - (a) Generally, both the P - Δ and P - δ effects on the nodal displacements must be considered in the DM. The AISC *Specification* Appendix 7, Section 7.3(1), indicates that if $\alpha P_r < 0.15P_{eL}$ for all members whose flexural stiffnesses are considered to contribute to the lateral stability of the structure, the P - δ effect on the lateral displacements may be neglected in the analysis. Although not defined in the AISC *Specification*, “members whose flexural stiffnesses are considered to contribute to the lateral stability” in this context is intended to apply to both beams and columns in unbraced frames. For

nonrectangular structures such as gable frames, the term “lateral displacements” may be interpreted as the general nodal displacements in the frame analysis model.

With the exception of sway columns without significant transverse member loads, where both ends have substantial rotational restraints, this Guide recommends that when $\alpha P_r > 0.05 \bar{P}_{el}$, where $\bar{P}_{el}$ is the elastic buckling load based on the overall member length determined as discussed in Section 4.5.2 but using the reduced elastic stiffness of the DM analysis model discussed later, the member should be subdivided with intermediate nodes along its length when a P - Δ only analysis is employed. This ensures better accuracy of the element nodal displacements and moments along the member length than will be achieved using the previous AISC *Specification* rule when αP_r exceeds $0.05 \bar{P}_{el}$. The sidesway moments in fixed-base columns, and columns with top and bottom rotational restraint from adjacent framing, may be analyzed sufficiently with a P - Δ only analysis and a single element per member when $\alpha P_r \leq 0.12 \bar{P}_{el}$. Second-order analysis procedures that include both P - Δ and P - δ effects in the formulation require fewer elements. Detailed guidelines for the necessary number of elements are provided subsequently in Sections 6.2.1 and 6.2.2 of this Guide. These guidelines and the above recommendations are based on Guney and White (2007). In many cases a sufficient subdivision will occur naturally with tapered members due to the frequency of plate and/or geometry changes. However, extra nodes may be required for prismatic members and long tapered members without changes of plates or taper.

- (b) The P - δ effects on the internal element moments (between the nodes) may be neglected in individual elements in load combinations when $\alpha P_r \leq 0.02 \bar{P}_{el}$ for that element (Guney and White, 2007),

where

$\bar{P}_{el}$ = flexural buckling load based on the cross-section geometry and the element length between the nodal locations with idealized simply supported nodal end conditions, determined using the reduced elastic stiffnesses of the DM analysis model discussed later, kips

- (c) Internal P - δ moments may be included by performing a second-order analysis to determine the nodal displacements, forces and moments, then calculating the second-order internal moments in

each element using the forces, moments, and displacements calculated with the reduced stiffness from the DM analysis as follows:

- (i) Calculate δ_{1st} , the first-order displacement perpendicular to the element chord caused by the second-order nodal forces and any applied loads within the element length, at any locations of interest.
- (ii) Calculate the second-order displacement at each of the preceding locations as

$$\delta_{2nd} = \frac{\delta_{1st}}{1 - \alpha P_r / \bar{P}_{el}} \quad (4.6-4)$$

where $\bar{P}_{el}$ may be estimated for linearly-tapered segments using $\bar{P}_{el}$ from Equation 4.5-4, but applied to the element length, ℓ , and using the reduced elastic stiffness of the DM analysis model.

- (iii) Calculate the required internal second-order moment at each of the above locations as

$$M_r = M_{1st} + \alpha P_r \delta_{2nd} \quad (4.6-5)$$

where

M_{1st} = the first-order moment at a given position along the element length, caused by the second-order nodal forces and any applied loads within the element length, kip-in.

This procedure provides good accuracy for general cases involving prismatic or nonprismatic member geometry for values of $\alpha P_r / \bar{P}_{el} \leq 0.7$ (Guney and White, 2007). This limit is satisfied in all cases when a sufficient number of elements is employed in a P - Δ -only analysis or a second-order analysis using an element geometric stiffness based on element cubic transverse displacements using the guidelines discussed subsequently in Section 6.2. Alternatively, the AISC (2005) nonsway amplifier

$$B_1 = \frac{C_m}{1 - \alpha P_r / \bar{P}_{el}} \geq 1.0 \quad (4.6-6a, \text{ from Spec. Eq. C2-2})$$

may be applied to all the moments, M_{1st} , throughout the length of a given element, except those at the ends. Equation 4.6-6a is useful for elements in linearly tapered members that do not have any transverse applied loads. In this case, the equivalent uniform moment factor, C_m , may be expressed approximately as

$$C_m = 0.6 + 0.4(f_1/f_2) \quad (4.6-6b)$$

where

$$\begin{aligned} f_2 &= \text{the absolute value of the largest compressive flexural stress at either element end node, ksi} \\ f_1 &= 2f_{mid} - f_2, \text{ ksi} \\ f_{mid} &= \text{flexural stress due to } M_{1st} \text{ at the mid-length of the element in the flange containing the stress } f_2, \text{ taken as positive for compression and negative for tension, ksi} \end{aligned} \quad (4.6-6c)$$

Equation 4.6-6b accounts for the fact that a linear variation in M_{1st} produces a nonlinear variation in the corresponding flexural stress along the length of a tapered member. The value f_1 is the flange stress obtained by extending a line through f_2 and f_{mid} to the opposite element end node.

In many cases, Equation 4.6-6a gives $B_1 = 1.0$, indicating that the second-order amplification of the internal moments may be neglected. Equation 4.6-4 in conjunction with Equation 4.6-5 generally provides better accuracy for both prismatic and nonprismatic members compared to the application of Equation 4.6-6a. This is particularly true for elements with transverse applied loads, where the AISC *Specification* gives a conservative value of $C_m = 1.0$ and Table C-C2.1 in the AISC Commentary gives refined equations for C_m that are applicable only for prismatic members with ideally pinned or ideally fixed end conditions. The use of $C_m = 1.0$ is recommended for general cases with transverse applied loads.

- (d) The accuracy of any second-order analysis program used should be tested using appropriate benchmark problems such as those provided in Appendix C. If the benchmark tests are satisfied, the software may be assumed to provide adequate results without subdividing the members into multiple elements as recommended in item 2(a).
3. Given the satisfaction of the preceding requirements for the second-order elastic analysis calculations, the DM analysis model must include the following attributes:
 - (a) The analysis must be conducted with elastic stiffness reductions for all members whose flexural stiffness is considered to contribute to the lateral stability of the structure. Although not defined in the AISC *Specification*, “members whose flexural stiffness is considered to contribute to the lateral

stability” in this context is intended to apply only to columns in unbraced frames. This is accomplished by reducing the value of EI and/or EA in the formulation of the member stiffnesses.

For members whose flexural stiffnesses contribute to the lateral stability:

If $\alpha P_r/P_y \leq 0.5$, use $0.8EI$ in the flexural stiffness terms of the second-order analysis.

If $\alpha P_r/P_y > 0.5$, use $0.8\tau_b EI$ in the flexural stiffness terms of the second-order analysis,

where

$$\tau_b = 4 \frac{\alpha P_r}{P_y} \left(1 - \frac{\alpha P_r}{P_y} \right) \quad (4.6-7)$$

P_y = smallest value of $A_g F_y$ of the member, kips

This reduction need only be applied to the portion of a member where $\alpha P_r/P_y > 0.5$. Alternatively, notional loads of $0.001Y_i$, in addition to those required by item 3(b) (following), may be used along with a stiffness of $0.8EI$ in lieu of reducing the stiffness to $0.8\tau_b EI$.

For members whose axial stiffnesses contribute to the lateral stability (primarily members of braced frames), use $0.8EA$ in the axial stiffness terms of the second-order analysis.

In lieu of modifying the cross-section properties, A and I , by 0.8, it is acceptable (and recommended) to reduce the modulus of elasticity, E , by the factor 0.8 for all members in the second-order elastic analysis. This avoids small problems that can occur in some cases, such as unintended additional drift of a frame due to differential column axial shortening between gravity columns and lateral-load resisting columns when beams or rafters from the lateral-load resisting system are framed into the gravity columns. This approach also gives results that more closely match those from the more advanced methods to which the DM was calibrated. Note that the value of E is not reduced when applying other *Specification* provisions, such as slenderness limit checks (2005 AISC *Specification* Table B4.1) or column strength equations.

- (b) Minimum lateral loads of 0.002 times the vertical load, Y_i , applied at each level are required for gravity-only load combinations when $\Delta_{2nd}/\Delta_{1st} \leq 1.5$ ($\Delta_{2nd}/\Delta_{1st} \leq 1.71$ based on the reduced stiffness). Alternatively, explicit out-of-plumbness may be

modeled in lieu of notional loads.

For load combinations where $\Delta_{2nd}/\Delta_{1st} > 1.5$ ($\Delta_{2nd}/\Delta_{1st} > 1.71$ based on the reduced stiffness), the notional lateral loads must be added to any lateral loads already present in the combination.

For gable frames and for frames with stories having unequal height columns, it is recommended that individual notional lateral loads equal to $0.002y_i$ be applied at the top of each column, where y_i is the vertical load transferred to the column at its top. Also, for columns with intermediate vertical loads along their length, a notional lateral load of $0.002y_i$ should be applied at the location of the intermediate vertical loads, where y_i is the intermediate vertical load applied to the column.

These notional load and out-of-plumbness magnitudes are based on a specified maximum out-of-plumbness of $L/500$. For structures where a different out-of-plumbness is specified, the notional loads should be scaled linearly. Further discussions of this implementation of the notional lateral loads are provided in item 3(b) of Section 4.6.1 and in Section 4.4.3.

- (c) For ASD, the analysis is conducted using loads of 1.6 times those from ASD load combinations. The resulting member forces and moments are divided by 1.6 for member design calculations. The 1.6 multiplier also applies to any notional loads added to satisfy item 3(b).
4. The in-plane flexural buckling strength of columns and beam-columns, P_{ni} , is calculated based on the actual unbraced length with $K = 1.0$ except as noted for the three cases discussed here, where simplifying extensions to the AISC *Specification* are provided. The reduced member stiffnesses in item 3(a) should not be used in the member strength calculations. The member resistances are always calculated using nominal (unreduced) stiffnesses.
 - (a) For members with $\alpha P_r \leq 0.10P_{eL}$ at all locations along their length, or stated more simply, for $\alpha/\gamma_{eL} \leq 0.10$, P_{ni} may be taken as the equivalent cross-section axial yield strength accounting for local buckling effects, QP_y . This simplification is permissible because the in-plane stability effects are very minor at the member level for columns or beam-columns that satisfy the preceding limit. Many members in a typical single-story metal building frame will satisfy this limit. Note that P_{eL} and γ_{eL} in these expressions do not contain an overbar, i.e., these limits are checked using the nominal

elastic stiffness.

- (b) If P - δ effects are included in the analysis model and an appropriate member out-of-straightness between nodes is also included in the model, P_{ni} may be taken as QP_y , even when $\alpha/\gamma_{eL} > 0.10$. This is permissible because the reduced stiffness and out-of-straightness in the analysis account sufficiently for the in-plane stability effects at the member level. The appropriate member out-of-straightness is an imperfection of $0.001L$ in the direction that the member deforms relative to a chord between its support points or points of connection to other members. A chorded representation of the out-of-straightness with a maximum amplitude at the middle of the unsupported length is considered sufficient.
- (c) For gable rafters, when the midspan work point (cross-section centroid) is offset above the rafter chord by $L_{chord}/50$ or more, where L_{chord} is the span length along the rafter chord between the cross-section centroids at the tops of the columns, P_{ni} may be taken as QP_y . This is permissible because the offset of the midspan work point for these types of members nullifies the importance of any out-of-straightness relative to the chord between the ends of the on-slope length of the rafters. For rafters framing between equal height columns, this requirement is satisfied in all cases when the pitch of the rafter centroidal axis is at least $1/2$ in 12 throughout the span length.

4.6.3 The First-Order Method (FOM)

1. The FOM is only permitted for load combinations where $\Delta_{2nd}/\Delta_{1st} \leq 1.5$. Because the objective of using the FOM is likely to be the avoidance of a second-order analysis, it is suggested that the ratio $\Delta_{2nd}/\Delta_{1st}$ be determined using the AISC *Specification* Equation C2-3 for B_2 with ΣP_{e2} taken from Equation C2-6b.

In addition, for all members whose flexural stiffness contributes to the lateral stability, αP_r must be less than or equal to $0.5P_y$, where P_y is the lowest axial yield strength of the member.

2. A first-order analysis is performed as follows:
 - (a) The analysis is conducted without member stiffness reductions.
 - (b) Notional loads must be applied in addition to any lateral loads in each load combination. These are calculated as:

$$N_i = 2.1(\Delta/L)Y_i \geq 0.0042Y_i \quad (4.6-8, \text{Spec. Eq. C2-8})$$

where

Δ/L = highest ratio of first-order story drift under the strength load combination, Δ , to the story height, L , for all stories of the structure calculated using first-order deflection results

Contrary to the user note in AISC *Specification* Section C2.2b, for the FOM it is not necessary to multiply the gravity loads in the ASD load combinations by 1.6 prior to the analysis and then subsequently divide the results by 1.6, because the analysis is linear. Therefore, for design by ASD, Δ in Equation 4.6-8 should be based on 1.0 times the ASD load combinations. It is emphasized that this is the maximum first-order drift of all the stories under the strength load combination being considered.

Y_i = vertical load introduced at each level for each load combination, kips. For ASD, multiply the vertical loads by 1.6.

For gable frames or frames with stories with unequal height columns, Equation 4.6-8 should be used to determine a notional lateral load, N_i , ap-

plied at the top of each column; Y_i is defined as the vertical load transferred to each column at its top; and Δ/L is the maximum ratio of the individual column Δ values to the individual column heights, L , throughout the structure. For columns with intermediate vertical loads along their length, the equation should be used to determine a notional lateral load, N_i , applied at the location of the intermediate vertical loads, where Y_i is the intermediate vertical load applied to the column.

- (c) The first-order analysis is carried out using the normal LRFD or ASD combinations. For ASD, do not use the 1.6 factor on the loads or results other than as required in the calculation of the notional loads in item 2(b).
 - (d) All moments from the first-order analysis must be multiplied by B_1 . For web-tapered members, the amplification factor in Equation 4.6-1 is recommended for the calculation of B_1 .
3. The in-plane flexural buckling strength of columns and beam-columns, P_{ni} , is determined based on the actual unbraced length between stories and idealized pinned-pinned end conditions ($K = 1.0$).

Chapter 4

Stability Design Requirements

The most significant and possibly the most challenging changes in the AISC *Specification* are in the area of stability design, that is, the analysis of framing systems and the application of rules for proportioning of the frame components accounting for stability effects. With a few exceptions, designers using the 1989 AISC *Specification for Structural Steel Buildings—Allowable Stress and Plastic Design* (AISC, 1989) have conducted linearly elastic structural analysis without any explicit consideration of second-order effects, geometric imperfections, residual stresses, or other nonideal conditions. Changes in the AISC *Specification* make explicit consideration of some, or all, of these factors mandatory in the analysis phase.

4.1 KEY TERMINOLOGY

The following key terms are used in the AISC *Specification* and this document.

P-Δ effect. Additional force or moment (couple) due to axial force acting through the relative transverse displacement of the member (or member segment) ends (see Figure 4-1).

P-δ effect. Additional bending moment due to axial force acting through the transverse displacement of the cross-section centroid relative to a chord between the member (or member segment) ends (see Figure 4-2). In singly symmetric web-tapered I-shaped members, and in members with steps in the cross-section geometry along their length, this transverse displacement includes both the deflections relative to the chord between the member or element ends, due to applied loads, as well as the offset of the (nonstraight)

cross-section centroidal axis from the chord. When members are subdivided into shorter-length elements in a second-order matrix analysis, the *P-δ* effects at the member level are captured partly by *P-Δ* effects at the individual member or segment level (see Figure 4-3).

Second-order analysis. Structural analysis in which the equilibrium conditions are formulated on the deformed structure. Second-order effects (both *P-δ* and *P-Δ*, unless specified otherwise) are included. First-order elastic analysis with appropriate usage of amplification factors is a second-order analysis. Other methods of second-order elastic analysis include matrix formulations based on the deformed geometry and *P-Δ* analysis procedures applied with a sufficient number of elements per member. See Chapter 6, Section 6.2, for a brief summary and assessment of different methods of second-order analysis. See Chapter 6, Section 6.2.1, for a discussion of the required number of elements per member for various types of second-order matrix analysis.

Second-order effect. Effect of loads acting on the deformed configuration of a structure; includes *P-δ* effect and *P-Δ* effect.

4.2 ASCE 7 AND IBC SEISMIC STABILITY REQUIREMENTS

Requirements for consideration of second-order effects under some conditions were introduced into the seismic provisions of the American Society of Civil Engineers' standard, ASCE 7, beginning in 1998 (ASCE, 1998) and the International Building Code (IBC) beginning in 2000 (IBC,

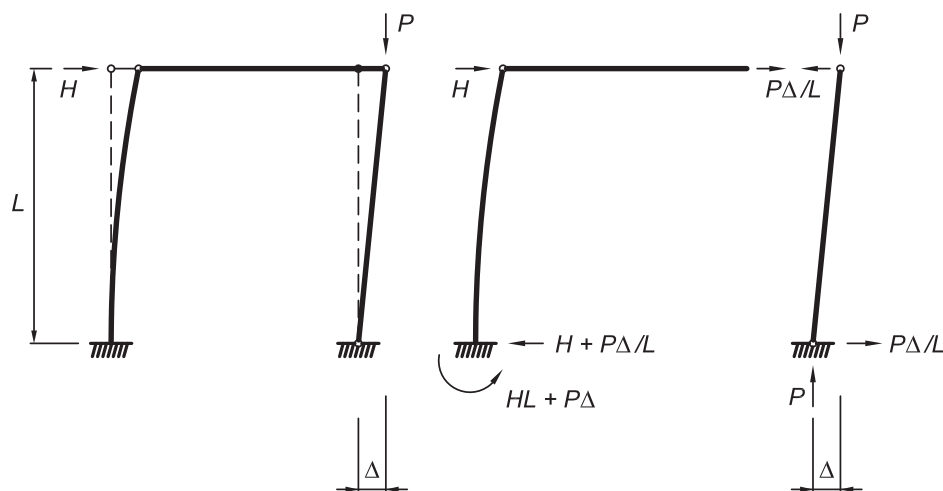


Fig. 4-1. Illustration of *P-Δ* effect.

2000). These provisions established limits on the maximum P - Δ effects and imposed second-order analysis requirements in some cases. The current provisions, summarized from ASCE/SEI 7-05 (ASCE, 2005), are as follows:

Section 12.8.7 requires the calculation of a *seismic stability coefficient*, θ , for each seismic load combination:

$$\theta = \frac{P_x \Delta}{V_x h_{sx} C_d} \quad (4.2-1, \text{ASCE/SEI 7 12.8-16})$$

where

P_x = gravity load in the combination (with a maximum load factor of 1.0), kips

$\frac{\Delta}{V_x C_d}$ = elastic sidesway flexibility of the structure under a lateral load, V_x , calculated using the nominal elastic (unreduced) structural stiffness, in./kip

h_{sx} = story height at the level being considered, in.

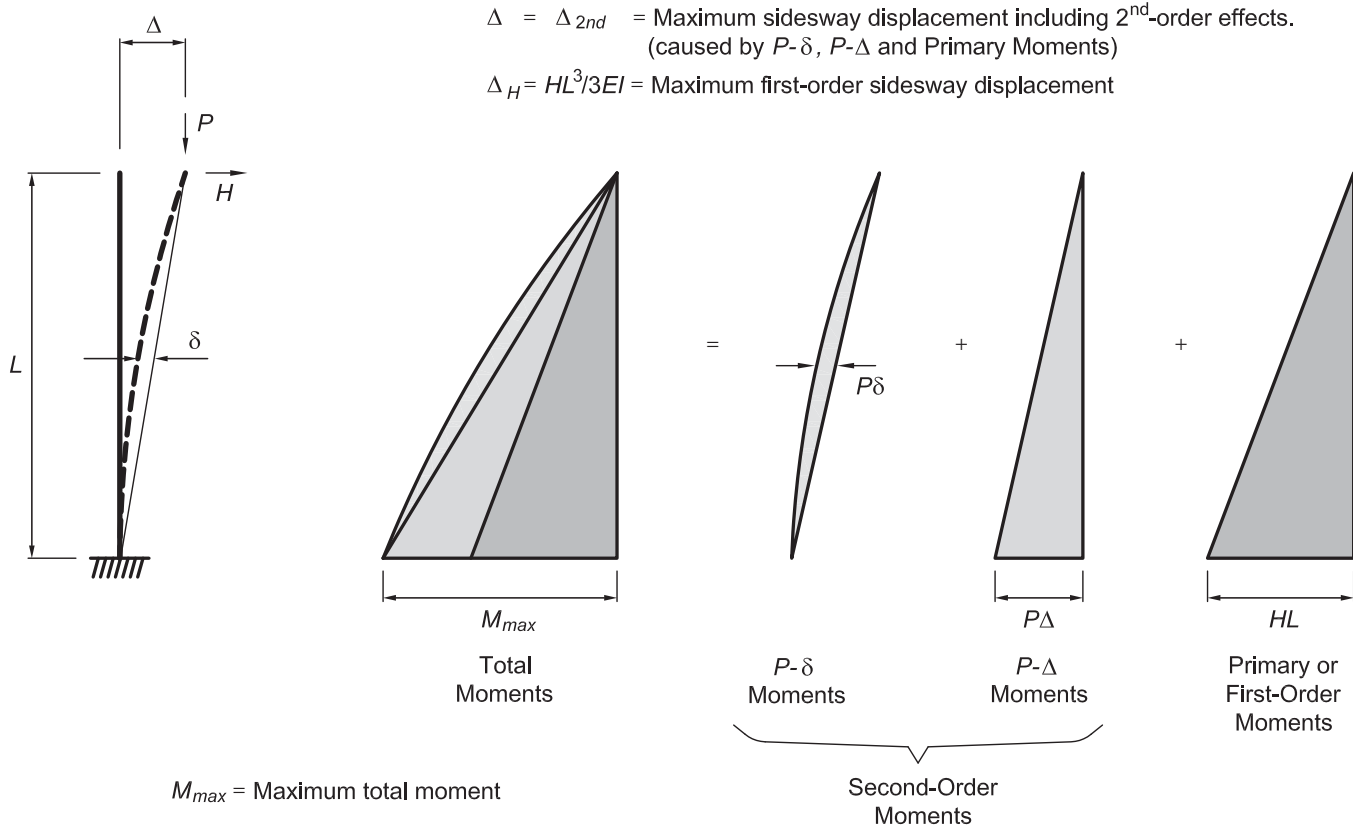


Fig. 4-2. Illustration of combined P - δ and P - Δ effects on sidesway moments and displacements.

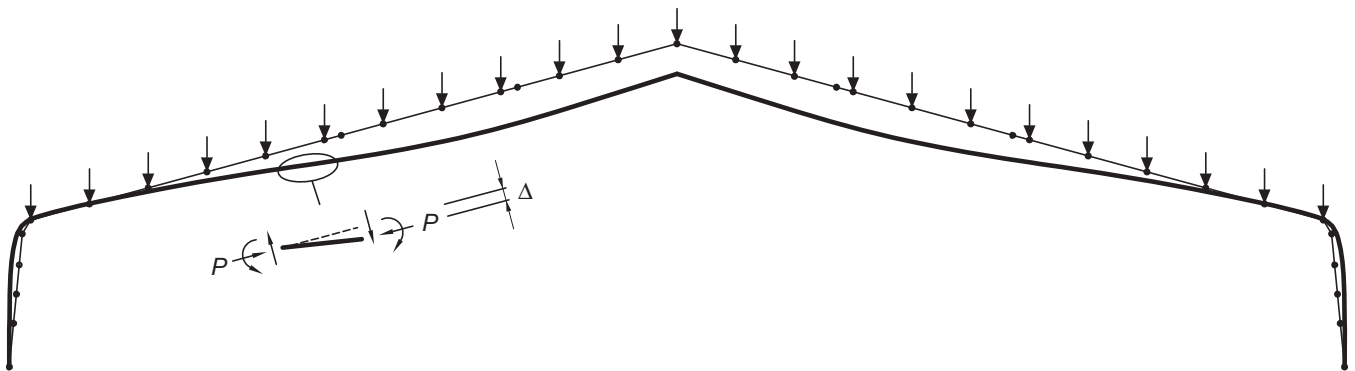


Fig. 4-3. Capture of member P - δ effects by subdivision into shorter-length elements.

Here, θ is an estimate of the ratio of the gravity load to the elastic sidesway buckling strength of the frame and is an indicator of the magnitude of the expected P - Δ effects. Structures with θ less than or equal to 0.10 have small P - Δ effects and are exempt from any ASCE 7 second-order analysis requirement. Structures with θ between 0.10 and an upper limit that can range as high as 0.25 are permitted, but must be designed using an analysis that includes P - Δ effects. Structures with θ above the upper limit of 0.25, corresponding to a P - Δ amplification of the sidesway deflections and moments of $1/(1 - 0.25) = 1.33$, are not permitted.

These provisions have been interpreted to apply only to seismic load combinations. Bachman et al. (2004) indicate that the calculation of θ need never include the roof live load or snow load except in the case of flat roof snow loads of greater than 30 psf, where 20% of the snow load is to be included unless otherwise required by the authority having jurisdiction. This usually limits P_x to a fairly small percentage of the full gravity design load. As a result, for single-story metal building frames, θ seldom exceeds the upper limit. Wide modular frames can have θ exceeding 0.10, but θ can usually be brought down to 0.10 or less by increasing the frame lateral stiffness slightly.

These provisions require consideration of significant P - Δ effects under seismic loading but do not provide any assurance of adequate second-order response under other load combinations that have much higher gravity loading. These conditions are addressed in Section 4.3.

4.3 AISC STABILITY REQUIREMENTS

Section C1 of the AISC *Specification* requires that “Stability shall be provided for the structure as a whole and for each of its elements.” Stability for the individual members of the structure is provided by compliance with the design provisions of Chapters E, F, G, H and I along with the member bracing requirements of Appendix 6. Overall stability of the structure is provided by selecting an appropriate analysis approach combined with a corresponding set of member (or component) design constraints.

Any method of design that considers the following effects is permitted by the AISC *Specification*.

1. Second-order effects
 - P - Δ effects
 - P - δ effects
2. Geometric imperfections
 - System out-of-plumbness
 - Member out-of-straightness
3. Member stiffness reductions due to residual stress
4. Member flexural, shear and axial deformations
5. Connection flexibility

The second-order effects required for the design calculations are those from the geometric nonlinearity of the elastic structure. In essence, this means that equilibrium must be considered in the deflected elastic configuration of the structure rather than in the initial geometry, as is the case for first-order elastic analysis. A wide variety of approaches for handling elastic geometric nonlinearity are available in commercial and in-house software, some of which are discussed in Chapter 6. Various approximate hand methods are also available and are satisfactory in certain cases.

Overall geometric imperfections in a frame can be handled in the preceding elastic analysis in two ways. The most intuitively obvious approach is to incorporate the maximum expected or permitted out-of-plumbness of the structure in the initial modeling of the geometry of the structure. An alternative approach is to include notional loads, which are lateral loads calibrated to produce the same sidesway as the expected out-of-plumbness. Member out-of-straightness has traditionally been handled in the column strength curves but can alternatively be handled by explicit modeling of out-of-straightness between member ends, if preferred. For members and frames subjected predominantly to in-plane bending, the geometric imperfections represented by explicit modeling or notional loads are those in the plane of the member and/or frame.

The effect of member stiffness reduction due to residual stress has traditionally been incorporated in the column strength equations in conjunction with the use of member effective lengths, rather than being considered directly in the analysis. This approach is still permitted in the Effective Length Method. However, it is now possible to consider this effect directly in the analysis. This is the approach taken in the direct analysis method and the first-order analysis method procedures outlined later, which do not require calculation of effective length factors.

The calculation of axial and flexural deformations is a basic component of the direct stiffness approach used in most modern elastic frame analysis software. Shear deformations are not often included in the analysis because their influence on the results is usually small, and therefore, the extra required calculations are not justified. For cases in which shear deformations are significant, they are an option to include in most general analysis programs and can be incorporated into in-house software.

Connection flexibility is routinely handled in elastic analysis software for cases in which the connections are fully restrained (FR) moment connections or simple shear connections by specifying ideally rigid or ideally pinned connections, respectively. For prismatic members, the AISC *Specification* Commentary (AISC, 2005) suggests that a connection with a rotational secant spring stiffness of at least $20EI/L$ at full-service loads can be considered rigid and one with a stiffness below $2EI/L$ can be considered pinned with

respect to stiffness. However, connection strength must also be considered when evaluating whether connections may be considered as ideally rigid or ideally pinned.

Bjorhovde, Colson and Brozzetti (1990) propose a connection classification system that may be interpreted as follows: Connections may be considered rigid when they have a secant rotational stiffness greater than $0.5EI/d$ at $0.7M_p$ of the connecting member, where d is the member depth. Connections with a secant rotational stiffness less than $0.1EI/d$ at $0.2M_p$ of the connecting member should be considered as pinned. Although the Bjorhovde et al. (1990) system was originally developed with prismatic members in mind, it may be applied as an approximate classification approach in frames composed of web-tapered members, using d as the depth of the member at the connection. Connections with stiffnesses between these limits are classified as partially restrained (PR). Inclusion of PR connection stiffness and strength in the analysis is required by Section B3.6b of the *AISC Specification*. Including PR spring stiffnesses in off-the-shelf or in-house software is technically straightforward, but is complicated by shakedown behavior in PR connections and the fact that connections cannot be designed until after the members are selected. A number of commercial software programs currently allow the engineer to also address connection strength by defining the connection's full moment-rotation response. Section B3.6 of the *Specification* also requires that the ductility of simple and PR connections be checked.

4.4 STABILITY DESIGN METHODS

The *AISC Specification* provides three stability design methods that account for items 1 through 3 in Section 4.3. In the following discussions these methods are referred to as:

1. The Effective Length Method (ELM), referred to as "design by second-order analysis" in Section C2.2a of the *AISC Specification*.
2. The Direct Analysis Method (DM) in Appendix 7 of the *AISC Specification*.
3. The First-Order Analysis Method (FOM), referred to as "design by first-order analysis" in Section C2.2b of the *AISC Specification*.

Each of the methods holds certain advantages. The *AISC Specification* also permits the use of any other design method that accounts for all of the elements listed in Section 4.3; however, selecting from the three methods included in the *AISC Specification* is the most practical approach for most engineers.

The primary advantage of the ELM is that experienced steel engineers will already be familiar with many of its

elements. The DM holds the advantages that (1) it may be used for all structures and load combinations, (2) it provides the most accurate assessment of internal forces and moments, and (3) columns may be designed without calculation of K factors. The virtues of the FOM are that (1) it permits design without a second-order analysis (an assumed second-order amplification is implicit in this method), and (2) it permits the design of columns without the calculation of K factors.

Theoretical details of the differences between the methods are covered in the *AISC Specification* Commentary and numerous research papers (Maleck and White, 2003; Deierlein, 2003, 2004; Kuchenbecker et al., 2004; Surovek-Maleck and White, 2004a, 2004b; Nair, 2005; Martinez-Garcia and Ziemian, 2006; White et al., 2007a; and White et al., 2007b) and are not discussed in this document. From an implementation viewpoint, the differences between the methods are in the areas of:

1. Limits on structural characteristics that establish the applicability of the methods
2. The type of structural analysis to be employed (first-order or second-order)
3. The method of accounting for nominal out-of-straightness and out-of-plumbness (use of notional loads or explicit modeling of imperfections in the analysis, or implicit inclusion in column strength equations via effective lengths)
4. The method for considering stiffness reduction from residual stress effects (directly in the analysis or implicitly in column strength equations via effective lengths)
5. Corresponding design constraints

The three methods differ (in analysis details, notional loads, and stiffness reductions, for example) and result in somewhat different required strengths for use in design. In general, for structures with significant second-order effects, the DM and the FOM will generate larger and more realistic sidesway moments than those determined using the ELM. On the other hand, the calculated in-plane column available strengths are larger and more easily determined using the DM and FOM. This is because these methods increase the member required flexural strengths, M_r , rather than reduce the member required axial strengths, P_c , to account for sidesway instability effects. Conversely, the ELM accounts for sidesway stability effects by reducing P_c via the use of $K > 1$ in moment frames, where K is the effective length factor, or by explicit use of buckling analyses to determine the theoretical column buckling loads.

The following sections provide an overview of the major implementation differences between the three methods.

4.4.1 Limits of Applicability

The DM is permitted for all structures and all load combinations. The usage of the other two methods is restricted to load combinations in which $\Delta_{2nd}/\Delta_{1st} \leq 1.5$, where Δ_{2nd} is the second-order drift and Δ_{1st} is the first-order drift for the strength combination being considered (the ASD load combinations multiplied by 1.6 or the LRFD load combinations). The limit of $\Delta_{2nd}/\Delta_{1st} \leq 1.5$ is applicable to an analysis conducted using unreduced stiffness, EA and EI . If reduced member stiffness is used for the analysis, as discussed in Section 4.4.4, this limit is $\Delta_{2nd}/\Delta_{1st} \leq 1.71$. Clear span frames often meet this restriction for all load combinations, but wide modular frames will often exceed this limit under the maximum gravity load combinations. As a result, the DM is the only method suitable for some load combinations of many metal building frames unless the designer is willing to limit $\Delta_{2nd}/\Delta_{1st}$ to no more than 1.5 for every load combination.

4.4.2 Type of Analysis

Both the ELM and the DM require that a second-order analysis be performed. As the name implies, the FOM does not require a second-order analysis. It provides a second-order amplification indirectly through the use of larger notional loads. The detailed requirements for the second-order analysis calculations required by the ELM and DM differ somewhat and are covered later in the sections that describe each method in detail.

4.4.3 Out-of-Plumbness

Each of the three methods requires the application of notional loads, or explicit modeling of the out-of-plumbness on which the notional loads are based, for at least some load combinations in the structural analysis. Notional loads are artificial lateral loads applied to the structure to account for geometric imperfections and other nonideal conditions that can induce or increase the sway of a structure.

Notional Loads

In all the methods of the AISC *Specification*, the equations for calculating notional loads are based on an assumed uniform out-of-plumbness of $L/500$. However, Appendix 7, Section 7.3(2), of the *Specification* states that this can be adjusted by the ratio of the expected out-of-plumbness to $L/500$ to account for a lesser assumed out-of-plumbness. Prior to the 2007 edition, Section 6.8 of Common Industry Standards in the *Metal Building Systems* (MBMA, 2002) specified an out-of-plumbness erection tolerance of $L/300$. Structures to be built to this standard should have their notional loads or nominal out-of-plumbness increased by the factor of $(L/300)/(L/500) = 1.67$ relative to the specified AISC value to account for this more liberal tolerance. The latest edition

of the *Metal Building Systems Manual* (MBMA, 2007) has eliminated this exception; therefore, structures to be built to the 2007 MBMA standard should be designed using the notional loads specified in the AISC *Specification*.

Notional loads are calculated for each load combination as a percentage of the vertical load acting at each level for that load combination. Although the text of the AISC *Specification* defines notional loads as a percentage of the *gravity* load, the notional loads are more properly defined as a percentage of the *vertical* load, regardless of the source. This is apparent based on the direct correspondence between notional loads and the alternative explicit modeling of out-of-plumbness.

The physical out-of-plumb imperfections in the structure may be in either sidesway direction. However, the direction of application of the notional loads for each load combination is selected to increase the overall destabilizing effect for that combination. For gravity-only load combinations that cause a net (i.e., weighted average) sidesway either due to nonsymmetry of loads or geometry, the notional loads should be applied in the direction that increases the net sidesway. For structures with multiple stories or levels, and in which the sidesway deformations are in different directions in different stories or levels, it is necessary to include a pair of load combinations, separately considering the notional loads associated with a uniform out-of-plumbness in each direction. For load combinations involving lateral loads, the notional loads should be applied only in the direction that adds to the effect of the lateral loads. One need not apply notional loads in a direction opposite from the total lateral loads to minimize the reduction in internal forces in certain components due to the lateral load. For gravity load combinations with no sidesway, it is necessary to include a pair of load combinations, separately considering notional loads in each direction, unless symmetry of the frame is enforced by other means.

Separate notional loads should be applied at the top of each of the columns in proportion to the vertical load transferred to each column. In columns with axial forces applied at an intermediate location along the length, a proportional notional load should be placed at that location on the column. For any instances in which questions arise about the calculation and application of notional loads, the question may be answered by determining the lateral forces that are equivalent to the effect of the intended uniform nominal out-of-plumbness.

For ASD designs, notional loads calculated based on ASD load combination factors must be increased by a factor of 1.6 in all three methods. The 1.6 factor overestimates the second-order effects somewhat in ASD designs relative to LRFD, particularly where second-order effects are significant.

Explicit Modeling of Out-of-Plumbness

The AISC *Specification* permits explicit modeling of out-of-plumbness in the structural analysis in lieu of the use of notional loads for the DM. This avoids the need to determine how to apply notional loads in buildings with sloping roofs or floors, where the building geometry is nonrectangular or nonregular, or in structures where axial loads are applied at intermediate positions along the length of a member. This approach is easy to automate in computer-based design, and it allows the designer to better understand the true nature of internal forces set up in the structure from out-of-plumbness effects. However, unless automated methods of specifying out-of-plumbness are available in analysis software, it is often easier to apply notional loads along with the other applied loads on the structure rather than to modify the structure geometry.

The modeled out-of-plumbness should be consistent with the erection tolerances specified for the structure. Therefore, if the erection tolerances are smaller than $L/500$, a reduced uniform out-of-plumbness equal to the specified erection tolerance may be employed. Also, where a larger erection tolerance is permitted, this larger tolerance should be used as the modeled uniform out-of-plumbness in the structural analysis.

The physical out-of-plumb imperfections in the structure may be in either sidesway direction. However, the direction of the modeled uniform out-of-plumbness for each load combination is selected to increase the overall destabilizing effect for that combination. For gravity-only load combinations that induce a net sidesway, the modeled out-of-plumbness should be in the direction of the net sidesway. For structures with multiple stories or levels and in which the sidesway deformations are in different directions in different stories or levels, two different uniform out-of-plumb geometries are required to capture the potential overall destabilizing effects in both directions. For load combinations involving lateral loads, the out-of-plumbness should be in the direction of the lateral loads. For a gravity load combination with no sidesway, it is necessary to consider a uniform out-of-plumbness in both directions unless any symmetry of the design is enforced by other means.

Only two different out-of-plumb geometries are typically required to cover the overall destabilizing effects for all load combinations. In contrast, the corresponding notional loads discussed in the previous section are, in general, different for each load combination (although the notional loads may be taken conservatively as the maximum values from all the load combinations).

For the ELM in all cases, and the DM in some cases, the notional loads are specified as minimum lateral loads in the gravity-only load combinations. That is, they are not used in combination with other lateral loads. Consequently,

out-of-plumbness need not be included in the model for these load combinations. Modeling out-of-plumbness for all load combinations is permitted but will result in conservative results for lateral load combinations in which notional loads are not required. In cases where the notional loads are specified as additive rather than minimum, the model must include out-of-plumbness (or out-of-plumbness effects via notional loads) regardless of the magnitude of the lateral loads.

4.4.4 Stiffness Reduction

Only the DM requires explicit consideration of member stiffness reduction due to the combined effects of residual stresses and distributed yielding with member axial forces and moments in the structural analysis. This is handled by reducing the flexural stiffness in moment frames and the axial stiffness in braced frames to 80% of their nominal elastic values in the second-order structural analysis. Where the axial compression load in a flexural member contributing to lateral stability exceeds 50% of the yield load, the flexural stiffness is further reduced. This is discussed in detail later. The other two methods consider member stiffness reductions only implicitly, either via the calculation of the column strengths using effective lengths or by the calculation of a larger notional lateral load.

4.4.5 Design Constraints

The DM and the FOM permit the design of columns for in-plane buckling using a length equal to the actual unsupported length ($K = 1$), or a smaller length in some cases. For moment frames, the ELM requires the calculation and use of elastic buckling load values (or the corresponding K factors) determined using a sidesway buckling analysis of the structure unless $\Delta_{2nd} / \Delta_{1st} \leq 1.1$. If $\Delta_{2nd} / \Delta_{1st} \leq 1.1$, the in-plane flexural buckling load may be calculated based on the actual member length and idealized pinned-pinned end conditions (i.e., $K = 1$).

4.5 COMMON ANALYSIS PARAMETERS

Several parameters are used throughout the text of the stability requirements in the AISC *Specification* to establish limits of applicability of various provisions and in other calculations. These are defined as follows:

4.5.1 αP_r

αP_r is the required axial compressive strength, P_r , multiplied by the factor α where $\alpha = 1.6$ for ASD and $\alpha = 1.0$ for LRFD.

For LRFD, αP_r is simply the required axial compressive strength determined using the LRFD load combination factors for the load combination under consideration. For ASD, αP_r is the required axial compression strength determined

using ASD load combination factors multiplied by 1.6, to give an approximation of the required strength under ultimate conditions.

αP_r is used in the determination of whether the FOM may be used. It is also used in the DM to determine whether P - δ effects can be neglected in the calculation of the sidesway displacements, and to calculate the required stiffness reduction. αP_r is also used in approximate second-order analysis techniques such as the B_1 - B_2 method.

4.5.2 P_{eL} or $\gamma_{eL}P_r$

P_{eL} is the nominal in-plane elastic flexural buckling strength of a member subjected to axial compression force and having assumed ideally pinned-pinned end conditions. This parameter is used extensively in the provisions for both the ELM and the DM as well as in the member design provisions. Consideration of the actual member rotational and sidesway restraint end conditions is handled subsequently within the AISC design procedures, via the calculation of other buckling loads or the corresponding effective length factors in the ELM, and via the modeling of the structure for the structural analysis.

In many cases it is more convenient to work with the equivalent parameter $\gamma_{eL}P_r$, which is the required strength, P_r , multiplied by the elastic buckling load ratio, $\gamma_{eL} = P_{eL}/P_r$. That is,

$$P_{eL} = \gamma_{eL}P_r \quad (4.5-1)$$

where

P_{eL} = Euler buckling load, evaluated in the plane of bending, kips. This is the internal axial force at elastic buckling of the member, assuming simply supported end conditions.

P_r = required axial strength for the column, kips

γ_{eL} = a scalar ratio

Regardless of the complexity of the loadings (e.g., stepped loading or distributed axial loading) or the member geometry (e.g., tapered and/or stepped geometry), there is only one γ_{eL} corresponding to the member elastic flexural buckling strength. However, for stepped or distributed axial loading, P_r and P_{eL} vary along the member length.

The elastic flexural buckling strength can also be expressed as:

$$F_{eL} = \gamma_{eL}F_r \quad (4.5-2)$$

where

F_{eL} = axial stress at elastic buckling of the member, assuming simply supported end conditions, ksi

F_r = required axial stress for the column, ksi

For a straight, geometrically perfect prismatic column with a constant axial loading,

$$P_{eL} = \frac{\pi^2 EI}{L^2} \quad (4.5-3)$$

For a tapered I-shaped member, there is no exact closed-form solution for P_{eL} . However, several approaches to a solution are available:

1. P_{eL} can be determined by an elastic eigenvalue buckling analysis. Many advanced finite element and/or frame analysis programs can be used to calculate elastic buckling multipliers, γ_{eL} , corresponding to a given required axial strength using numerical eigenvalue solution techniques. P_{eL} is then determined as the required axial compression strength, used in the analysis, multiplied by γ_{eL} . The quality of such solutions depends on the accuracy of tapered member modeling, element choice, and meshing. The engineer should run the benchmark problems provided in Appendix C to establish the appropriateness of the computer program and modeling techniques prior to use in design. Although this approach has the advantage of handling essentially any imaginable geometry and loading, it may not be practical in a production environment unless the finite element modeling is automated and integrated into analysis-design software.
2. P_{eL} can be determined by the method of successive approximations (Timoshenko and Gere, 1961). This technique uses an iterative beam analysis to find the axial load, $\gamma_{eL}P_r$, at which the beam deflections resulting from applied P - δ moments are a uniform multiple of the deflections assumed to calculate the P - δ moments. This is an iterative process in which (1) a load, P_r , and a deflected buckling mode shape are assumed; (2) the P - δ moments from the assumed deflections times the assumed axial load are calculated; (3) the calculated P - δ moments are applied in a beam analysis of the member to compute a new deflected shape; and (4) the new deflected shape is substituted as a new approximation for the buckled geometry. The process is continued iteratively until the calculated deflections everywhere along the beam are a uniform multiple, γ_{eL} , of the assumed deflections. P_{eL} is then determined as the assumed axial load, P_r , multiplied by γ_{eL} .

The method of successive approximations requires relatively few calculations compared with eigenvalue solution techniques, is easily programmed, and is adaptable to handle various tapers and steps in the member loading and geometry. The method is illustrated in Timoshenko and Gere (1961) with an example in a format easily adapted to a spreadsheet or procedural computer program. See Appendix C of this Guide for benchmark examples of web-tapered members.

3. P_{eL} can be approximated with good accuracy for a single linearly tapered member with simply supported conditions and supporting a constant internal axial force, and having no plate or taper changes, as:

$$P_{eL} = \frac{\pi^2 EI'}{L^2} \quad (4.5-4)$$

where

I' = moment of inertia calculated using the depth at $0.5L$ ($I_{\text{small}}/I_{\text{large}})^{0.0732}$ from the small end, in.⁴

This empirically derived expression gives results accurate to within several percentage points for the range of members addressed in this document. The preceding approximation should not be used for any buckling solution in which anything other than simply supported end conditions are assumed. That is, the preceding expression for I' is valid solely for idealized simply supported end conditions.

4. For linearly tapered members subjected to nonuniform axial compression, γ_{eL} can be calculated conservatively as $P_{eL}/(P_r)_{\text{max}}$, where P_{eL} is calculated using Equation 4.5-4 and $(P_r)_{\text{max}}$ is the largest internal axial compression along the member length.

4.5.3 $\Delta_{2nd}/\Delta_{1st}$

$\Delta_{2nd}/\Delta_{1st}$ is the ratio of story drifts calculated from a second-order and first-order analysis, respectively. This ratio is used to establish the applicability of the approved design methods, to establish the applicability of the $K = 1$ provisions of the ELM [2005 AISC *Specification* Section C2.2a(4)], to determine whether notional loads are additive to lateral loads in the DM, and also in the B_1 - B_2 method.

Unless otherwise noted, this ratio is calculated from analyses using unreduced member stiffnesses. For information on calculating $\Delta_{2nd}/\Delta_{1st}$ for gable frames, see Section 6.3.3.

$\Delta_{2nd}/\Delta_{1st}$ is calculated separately for each load combination. This parameter gives an indication of the significance of the second-order effects in a load combination. No maximum limit on $\Delta_{2nd}/\Delta_{1st}$ is established by the AISC *Specification* (AISC, 2005). Values below 1.1 are considered insignificant. Values above 1.5 are considered large second-order effects. As such, the design must be conducted using the DM when this threshold is exceeded. Values between 1.1 and 1.5 are considered moderate second-order effects. The design may be conducted either by the DM or by the ELM in these cases.

The reader is cautioned against using the ratio M_{2nd}/M_{1st} as a substitute for $\Delta_{2nd}/\Delta_{1st}$. The moments usually include significant first-order gravity components that will obscure the magnitude of the second-order effects.

4.6 DETAILED REQUIREMENTS OF THE STABILITY DESIGN METHODS

The following sections summarize the detailed requirements for each of the three stability design methods discussed earlier. Additional information on first-order and second-order frame analysis is given in Chapter 6.

4.6.1 The Effective Length Method (ELM)

1. The ELM is only permitted for load combinations where $\Delta_{2nd}/\Delta_{1st} \leq 1.5$.
2. A second-order analysis, considering both P - Δ and P - δ effects as detailed below, is required:
 - (a) The P - Δ effects on the nodal displacements must be considered. The P - δ effects on the nodal displacements may be neglected in the calculation of required strengths because the ELM beam-column unity checks are insensitive to these effects.
 - (b) The P - δ effects on the internal element moments (between the nodes) may be neglected in individual elements in load combinations when $\alpha P_r \leq 0.02P_{e\ell}$ for that element,

where

$P_{e\ell}$ = flexural column buckling load based on the cross-section geometry and the element length between the nodal locations with idealized simply supported nodal end conditions, kips

Otherwise, they must be considered.

- (c) Internal P - δ moments may be included by performing a second-order analysis to determine the nodal displacements, forces and moments, and then calculating the second-order internal moments in each element as follows (Guney and White, 2007):
 - (i) Calculate δ_{1st} , the first-order displacement perpendicular to the element chord caused by the second-order nodal forces and any applied loads within the element length, at any locations of interest.
 - (ii) Calculate the second-order displacement at each of the preceding locations as

$$\delta_{2nd} = \frac{\delta_{1st}}{1 - \alpha P_r / P_{e\ell}} \quad (4.6-1)$$

where $P_{e\ell}$ may be estimated for linearly tapered segments using P_{eL} from Equation 4.5-4, but applied to the element length, ℓ .

- (iii) Calculate the required internal second-order moment at each of the above locations as

$$M_r = M_{1st} + \alpha P_r \delta_{2nd} \quad (4.6-2)$$

where

M_{1st} = first-order moment at a given position along the element length, caused by the second-order nodal forces and any applied loads within the element length, kip-in.

This procedure provides good accuracy for general cases involving prismatic or nonprismatic member geometry for values of $\alpha P_r / P_{el} \leq 0.7$. This limit is satisfied in all cases when (1) a P - Δ only analysis or a second-order analysis using an element geometric stiffness based on element cubic transverse displacements is used, and (2) the number of elements per member is greater than or equal to that specified by the guidelines discussed subsequently in Section 6.2. Alternatively, the nonsway amplifier

$$B_1 = \frac{C_m}{1 - \alpha P_r / P_{el}} \geq 1.0 \quad (4.6-3a)$$

may be applied to all the moments M_{1st} throughout the length of a given element, except those at the ends. Equation 4.6-3a is useful for elements in linearly tapered members that do not have any applied transverse loads. In this case, the equivalent uniform moment factor, C_m , may be expressed approximately as

$$C_m = 0.6 + 0.4(f_1/f_2) \quad (4.6-3b)$$

where

f_2 = the absolute value of the largest compressive flexural stress at either element end node, ksi

$$f_1 = 2f_{mid} - f_2, \text{ ksi} \quad (4.6-3c)$$

f_{mid} = flexural stress due to M_{1st} at the mid-length of the element in the flange containing the stress f_2 , taken as positive for compression and negative for tension, ksi

Equation 4.6-3b accounts for the fact that a linear variation in M_{1st} produces a nonlinear variation in the corresponding flexural stress along the length of a tapered member. The value f_1 is the flange stress obtained by extending a line through f_2 and f_{mid} to the opposite element end node.

In many cases, Equation 4.6-3a gives $B_1 = 1.0$, indicating that the second-order amplification of the internal moments may be neglected. Equations 4.6-1 and 4.6-2 generally provide better accuracy for both prismatic and nonprismatic members compared to the amplified moments determined using Equation 4.6-3a. This is particularly true for elements with transverse applied loads, where the AISC *Specification* gives a conservative value of $C_m = 1.0$ and Table C-C2.1 in the AISC *Specification* Commentary gives refined equations for C_m that are applicable only for prismatic members with ideally pinned or ideally fixed end conditions. The use of $C_m = 1.0$ is recommended for general cases with transverse applied loads.

- (d) P - δ effects must be included in the calculation of elastic column buckling strengths, P_{el} , when using either an eigenvalue or the successive approximation approach. See Appendix B1.2 for guidance on subdividing the members into a sufficient number of smaller-length elements for matrix eigenvalue analyses.
- (e) The accuracy of any second-order analysis program used should be tested using appropriate benchmark problems such as those provided in Appendix C. Particular care should be taken to establish whether P - δ effects are correctly included in the analysis. Sections 6.2.1 and 6.2.2 provide guidelines for subdivision of members to ensure sufficient accuracy with respect to these effects.

The *amplified first-order elastic analysis approach* (e.g., the B_1 - B_2 approach) is an acceptable second-order analysis method. If this approach is employed, Equation C2-6b in the AISC *Specification* (or the more refined Commentary Equation C-C2-6 not including the limit $1.7HL/\Delta_H$) is recommended for the definition of ΣP_{e2} in Equation C2-3.

Implementation of the B_1 - B_2 may involve more work compared to other alternative approaches [e.g., a general P - Δ analysis as discussed earlier, or the alternate amplifier-based method discussed by White et al. (2007a, 2007b)]. The amplifier-based methods are particularly difficult to implement and lose accuracy for gable frames, where the sidesway column displacements are generally not the same, and for frames with unequal height columns, where the methods must be modified to account for the different column heights (White and Kim, 2006).

The reader should note that the term *amplified first-order elastic analysis* is typically used to refer to the specific B_1 - B_2 method of calculating the second-order forces and moments. It is important to distinguish this term from the terms used for the different design methods, i.e., the ELM, the DM and the FOM. The B_1 - B_2 second-order analysis method is one of many methods of second-order analysis that may be used for the calculation of the forces and moments in either of the design methods that require a second-order analysis (the ELM and the DM).

3. Given the satisfaction of the preceding requirements for the second-order elastic analysis calculations, the ELM structural analysis model must include the following attributes:

- (a) The analysis is conducted with nominal elastic stiffnesses, i.e., no member stiffness reductions.
- (b) Minimum lateral loads of 0.002 times the vertical load, Y_i , applied at each level are required for all gravity-only load combinations. For gable frames and for frames with stories having unequal height columns, it is recommended that individual notional lateral loads equal to $0.002y_i$ should be applied at the top of each column, where y_i is the vertical load transferred to the column at its top. Also, for columns with intermediate vertical loads along their length, a notional lateral load of $0.002y_i$ should be applied at the location of the intermediate vertical loads, where y_i is the intermediate vertical load applied to the column. This is necessary to capture the geometric imperfection effects on different height columns, as well as to capture the behavior in cases where the lateral displacements are generally different at the different column locations.

In lieu of applying the notional lateral loads, one can impose an out-of-plumbness of $0.002H$ on the structure for analysis of the gravity-only load combinations, where H is the vertical height above the base, or in general, the node(s) having the minimum vertical coordinate. This may be implemented by shifting all the nodes of the analysis model horizontally by $0.002H$ relative to the node(s) at the base of the structure. For cases in which questions arise about the appropriate application of the notional lateral loads, one should always return to the model where the uniform out-of-plumbness is represented explicitly in the structural model. The appropriate notional loads are the ones that are equivalent to the effect of this out-of-plumbness.

For both notional loads and explicit modeling of out-of-plumbness, the factor 0.002 is based on an assumed erection tolerance of $L/500$. For adjustments to this factor to account for structures built to different tolerances, see Section 4.4.3.

- (c) For ASD, the analysis is conducted using loads of 1.6 times those from ASD load combinations. The resulting member forces and moments are divided by 1.6 for the member design calculations. The 1.6 multiplier also applies to any notional loads added to satisfy item 3(b).
4. The in-plane flexural buckling strength of columns and beam-columns, P_{ni} , is determined as follows:
 - (a) For members in load combinations where $\Delta_{2nd}/\Delta_{1st} \leq 1.1$, calculate P_{ni} based on the actual unbraced length with $K = 1.0$, i.e., assuming idealized pinned-pinned end conditions on the actual unbraced length.
 - (b) For other cases, P_{ni} must be calculated using an effective length factor, K , or the corresponding column buckling stress, F_e , determined from a sidesway buckling analysis of the structure. Because member taper violates one of the inherent assumptions of the traditional alignment charts, more advanced methods of determining K or F_e are normally required. See Appendix B for further information on the calculation of elastic buckling strengths of tapered columns and frames.

4.6.2 The Direct Analysis Method (DM)

1. The DM is permitted for all structures and load combinations.
2. A second-order analysis with characteristics similar to those discussed in item 2 of Section 4.6.1 is required. However, there are a few important differences. The following discussion repeats much of the discussion in item 2 of Section 4.6.1 with an emphasis on the specific requirements in the context of the DM.
 - (a) Generally, both the P - Δ and P - δ effects on the nodal displacements must be considered in the DM. The AISC *Specification* Appendix 7, Section 7.3(1), indicates that if $\alpha P_r < 0.15P_{eL}$ for all members whose flexural stiffnesses are considered to contribute to the lateral stability of the structure, the P - δ effect on the lateral displacements may be neglected in the analysis. Although not defined in the AISC *Specification*, “members whose flexural stiffnesses are considered to contribute to the lateral stability” in this context is intended to apply to both beams and columns in unbraced frames. For

nonrectangular structures such as gable frames, the term “lateral displacements” may be interpreted as the general nodal displacements in the frame analysis model.

With the exception of sway columns without significant transverse member loads, where both ends have substantial rotational restraints, this Guide recommends that when $\alpha P_r > 0.05 \bar{P}_{el}$, where $\bar{P}_{el}$ is the elastic buckling load based on the overall member length determined as discussed in Section 4.5.2 but using the reduced elastic stiffness of the DM analysis model discussed later, the member should be subdivided with intermediate nodes along its length when a P - Δ only analysis is employed. This ensures better accuracy of the element nodal displacements and moments along the member length than will be achieved using the previous AISC *Specification* rule when αP_r exceeds $0.05 \bar{P}_{el}$. The sidesway moments in fixed-base columns, and columns with top and bottom rotational restraint from adjacent framing, may be analyzed sufficiently with a P - Δ only analysis and a single element per member when $\alpha P_r \leq 0.12 \bar{P}_{el}$. Second-order analysis procedures that include both P - Δ and P - δ effects in the formulation require fewer elements. Detailed guidelines for the necessary number of elements are provided subsequently in Sections 6.2.1 and 6.2.2 of this Guide. These guidelines and the above recommendations are based on Guney and White (2007). In many cases a sufficient subdivision will occur naturally with tapered members due to the frequency of plate and/or geometry changes. However, extra nodes may be required for prismatic members and long tapered members without changes of plates or taper.

- (b) The P - δ effects on the internal element moments (between the nodes) may be neglected in individual elements in load combinations when $\alpha P_r \leq 0.02 \bar{P}_{el}$ for that element (Guney and White, 2007),

where

$\bar{P}_{el}$ = flexural buckling load based on the cross-section geometry and the element length between the nodal locations with idealized simply supported nodal end conditions, determined using the reduced elastic stiffnesses of the DM analysis model discussed later, kips

- (c) Internal P - δ moments may be included by performing a second-order analysis to determine the nodal displacements, forces and moments, then calculating the second-order internal moments in

each element using the forces, moments, and displacements calculated with the reduced stiffness from the DM analysis as follows:

- (i) Calculate δ_{1st} , the first-order displacement perpendicular to the element chord caused by the second-order nodal forces and any applied loads within the element length, at any locations of interest.
- (ii) Calculate the second-order displacement at each of the preceding locations as

$$\delta_{2nd} = \frac{\delta_{1st}}{1 - \alpha P_r / \bar{P}_{el}} \quad (4.6-4)$$

where $\bar{P}_{el}$ may be estimated for linearly-tapered segments using $\bar{P}_{el}$ from Equation 4.5-4, but applied to the element length, ℓ , and using the reduced elastic stiffness of the DM analysis model.

- (iii) Calculate the required internal second-order moment at each of the above locations as

$$M_r = M_{1st} + \alpha P_r \delta_{2nd} \quad (4.6-5)$$

where

M_{1st} = the first-order moment at a given position along the element length, caused by the second-order nodal forces and any applied loads within the element length, kip-in.

This procedure provides good accuracy for general cases involving prismatic or nonprismatic member geometry for values of $\alpha P_r / \bar{P}_{el} \leq 0.7$ (Guney and White, 2007). This limit is satisfied in all cases when a sufficient number of elements is employed in a P - Δ -only analysis or a second-order analysis using an element geometric stiffness based on element cubic transverse displacements using the guidelines discussed subsequently in Section 6.2. Alternatively, the AISC (2005) nonsway amplifier

$$B_1 = \frac{C_m}{1 - \alpha P_r / \bar{P}_{el}} \geq 1.0 \quad (4.6-6a, \text{ from Spec. Eq. C2-2})$$

may be applied to all the moments, M_{1st} , throughout the length of a given element, except those at the ends. Equation 4.6-6a is useful for elements in linearly tapered members that do not have any transverse applied loads. In this case, the equivalent uniform moment factor, C_m , may be expressed approximately as

$$C_m = 0.6 + 0.4(f_1/f_2) \quad (4.6-6b)$$

where

$$\begin{aligned} f_2 &= \text{the absolute value of the largest compressive flexural stress at either element end node, ksi} \\ f_1 &= 2f_{mid} - f_2, \text{ ksi} \\ f_{mid} &= \text{flexural stress due to } M_{1st} \text{ at the mid-length of the element in the flange containing the stress } f_2, \text{ taken as positive for compression and negative for tension, ksi} \end{aligned} \quad (4.6-6c)$$

Equation 4.6-6b accounts for the fact that a linear variation in M_{1st} produces a nonlinear variation in the corresponding flexural stress along the length of a tapered member. The value f_1 is the flange stress obtained by extending a line through f_2 and f_{mid} to the opposite element end node.

In many cases, Equation 4.6-6a gives $B_1 = 1.0$, indicating that the second-order amplification of the internal moments may be neglected. Equation 4.6-4 in conjunction with Equation 4.6-5 generally provides better accuracy for both prismatic and nonprismatic members compared to the application of Equation 4.6-6a. This is particularly true for elements with transverse applied loads, where the AISC *Specification* gives a conservative value of $C_m = 1.0$ and Table C-C2.1 in the AISC Commentary gives refined equations for C_m that are applicable only for prismatic members with ideally pinned or ideally fixed end conditions. The use of $C_m = 1.0$ is recommended for general cases with transverse applied loads.

- (d) The accuracy of any second-order analysis program used should be tested using appropriate benchmark problems such as those provided in Appendix C. If the benchmark tests are satisfied, the software may be assumed to provide adequate results without subdividing the members into multiple elements as recommended in item 2(a).
3. Given the satisfaction of the preceding requirements for the second-order elastic analysis calculations, the DM analysis model must include the following attributes:
 - (a) The analysis must be conducted with elastic stiffness reductions for all members whose flexural stiffness is considered to contribute to the lateral stability of the structure. Although not defined in the AISC *Specification*, “members whose flexural stiffness is considered to contribute to the lateral

stability” in this context is intended to apply only to columns in unbraced frames. This is accomplished by reducing the value of EI and/or EA in the formulation of the member stiffnesses.

For members whose flexural stiffnesses contribute to the lateral stability:

If $\alpha P_r/P_y \leq 0.5$, use $0.8EI$ in the flexural stiffness terms of the second-order analysis.

If $\alpha P_r/P_y > 0.5$, use $0.8\tau_b EI$ in the flexural stiffness terms of the second-order analysis,

where

$$\tau_b = 4 \frac{\alpha P_r}{P_y} \left(1 - \frac{\alpha P_r}{P_y} \right) \quad (4.6-7)$$

P_y = smallest value of $A_g F_y$ of the member, kips

This reduction need only be applied to the portion of a member where $\alpha P_r/P_y > 0.5$. Alternatively, notional loads of $0.001Y_i$, in addition to those required by item 3(b) (following), may be used along with a stiffness of $0.8EI$ in lieu of reducing the stiffness to $0.8\tau_b EI$.

For members whose axial stiffnesses contribute to the lateral stability (primarily members of braced frames), use $0.8EA$ in the axial stiffness terms of the second-order analysis.

In lieu of modifying the cross-section properties, A and I , by 0.8, it is acceptable (and recommended) to reduce the modulus of elasticity, E , by the factor 0.8 for all members in the second-order elastic analysis. This avoids small problems that can occur in some cases, such as unintended additional drift of a frame due to differential column axial shortening between gravity columns and lateral-load resisting columns when beams or rafters from the lateral-load resisting system are framed into the gravity columns. This approach also gives results that more closely match those from the more advanced methods to which the DM was calibrated. Note that the value of E is not reduced when applying other *Specification* provisions, such as slenderness limit checks (2005 AISC *Specification* Table B4.1) or column strength equations.

- (b) Minimum lateral loads of 0.002 times the vertical load, Y_i , applied at each level are required for gravity-only load combinations when $\Delta_{2nd}/\Delta_{1st} \leq 1.5$ ($\Delta_{2nd}/\Delta_{1st} \leq 1.71$ based on the reduced stiffness). Alternatively, explicit out-of-plumbness may be

modeled in lieu of notional loads.

For load combinations where $\Delta_{2nd}/\Delta_{1st} > 1.5$ ($\Delta_{2nd}/\Delta_{1st} > 1.71$ based on the reduced stiffness), the notional lateral loads must be added to any lateral loads already present in the combination.

For gable frames and for frames with stories having unequal height columns, it is recommended that individual notional lateral loads equal to $0.002y_i$ be applied at the top of each column, where y_i is the vertical load transferred to the column at its top. Also, for columns with intermediate vertical loads along their length, a notional lateral load of $0.002y_i$ should be applied at the location of the intermediate vertical loads, where y_i is the intermediate vertical load applied to the column.

These notional load and out-of-plumbness magnitudes are based on a specified maximum out-of-plumbness of $L/500$. For structures where a different out-of-plumbness is specified, the notional loads should be scaled linearly. Further discussions of this implementation of the notional lateral loads are provided in item 3(b) of Section 4.6.1 and in Section 4.4.3.

- (c) For ASD, the analysis is conducted using loads of 1.6 times those from ASD load combinations. The resulting member forces and moments are divided by 1.6 for member design calculations. The 1.6 multiplier also applies to any notional loads added to satisfy item 3(b).
4. The in-plane flexural buckling strength of columns and beam-columns, P_{ni} , is calculated based on the actual unbraced length with $K = 1.0$ except as noted for the three cases discussed here, where simplifying extensions to the AISC *Specification* are provided. The reduced member stiffnesses in item 3(a) should not be used in the member strength calculations. The member resistances are always calculated using nominal (unreduced) stiffnesses.
 - (a) For members with $\alpha P_r \leq 0.10P_{eL}$ at all locations along their length, or stated more simply, for $\alpha/\gamma_{eL} \leq 0.10$, P_{ni} may be taken as the equivalent cross-section axial yield strength accounting for local buckling effects, QP_y . This simplification is permissible because the in-plane stability effects are very minor at the member level for columns or beam-columns that satisfy the preceding limit. Many members in a typical single-story metal building frame will satisfy this limit. Note that P_{eL} and γ_{eL} in these expressions do not contain an overbar, i.e., these limits are checked using the nominal

elastic stiffness.

- (b) If $P-\delta$ effects are included in the analysis model and an appropriate member out-of-straightness between nodes is also included in the model, P_{ni} may be taken as QP_y , even when $\alpha/\gamma_{eL} > 0.10$. This is permissible because the reduced stiffness and out-of-straightness in the analysis account sufficiently for the in-plane stability effects at the member level. The appropriate member out-of-straightness is an imperfection of $0.001L$ in the direction that the member deforms relative to a chord between its support points or points of connection to other members. A chorded representation of the out-of-straightness with a maximum amplitude at the middle of the unsupported length is considered sufficient.
- (c) For gable rafters, when the midspan work point (cross-section centroid) is offset above the rafter chord by $L_{chord}/50$ or more, where L_{chord} is the span length along the rafter chord between the cross-section centroids at the tops of the columns, P_{ni} may be taken as QP_y . This is permissible because the offset of the midspan work point for these types of members nullifies the importance of any out-of-straightness relative to the chord between the ends of the on-slope length of the rafters. For rafters framing between equal height columns, this requirement is satisfied in all cases when the pitch of the rafter centroidal axis is at least $1/2$ in 12 throughout the span length.

4.6.3 The First-Order Method (FOM)

1. The FOM is only permitted for load combinations where $\Delta_{2nd}/\Delta_{1st} \leq 1.5$. Because the objective of using the FOM is likely to be the avoidance of a second-order analysis, it is suggested that the ratio $\Delta_{2nd}/\Delta_{1st}$ be determined using the AISC *Specification* Equation C2-3 for B_2 with ΣP_{e2} taken from Equation C2-6b.

In addition, for all members whose flexural stiffness contributes to the lateral stability, αP_r must be less than or equal to $0.5P_y$, where P_y is the lowest axial yield strength of the member.

2. A first-order analysis is performed as follows:
 - (a) The analysis is conducted without member stiffness reductions.
 - (b) Notional loads must be applied in addition to any lateral loads in each load combination. These are calculated as:

$$N_i = 2.1(\Delta/L)Y_i \geq 0.0042Y_i \quad (4.6-8, \text{Spec. Eq. C2-8})$$