

Bulletin on Stability Design of Cylindrical Shells

API BULLETIN 2U
THIRD EDITION, JUNE 2004



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Upstream Segment

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FOREWORD

This Bulletin is under jurisdiction of the API Subcommittee on Offshore Structures.

This Bulletin contains semi-empirical formulations for evaluating the buckling strength of stiffened and unstiffened cylindrical shells. Used in conjunction with API RP 2T or other applicable codes and standards, this Bulletin will be helpful to engineers involved in the design of offshore structures which include large diameter stiffened or unstiffened cylinders.

The buckling formulations and design considerations contained herein are based on classical buckling formulations, the latest available test data, and analytical studies. This third edition of the Bulletin provides buckling formulations and design considerations based on classical buckling solutions. It also incorporates user experience and feedback from users. It is intended for design and/or review of large diameter cylindrical shells, typically identified as those with D/t ratios greater than or equal to 300. Equations are provided for the prediction of stresses at which typical modes of buckling failures occur for unstiffened and stiffened cylindrical shells, from which the design of the shell plate and the stiffeners may be developed. Used in conjunction with API RP 2T or other applicable codes and standards, this Bulletin will be helpful to engineers involved in the design of offshore structures that include large diameter unstiffened and stiffened cylindrical shells.

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Suggested revisions are invited and should be submitted to API, Standards Department, 1220 L Street, NW, Washington, DC 20005

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Stability Design of Cylindrical Shells

Nomenclature

Note: The terms not defined here are uniquely defined in the sections in which they are used.

I	=	subscript denoting direction and load.
ϕ	=	longitudinal direction and any load combination
θ	=	hoop direction and any load combination
x	=	longitudinal direction and axial compression load only ($N_\theta = 0$).
h	=	hoop direction and hydrostatic external pressure ($N_\phi / N_\theta = 0.5$).
r	=	hoop direction and radical external pressure ($N_\phi = 0$).
j	=	subscript denoting buckling failure mode.
L	=	local shell buckling.
B	=	bay instability.
G	=	general instability.
C	=	column buckling.
$\bar{A}_r$	=	$A_r / (L_r t)$.
A_r	=	cross-sectional area of one ring stiffener, [in. ²].
A_s	=	cross-sectional area of one stringer stiffener, [in. ²].
A_t	=	total cross-sectional area of cylinder, [in. ²].
A_t	=	$2\pi R t + N_s A_s$, [in. ²].
b	=	stringer spacing as an arc dimension on the shell centerline, [in].
b_e	=	effective width of shell in at the shell centerline in the circumferential direction, [in].
B	=	mean bias factor.
C_m	=	end moment coefficient in interaction equation.
C_s	=	ratio of structural proportional limit to yield strength for a material.
C_x	=	elastic buckling coefficient = $\sigma_{xeL} R / E t$.
D_{max}, D_{min}	=	maximum and minimum shell diameters used to determine the out-of-roundness factor, γ , at a particular cross-section, in. The location giving the largest γ factor should be used.
D	=	centerline diameter of shell, [in].

D_o	=	outside diameter of shell, [in].
D_{nom}	=	nominal outside diameter of cylinder, [in].
E	=	modulus of elasticity, [ksi].
E_s	=	secant modulus
E_t	=	tangent modulus
f_a	=	applied (computed) axial stress, [ksi].
f_b	=	applied (computed) bending stress, [ksi].
f_θ	=	applied (computed) hoop stress, [ksi].
F_a	=	axial compressive stress permitted in the absence of bending moment, [ksi].
F_b	=	bending stress permitted in a cylinder in the absence of axial force, [ksi].
F_θ	=	hoop compressive stress permitted in a cylinder, [ksi].
F_{icj}	=	inelastic shell buckling stress for fabricated shell, [ksi].
F_{iej}	=	elastic shell buckling stress for fabricated shell, [ksi].
FS	=	factor of safety.
$\overline{F}_{iej}$	=	F_{iej}/β .
$\overline{F}_{icj}$	=	F_{icj}/β
F_y	=	minimum specified yield stress of material, [ksi].
F_{ys}	=	static yield stress of material (zero strain rate), [ksi].
G	=	shear modulus, $E/2(1 + \nu)$, [ksi].
g	=	$M_x M_\theta L_r t A_s / I_s$.
h_s	=	width (or depth) of stiffening element, [in].
I_{es}, I_{er}	=	moment of inertia of stringer and ring stiffener, respectively, plus effective width of shell about centroidal axis of combined section (see Fig. 2.2), [in. ⁴]
I_s, I_r	=	moment of inertia of stringer and ring stiffener, respectively, about its centroidal axis, [in. ⁴]
J_s, J_r	=	torsional stiffness constant of stringer and ring stiffener, respectively (for general non-circular shapes use $\Sigma h_s t_s^3/3$), [in. ⁴]
K	=	effective length factor for column buckling.

k	=	ratio of axial load to circumferential load (N_Φ / N_θ).
K_G	=	factor used in calculating ring stiffener stresses when a cylinder is subjected to external pressure.
K_L	=	factor used when calculating the shell stress at mid-bay, to account for the effects of a ring or end stiffeners, when a cylinder is subjected to external pressure.
K_p	=	effective pressure correction factor used in calculation of collapse pressure.
L	=	unsupported length of shell between rings, [in].
L_j	=	cylinder length for calculation of bay instability and general instability, [in].
L_e	=	effective width of shell in the longitudinal direction, [in].
L_b	=	length of cylinder between bulkheads or lines of support with sufficient stiffness to act as bulkheads. Lines of support which act as bulkheads include end ring stiffeners, [in].
L_r	=	ring spacing, [in].
L_t	=	unbraced length of cylinder, [in].
m	=	number of half waves into which the shell will buckle in the longitudinal direction.
M	=	applied bending moment.
M_1	=	the smaller of the moments at the ends of the unbraced length of a beam-column, [in-kips].
M_2	=	the larger of the moments at the ends of the unbraced length of a beam-column, [in-kips].
M_x	=	$L_r / \sqrt{Rt}$
M_θ	=	$b / \sqrt{Rt}$
n	=	number of waves into which the shell will buckle in the circumferential direction.
N_{iej}	=	theoretical elastic buckling load per unit length of shell (longitudinal or circumferential) for a perfect cylinder for both bay instability and general instability, kips per [in].

N_s	=	number of stringers.
N_ϕ	=	axial load per unit of circumference, [kips/in].
N_θ	=	circumferential load per unit of length, kips/in].
p	=	applied external pressure, [ksi].
p_{eB}	=	theoretical elastic failure pressure for bay instability mode, [ksi].
p_{eG}	=	theoretical elastic failure pressure for general instability mode, [ksi].
p_{eL}	=	theoretical failure pressure for local buckling mode, [ksi].
p_s	=	contribution of stringers to collapse pressure, [ksi].
p_{cG}	=	failure pressure for general instability mode, [ksi].
P	=	applied axial load, [ksi].
P_{cB}	=	inelastic axial compression bay instability load, kips.
p_{cB}	=	failure pressure for bay instability mode, [ksi].
p_{cL}	=	failure pressure for local buckling mode, [ksi].
r	=	radius of gyration, $r = (0.5R^2 + 0.125t^2)^{1/2} \approx 0.707R$, [in].
R	=	radius to centerline of shell, [in].
R_c	=	radius to centroidal axis of the combined ring stiffener and effective width of shell, [in].
R_o	=	radius to outside of shell, [in].
R_r	=	radius to centroid of ring stiffener, [in].
t	=	thickness of shell, [in].
t_w	=	thickness of web of ring stiffener, [in].
t_r, t_x	=	effective shell thickness, $t_r = (A_r + L_e t)/L_e$, $t_x = (A_s + b_e t)/b$, [in].
t_s	=	thickness of stiffening element, [in].
Z_r, Z_s	=	distance from centerline of shell to centroid of stiffener, for ring and stringer stiffeners, respectively (positive outward), [in].
α_{ij}	=	capacity reduction factor to account for the difference between classical theory and predicted instability stresses for fabricated shells.
β	=	a factor applied to the bay instability and general instability failure stresses to avoid interaction with the local buckling mode.
ρ_n, ρ_s	=	reduction factors used in computing collapse load for axial compression.

Δ_c, Δ_d	=	$F_{iej} / F_y, \bar{F}_{iej} / F_y$
η	=	plasticity reduction factor which accounts for the nonlinearity of material properties and the effects of residual stresses.
γ	=	$(D_{max} - D_{min}) 100 / D_{nom}$.
λ	=	$\pi R / L_r$.
λ_o, λ_e	=	slenderness parameters as defined in text and used for computing collapse load for axial compression.
λ_G	=	$\pi R / L_b$.
ν	=	Poisson's ratio.
σ_{iej}	=	theoretical elastic instability stress, [ksi].
ψ	=	partial safety factor.

SI Metric Conversion Factors

in. x 2.54	=	mm
ksi x 6.894757	=	MPa

Glossary

amplification reduction factor (Cm): Coefficient applied to bending term in interaction equation for members subjected to combined bending and axial compression to account for overprediction of secondary moment given by the amplification factor $1/(1 - f_a / F'_e)$.

asymmetric buckling: The buckling of the shell plate between the circumferential (i.e., ring) stiffeners characterized by the formation of two or more lobes (waves) around the circumference.

axial direction: Longitudinal direction of the member.

axisymmetric collapse: The buckling of the shell plate between the circumferential stiffeners characterized by accordion-like pleats around the circumference.

bay: The section of cylinder between rings.

bay instability: Simultaneous lateral buckling of the shell and stringers with the rings remaining essentially round.

capacity reduction factor (α_{ij}): Coefficient which accounts for the effects of shape imperfections, nonlinear behavior and boundary conditions (other than classical simply supported) on the buckling capacity of the shell.

critical buckling stress: The stress level associated with initiation of buckling. Critical buckling stress is also referred to as the inelastic buckling stress.

distortion energy theory: Failure theory defined by the following equation where the applied stresses are positive for tension and negative for compression.

$$f_a^2 - f_a f_\theta + f_\theta^2 = f_y^2$$

effective length (KL): The equivalent length used in compression formulas and determined by a bifurcation analysis.

effective length factor (K): The ratio between the effective length and the unbraced length of the member.

effective section: Stiffener together with the effective width of shell acting with the stiffener.

effective width: The reduced width of shell or plate which, with an assumed uniform stress distribution, produces the same effect on the behavior of a structural member as the actual width of shell or plate with its nonuniform stress distribution.

elastic buckling stress: The buckling stress of a cylinder based upon elastic behavior.

general instability: Buckling of one or more circumferential (i.e., ring) stiffeners with the attached shell plate in ring-stiffened cylindrical shells. For a ring- and stringer-stiffened cylindrical shell general instability refers to the buckling of one or more rings and stringers with the attached shell plate.

hierarchical order of instability: Refers to a design method that will ensure development of a design with the most critical instability mode (i.e., general instability) having a higher critical buckling stress than the less critical instability mode (i.e., local instability).

hydrostatic pressure: Uniform external pressure on the sides and ends of a member.

inelastic buckling stress: The buckling stress of a cylinder which exceeds the elastic stress limit of the member material. The inelastic material properties are accounted for, including effects of residual stresses due to forming and welding.

interaction of instability modes: Critical buckling stress determined for one instability mode may be affected (i.e., reduced) by another instability mode. Elastic buckling stresses for two or more instability modes should be kept apart to preclude an interaction between instability modes.

local instability: Buckling of the shell plate between the stiffeners with the stiffeners (i.e., rings or rings and stringers) remaining intact.

membrane stresses: The in-plane stresses in the shell; longitudinal, circumferential or shear.

maximum shear stress theory: Failure theory defined by the following equation:

$$\sigma_1 - \sigma_2 = F_y$$

where σ_1 is the maximum principal stress and σ_2 is the minimum principal stress, with tension positive and compression negative.

orthogonally stiffened: A member with circumferential (ring) and longitudinal (stringer) stiffeners.

radial pressure: Uniform external pressure acting only on the sides of a member.

residual stresses: The stresses that remain in an unloaded member after it has been formed and installed in a structure. Some typical causes are forming, welding and corrections for misalignment during installation in the structure. The misalignment stresses are not accounted for by the plasticity reduction factor η .

ring stiffened: A member with circumferential stiffeners.

shell panel: That portion of a shell which is bounded by two adjacent rings in the longitudinal direction and two adjacent stringers in the circumferential direction.

slenderness ratio (KL/r): The ratio of the effective length of a member to the radius of gyration of the member.

stress relieved: The residual stresses are significantly reduced by post weld heat treatment.

stringer stiffened: A member with longitudinal stiffeners.

yield stress: The yield stress of the material determined in accordance with ASTM A307.

SECTION 1—General Provisions

1.1 SCOPE

1.1.1 This Bulletin provides stability criteria for determining the structural adequacy against buckling of large diameter circular cylindrical members when subjected to axial load, bending, shear and external pressure acting independently or in combination. The cylinders may be unstiffened, longitudinally stiffened, ring stiffened or stiffened with both longitudinal and ring stiffeners. Research and development work leading to the preparation and issue of all three editions of this Bulletin is documented in References 1 through 16 and the Commentary.

1.1.2 The buckling capacities of the cylinders are based on linear bifurcation (classical) analyses reduced by capacity reduction factors which account for the effects of imperfections and nonlinearity in geometry and boundary conditions and by plasticity reduction factors which account for nonlinearity in material properties. The reduction factors were determined from tests conducted on fabricated steel cylinders. The plasticity reduction factors include the effects of residual stresses resulting from the fabrication process.

1.1.3 Fabricated cylinders are produced by butt-welding together cold or hot formed plate materials. Long fabricated cylinders are generally made by butt-welding together a series of short sections, commonly referred to as cans, with the longitudinal welds rotated between the cans. Long fabricated cylinders generally have D/t ratios less than 300 and are covered by AP RP 2A.

1.2 LIMITATIONS

1.2.1 The criteria given are for stiffened cylinders with uniform thickness between ring stiffeners or for unstiffened cylinders of uniform thickness. All shell penetrations must be properly reinforced. The results of experimental studies on buckling of shells with reinforced openings and some design guidance are given in Ref. 2. The stability criteria of this bulletin may be used for cylinders with openings that are reinforced in accordance with the recommendations of Ref. 2 if the openings do not exceed 10% of the cylinder diameter or 80% of the ring spacing. Special consideration must be given to the effects of larger penetrations.

1.2.2 The stability criteria are applicable to shells with diameter-to-thickness (D/t) ratios equal to or greater than 300 but less than 1200 and shell thicknesses of 5 mm (3/16 in.) or greater. The deviations from true circular shape and straightness must satisfy the requirements stated in this bulletin, refer to section 10.

1.2.3 Special considerations should be given to the ends of members and other areas of load application where the stress distribution may be nonlinear and localized stresses may exceed those predicted by linear theory. When the localized stresses extend over a distance equal to

one half wave length of the buckling mode, they should be considered as a uniform stress around the full circumference. Additional thickness or stiffening may be required.

1.2.4 Failure due to material fracture or fatigue and failure caused by dents resulting from accidental loads are not considered in the bulletin.

1.3 STRESS COMPONENTS FOR STABILITY ANALYSIS AND DESIGN

The internal stress field which controls the buckling of a cylindrical shell consists of the longitudinal membrane, circumferential membrane and in-plane shear stresses. The stresses resulting from a dynamic analysis should be treated as equivalent static stresses.

1.4 STRUCTURAL SHAPE AND PLATE SPECIFICATIONS

Unless otherwise specified by the designer, structural shapes and plates should conform to one of the specifications listed in Table 8.1.4-1/2 of API RP 2A, 20th edition, or Table 4 of API RP 2T.

1.5 HIERARCHICAL ORDER AND INTERACTION OF BUCKLING MODES

1.5.1 This Bulletin requires avoidance of failure in any mode, and recommends sizing of the cylindrical shell plate and the arrangement and sizing of the stiffeners to ensure that the buckling stress for the most critical general instability is higher than the less critical local instability buckling stress.

1.5.2 A hierarchical order of buckling stresses with adequate separation of general, bay and local instability stresses is also desirable for a cylindrical shell subjected to loading resulting in longitudinal and circumferential stresses to preclude any interaction of buckling modes. To prevent a reduction in buckling stress due to interaction of buckling modes, it is recommended that bay and general instability mode elastic buckling stresses remain at least 1.2 times the elastic buckling stress for local instability.

SECTION 2—Geometries, Failure Modes, and Loads

The maximum stresses corresponding to all of the failure modes will be referred to as buckling stresses. Buckling stress equations are given for the following geometries, failure modes and load conditions.

2.1 GEOMETRIES

- a. Unstiffened.
- b. Ring Stiffened.
- c. Stringer Stiffened.
- d. Ring and Stringer Stiffened.

The four cylinder geometries are illustrated in Figure 2.1 and the stiffener geometries in Figure 2.2.

2.2 FAILURE MODES

- a. **Local Shell Buckling**—buckling of the shell plate between stiffeners. The stringers remain straight and the rings remain round.
- b. **Bay Instability**—buckling of the stringers together with the attached shell plate between rings (or the ends of the cylinders for stringer stiffened cylinders). The rings and the ends of the cylinders remain round.
- c. **General Instability**—buckling of one or more rings together with the attached shell (shell plus stringers for ring and stringer stiffened cylinders).
- d. **Local Stiffener Buckling**—buckling of the stiffener elements.
- e. **Column Buckling**—buckling of the cylinder as a column.

The first four failure modes are illustrated in Figure 2.3.

2.3 LOADS AND LOAD COMBINATIONS

- a. **Determination of Applied Stresses Due to the Following Loads:**
 - 1. Longitudinal stress due to axial compression/tension and overall bending.
 - 2. Shear stress due to transverse shear and torsion.
 - 3. Circumferential stress due to external pressure.
 - 4. Combined (von Mises) stress due to combination of loads.

b. Determination of Utilization Ratios Based on Recommended Interaction Relationships for Combined Loads:

1. Longitudinal (axial) tension and circumferential (hoop) compression.
2. Longitudinal (axial) compression and circumferential compression.

Note: Stresses and stress combinations considered are for in-plane loads and do not account for secondary bending stresses due to out-of-plane pressure loading on shell plate.

Some of the external pressure on an orthogonally stiffened cylindrical shell will be directly transferred to the rings through the stringers and the resulting bending stresses in the stringers may be appreciable. Local, bay and general instability stresses compared against the applied axial and hoop stresses, whether obtained from a finite element analysis or determined based on equations in Section 11, may need to be supplemented by checking effective stringer column instability as an appropriate beam column element.

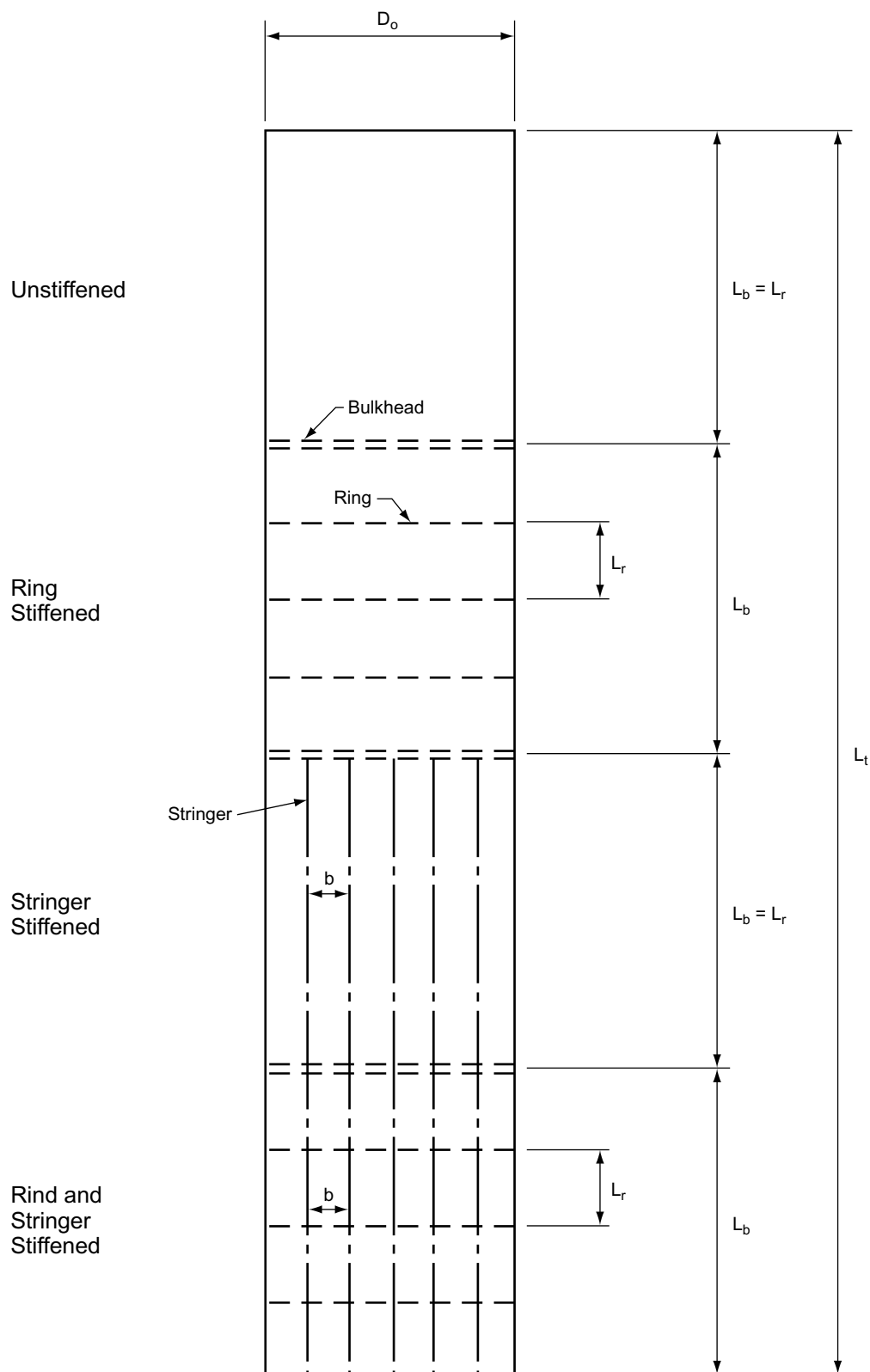
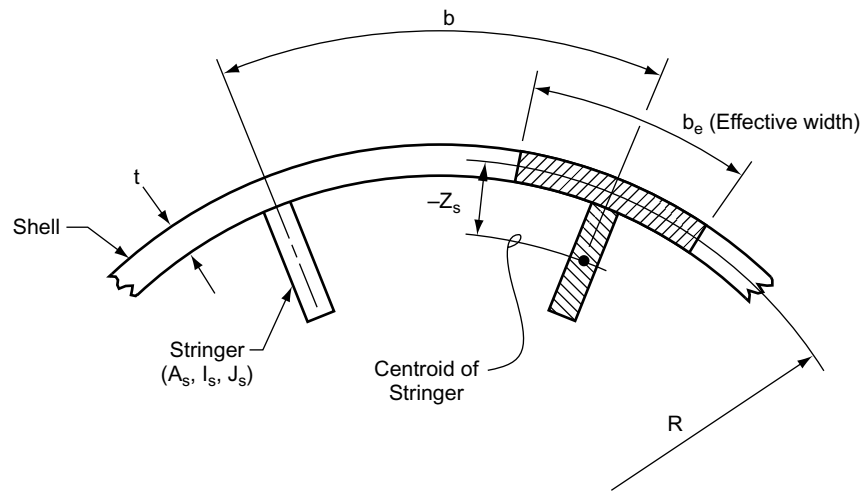
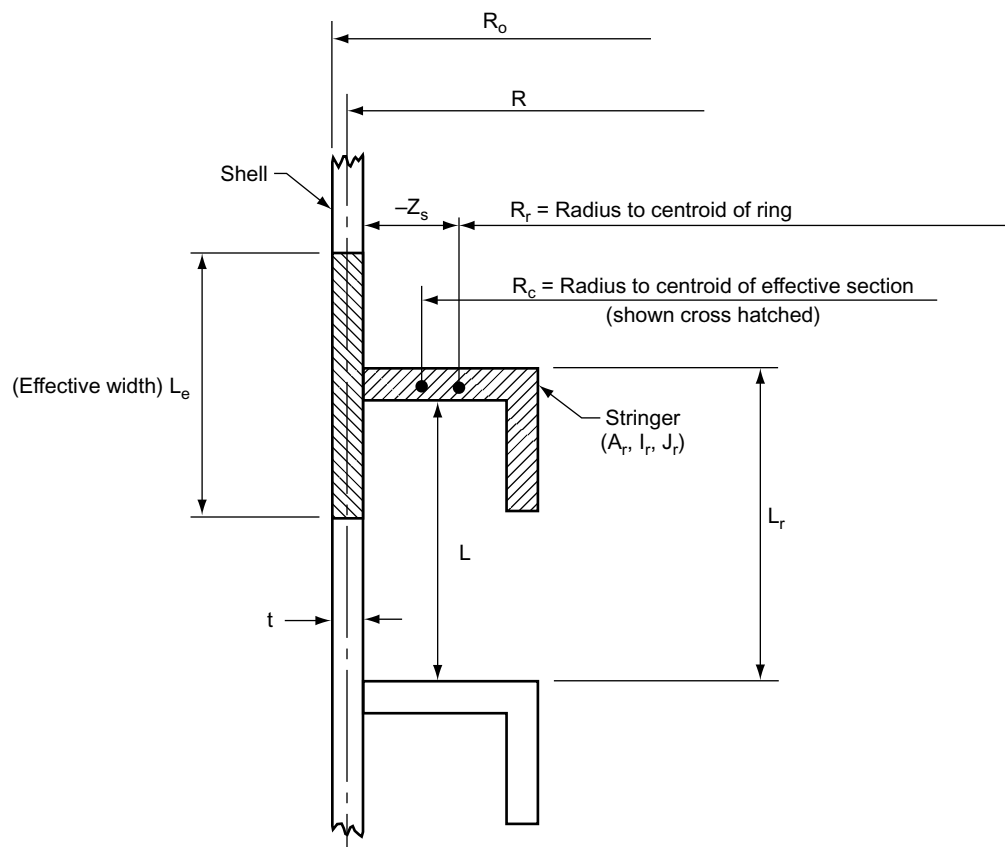


Figure 2.1--Geometry of Cylinder



Section Through Stringers



Section Through Rings

Figure 2.2--Geometry of Stiffeners

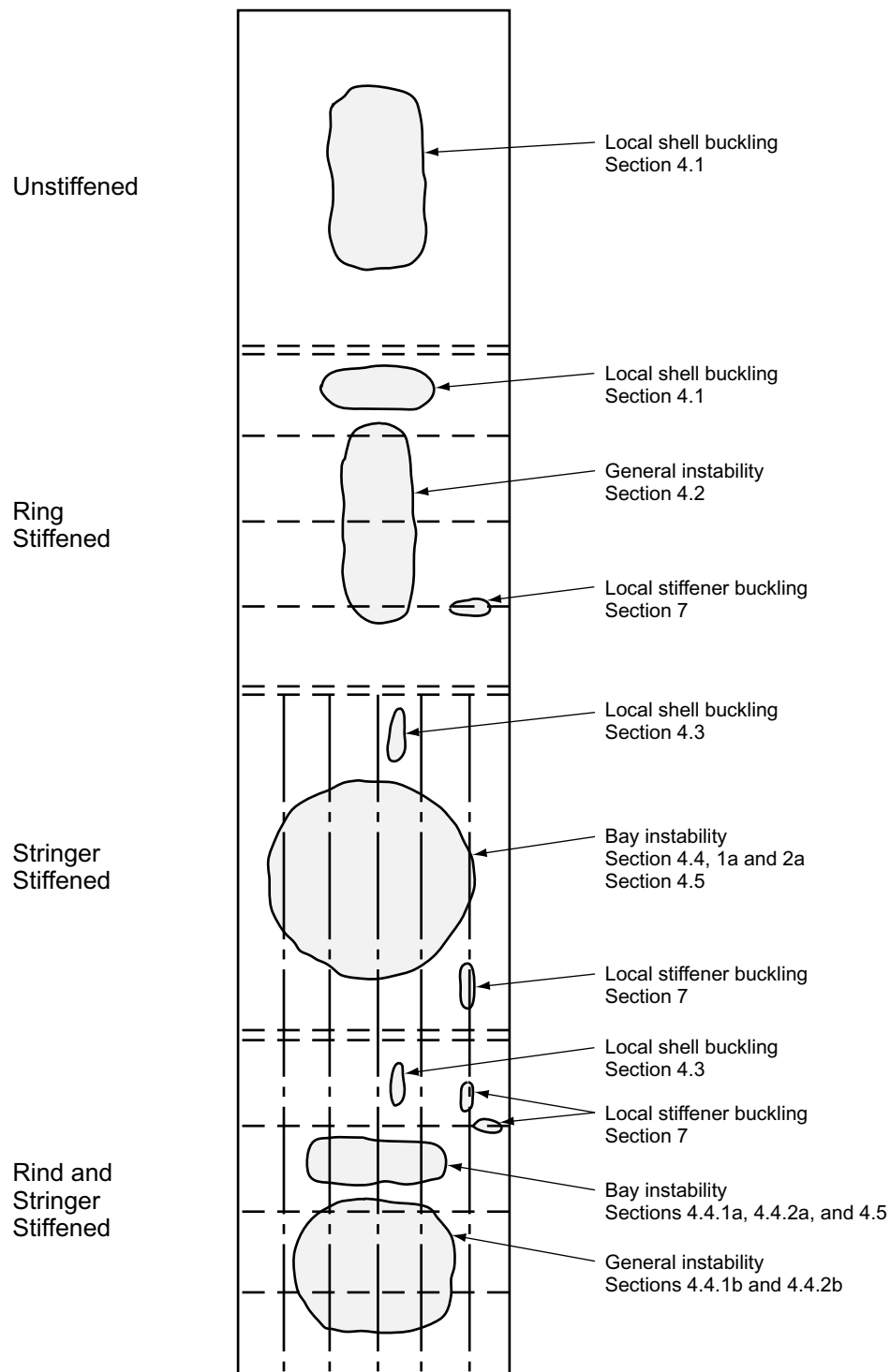


Figure 2.3--Shell Buckling Modes for Cylinders

SECTION 3—Buckling Design Method

3.1 The buckling strength formulations presented in this bulletin are based upon classical linear theory which is modified by reduction factors α_{ij} and η which account for the effects of imperfections, boundary conditions, nonlinearity of material properties and residual stresses. The reduction factors are determined from approximate lower bound values of test data of shells with initial imperfections representative of the specified tolerance limits given in Section 10.

3.2 The general equations for the predicted shell buckling stresses for fabricated steel cylinders subjected to the individual load cases of axial compression, bending and external pressure are given by Equations (3.2-1) and (3.2-2). The equations for α_{ij} and σ_{iej} are given in Section 4 and the equations for η are given in Section 5.

a. Elastic Shell Buckling Stress

$$F_{iej} = \alpha_{ij} \sigma_{iej} \quad (3.2-1)$$

b. Inelastic Shell Buckling Stress

$$F_{iej} = \eta F_{iej} \quad (3.2-2)$$

3.3 The bay instability stresses for cylinders with stringer stiffeners are given by orthotropic shell theory. This theory requires that the number of stringers must be greater than about three times the number of circumferential waves corresponding to the buckling mode. An alternate method is given for determining the bay instability stresses for cylinders which do not satisfy this requirement.

3.4 The buckling stress equations for cylinders under the individual load cases of axial compression, bending, radial external pressure ($N_\phi = 0$) and hydrostatic external pressure ($N_\phi/N_\theta = 0.5$) are given in Section 4. Interaction equations are given in Section 6 for cylinders subjected to combinations of axial load, bending and external pressure. The interaction between column buckling and shell buckling is considered in Section 8. The method for determining the size of stiffeners is given in Section 7.

3.5 A flow chart is given in Figure 3.1 for determining the allowable stresses. The equations for allowable stresses are given in Section 9 and equations for determining the stresses due to applied load are given in Section 11. A summary of the sections relating to the buckling modes for each of the different shell geometries is given on Table 3.1.

Table 3.1—Section Numbers Relating to Buckling Modes for Different Shell Geometries

	Geometry			
Buckling Mode	Unstiffened	Ring Stiff	Stringer Stiff	Ring and Stringer Stiff
Local Shell Buckling	4.1	4.1	4.3	4.3
Bay Instability			4.4 4.5	4.4 4.5
General Instability		4.2		4.4 (1b, 2b)
Local Stiffener Buckling		7.2	7.2	7.2
Column Buckling	8.0	8.0	8.0	8.0

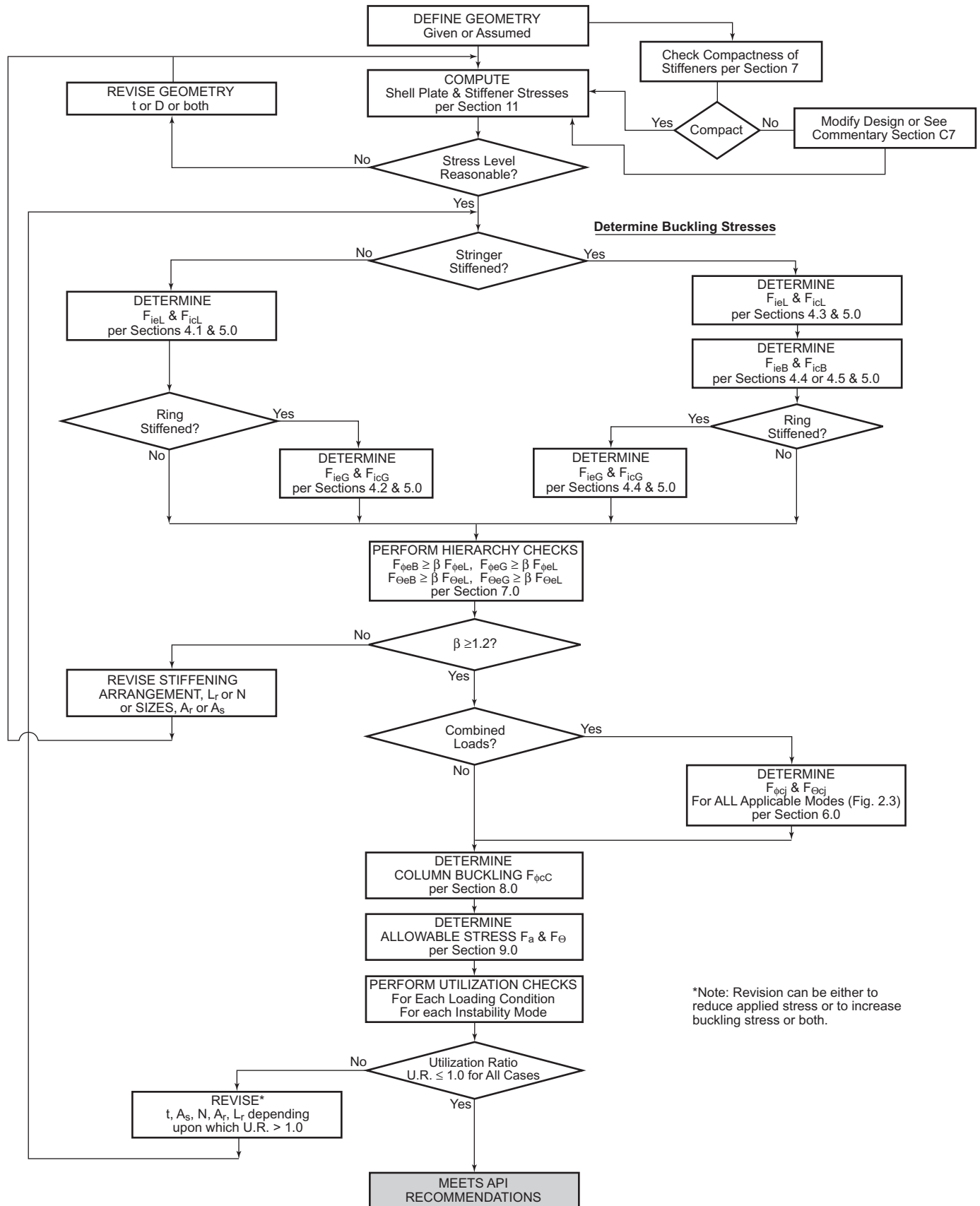


Figure 3.1--Flow Chart for Meeting API Recommendations



SECTION 4—Predicted Shell Buckling Stresses for Axial Load, Bending and External Pressure

This section gives equations for determining the shell buckling stresses for the load cases of axial compression, bending, radial external pressure ($N_\phi = 0$) and hydrostatic external pressure ($N_\phi = 0.5 N_\theta$). The general equations for predicting the elastic and inelastic buckling stresses for fabricated cylinders are given by Equations 3.2-1 and 3.2-2. The equations for determining α_{ij} and σ_{iej} are given in the following section. The equations for determining the plasticity reduction factors, η , are given in Section 5. Equations given in this section are based on the behavior of large diameter cylindrical shells and permit determination of local, bay and general instability mode buckling stresses. As illustrated in the Commentary, predicted stresses include imperfection/correction factors and are compatible with test data. Predicted stresses are based on the assumption that the instability modes are separated and do not interact. To ensure the assumption remains valid, a hierarchy among the instability modes is required. As shown in Section 7, ring and stringer stiffener spacing and sizes should be modified, as necessary, to achieve the desirable hierarchy.

The values of M_x and M_θ appearing in the following equations are defined as:

$$M_x = L_r / (Rt)^{0.5} \text{ and } M_\theta = b / (Rt)^{0.5} \quad (4-1a)$$

$$Z_x = M_x^2 (1 - \nu^2)^{0.5} \text{ and } Z_\theta = M_\theta^2 (1 - \nu^2)^{0.5} \quad (4-1b)$$

Where the term Z represents the classical definition of geometric parameter.

4.1 LOCAL BUCKLING OF UNSTIFFENED OR RING STIFFENED CYLINDERS

4.1.1 Axial Compression or Bending ($N_\theta = 0$)

The buckling stresses for cylinders subjected to axial compression or bending are assumed to be the same (see Commentary).

a. Elastic Buckling Stresses

$$F_{xeL} = C_{xL} \frac{\pi^2 E}{12(1 - \nu^2)} (t / L_r)^2 \quad (4.1-1)$$

where the buckling coefficient, C_{xL} , is expressed in terms of geometric curvature parameter, M_x , the D/t ratio and the imperfection factor, α_{xL} :

$$C_{xL} = \left[1 + \left(\frac{150}{(D/t)} \right) (\alpha_{xL})^2 (M_x^4) \right]^{0.5} \quad (4.1-2)$$

where, the imperfection factor is defined in paragraph 4.1.1(b).

b. Imperfection Factor, α_{xL}

$$\alpha_{xL} = 9.0 / (300 + D/t)^{0.4} \quad (4.1-3)$$

c. Inelastic Buckling Stresses: The buckling stress in the material elasto-plastic zone is determined following the empirical formulation given in Section 5.

$$F_{cL} = \eta F_{eL} \quad \text{for } F_{eL} > 0.5F_y$$

(i.e., $F_{eL} >$ material proportional limit)

$$F_{cL} = F_{eL} \quad \text{for } F_{eL} < 0.5F_y$$

(i.e., $F_{eL} <$ material proportional limit) (4.1-4)

4.1.2 External Pressure ($N_\phi N_\theta = 0$ or 0.5)

a. Elastic Buckling Stresses

$$F_{heL} = F_{reL} = C_{\theta L} \frac{\pi^2 E}{12(1-\nu^2)} (t/L_r)^2 \quad (4.1-5)$$

where the buckling coefficient, $C_{\theta L}$, will have a different definition for each geometry as defined by its asymmetric buckling mode (i.e., number of lobes, n).

The buckling coefficient, $C_{\theta L}$, can be readily obtained by defining the geometric curvature parameter, M_x , and the following formula based on Batdorf-introduced simplifications to Donnell's equations. A simple iterative approach is necessary to determine the number of half-waves (i.e., lobes) "n". Since API provides for determination of instability modes higher than that of local instability mode, local instability is considered not to interact with other instability modes. This is achieved by implementing the hierarchical failure order as required by Section 7. For unstiffened and ring-stiffened cylindrical shells and imperfection factor is defined in Section 4.1.2b.

Assuming a single mode, $m = 1$, between the rings, Batdorf's equation permits determination of the number of buckling lobes, n , from the following equation:

$$\frac{\beta^2(1+\beta^2)^4}{2+3\beta^2} = Z_m \quad (4.1-6)$$

where the modified geometric curvature parameter, Z_m , and β can be expressed as:

$$Z_m = \frac{12Z_x^2}{\pi^4} = \frac{12 \left[M_x^2 (1-\nu^2)^{1/2} \right]^2}{\pi^4}$$

$$Z_m = 0.112M_x^4 \quad \text{for Poisson's ratio of 0.3}$$

$$\beta = L_r / (\pi R / n)$$

The smallest “n” that causes that left and the right side of the equation (4.1-6) to be approximately equal defines the asymmetric buckling of the shell plate. Then the buckling coefficient, $C_{\theta L}$, can be directly computed from the following equation:

$$C_{\theta L} = \left[\frac{(1 + \beta^2)^2}{(0.5 + \beta^2)} + \frac{0.112M_x^4}{(1 + \beta^2)^2(0.5 + \beta^2)} \right] [\alpha_{\theta L}] \quad (4.1-7)$$

b. Imperfection Factor, $\alpha_{\theta L}$

For cylindrical shells with D/t ratios greater than 300, test-to-predicted stress ratios indicate that the use of an imperfection factor equal to 0.8 is too conservative. It is recommended that:

$$\begin{aligned} \alpha_{\theta L} &= 1.0 & \text{if } M_x < 5 \\ \alpha_{\theta L} &= 0.8 & \text{if } M_x > 5 \end{aligned} \quad (4.1-8)$$

c. Inelastic Buckling Stresses

Inelastic buckling stress definitions in terms of plasticity reduction factors to be applied on elastic buckling stresses are given in Section 5.

4.1.3 Transverse Shear

Panel instability due to transverse shear and torsion can be critical at interfaces. Critical buckling stress is affected not only by the shell thickness and the panel aspect ratio, but also by the boundary conditions.

As a minimum, it is necessary to incorporate the shear stress in a von Mises stress check to assess the overall effect of combined loads.

The local shear stress may become important when concentrated local load transfers occur due to attachments/appurtenances. Further discussion on this subject is presented in Section 4.3.3.

4.2 GENERAL INSTABILITY OF RING STIFFENED CYLINDERS

4.2.1 Axial Compression or Bending ($N_\theta = 0$)

a. Elastic Buckling Stresses

$$F_{xeG} = \alpha_{xG} \sigma_{xeG} = \alpha_{xG} 0.605E \frac{t}{R} (1 + \bar{A}_r)^{1/2} \quad (4.2-1)$$

$$\bar{A}_r = \frac{A_r}{L_r t}$$

where $\bar{A}_r$ is the ring area and L_r is the ring spacing.

b. Imperfection Factors

$$\alpha_{xG} = \begin{cases} 0.72 & \text{if } \bar{A}_r \geq 0.2 \\ (3.6 - 5.0\alpha_x)\bar{A}_r + \alpha_x & \text{if } 0.06 < \bar{A}_r < 0.2 \\ \alpha_x & \text{if } \bar{A}_r \leq 0.06 \end{cases} \quad (4.2-2)$$

$$\text{where } \alpha_x = 0.85 / [1 + 0.0025(D/t)] \quad (4.2-3)$$

c. Inelastic Buckling Stresses

Inelastic buckling stress definitions in terms of plasticity reduction factors to be applied on elastic buckling stresses are given in Section 5.

4.2.2 External Pressure ($N_\phi/N_\theta = 0$ or 0.5)

a. Elastic Buckling Stresses With or Without End Pressure

$$F_{reG} \text{ or } F_{heG} = \alpha_{\theta G} \frac{p_{eG} R_o}{t} K_{\theta G} \quad (4.2-4)$$

where $K_{\theta G}$ is given by Equation 11.3-12a

$$p_{eG} = \frac{E(t/R)\lambda_G^4}{(n^2 + k\lambda_G^2 - 1)(n^2 + \lambda_G^2)^2} + \frac{EI_{er}(n^2 - 1)}{L_r R_c^2 R_o} \quad (4.2-5)$$

where $\lambda_G = \pi R/L_b$, $k = 0$ for radial pressure and 0.5 for hydrostatic pressure, R_c is the radius to the centroid of the effective section, R_o is the radius to the outside of the shell and I_{er} is the moment of inertia of the effective section given by the following equation:

$$I_{er} = I_r + A_r Z_r^2 \frac{L_e t}{A_r + L_e t} + \frac{L_e t^3}{12} \quad (4.2-6)$$

where Z_r is the distance from the centerline of the shell to the centroid of the stiffener ring (positive outward).

The value of L_e can be approximated by $1.1\sqrt{Dt} + t_w$ when $M_x > 1.56$ and L_r when $M_x \leq 1.56$. The correct value for n is the value which gives the minimum value of p_{eG} in Equation 4.2-5. The minimum value of n is 2 and the maximum value will be less than 10 for most shells of interest. The minimum pressure will correspond to a noninteger value of n . The pressure p_{eG} is determined by trial and error.

- b. Imperfection Factors:** For fabricated cylinders which meet the fabrication tolerances given in Section 10, a constant value of $\alpha_{\theta G} = 0.8$ is adequate.
- c. Inelastic Buckling Stresses:** Inelastic buckling stress definitions in terms of plasticity reduction factors to be applied on elastic buckling stresses are given in Section 5.
- d. Failure Pressures**

$$p_{cG} = \eta \alpha_{\theta G} p_{eG} \quad (4.2-7)$$

See a, b, and c above for determination of the terms in Equation 4.2-7.

4.3 LOCAL BUCKLING OF STRINGER STIFFENED OR RING AND STRINGER STIFFENED CYLINDERS

The following equations are based upon the assumption that the stringers satisfy the compact section requirements of Section 7. A method is presented in the Commentary for noncompact sections.

4.3.1 Axial Compression or Bending ($N_{\theta} = 0$)

For the stringers to be effective in increasing the buckling stress, they must be spaced sufficiently close so that $M_{\theta} < 15$ and $b < 2L_r$. For values of $M_{\theta} > 15$ or $b > 2L_r$ the buckling stresses are computed as if the stringers were omitted. However, the stringers may be assumed to be effective in carrying part of the axial loading when computing the stresses due to applied loads.

- a. Elastic Buckling Stresses**

$$F_{xeL} = C_{xL} \frac{\pi^2 E}{12(1-\nu^2)} (t/b)^2 \quad (4.3-1)$$

where the buckling coefficient, C_{xL} , is expressed in terms of geometric curvature parameter, M_{θ} :

$$C_{xL} = 4.0 \quad M_{\theta} \leq 2 \quad (4.3.2)$$

$$C_{xL} = 4.0 \left\{ 1 + 0.038 [M_{\theta} - 2]^3 \right\} [\alpha_{xL}] \quad M_{\theta} > 2$$

where the stringer spacing, b , is defined as $\pi D / N$, and the imperfection reduction factor, α_{xL} , is set equal to 1.0 for geometries meeting API-recommended tolerances.

- b. **Inelastic Buckling Stresses:** Inelastic buckling stresses should be determined by applying plasticity reduction factors to elastic buckling stresses as recommended in Section 5.

4.3.2 External Pressure ($N_\phi / N_\theta = 0$ or 0.5)

The local buckling pressure of a stringer stiffened cylinder will be greater than a corresponding unstiffened or ring stiffened cylinder if $0.5N_s$ (N_s = Number of Stringers) is greater than the number of circumferential waves at buckling for the cylinder without stringers. This is based upon the assumption that one-half wave will form between stringers at buckling. For stringers with high torsional rigidity a full wave might form between stringers with a concurrent increase in the buckling pressure. This possible increase in buckling pressure is not considered.

- a. **Elastic Buckling Stresses With or Without End Pressure**

$$F_{\theta L} = C_{\theta L} \frac{\pi^2 E}{12(1-\nu^2)} (t / L_r)^2 \quad (4.3-3)$$

where the buckling coefficient, $C_{\theta L}$, is defined by:

$$C_{\theta L} = \frac{(1 + [L_r / b]^2)^2}{(L_r / b)^2} \left[1 + \frac{0.011 M_x^3}{0.5(1 + [L_r / b]^2)^2} \right] [\alpha_{\theta L}] \quad (4.3-4)$$

If the stringer spacing is large and the aspect ratio is small, the minimum number of buckling lobes, n , for an unstiffened cylindrical shell may yield a buckling coefficient larger than that obtained from above given buckling coefficient. In such instances, cylindrical shell should be treated as unstiffened and the buckling coefficient determined from equations in Section 4.1.2.

- b. **Imperfection Factors:** The test results indicate that no imperfection reduction factor is needed for stringer stiffened cylinders. Therefore:

$$\alpha_{\theta L} = 1.0$$

- c. **Inelastic Buckling Stresses:** Inelastic instability stresses are determined by applying plasticity reduction factors to elastic buckling stresses as recommended in Section 5.

4.4 BAY INSTABILITY OF STRINGER STIFFENED OR RING AND STRINGER STIFFENED CYLINDERS, AND GENERAL INSTABILITY OF RING AND STRING STIFFENED CYLINDERS BASED UPON ORTHOTROPIC SHELL THEORY

The theoretical elastic buckling loads for both bay instability and general instability are given by the following orthotropic shell equation (Equation 4.4-1). The elastic buckling load per

unit length of shell is denoted N_{iej} where i is the stress direction and j is the buckling mode with $j = B$ for bay instability and $j = G$ for general instability. The bay instability stress is determined by letting the cylinder length equal the ring spacing ($L_j = L_r$) and the general instability stress is determined by letting the cylinder length equal the distance between bulkheads or stiffener rings that are sufficiently sized to act as bulkheads ($L_j = L_b$).

When the rings and stringers are not sufficiently close together so that the shell plating is fully effective, the rigidity parameters (E_x , E_θ , D_x , D_θ , $D_{x\theta}$, $G_{x\theta}$) of Equation 4.4-1 are modified by the ratios of effective width to stiffener spacing. Equations are given for L_e and b_e which are the effective widths of plate in the x and θ directions, respectively. When $L_e < L_r$ or $b_e < b$, set $\nu = 0$; otherwise $\nu = 0.3$. In all cases, use $\nu = 0.3$ when calculating G .

The values of m and n to use in the following equation are those which minimize N_{iej} where $m \geq 1$ and $n \geq 2$. For the following equation to be valid, the number of stringers must be greater than about $3n$ and the bay instability stress should be less than 1.5 times the local shell buckling stress. When these conditions are not met, the equations in Section 4.5 should be used for sizing the stringers. Section 4.2 should be used for sizing the rings.

(4.4-1)

$$N_{iej} = \left[A_{33} + \frac{A_{12}A_{23} - A_{13}A_{22}}{A_{11}A_{22} - A_{12}^2} A_{13} + \frac{A_{12}A_{13} - A_{11}A_{23}}{A_{11}A_{22} - A_{12}^2} A_{23} \right] / Y$$

where

$$A_{11} = E_x \left(\frac{m\pi}{L_j} \right)^2 + G_{x\theta} \left(\frac{n}{R} \right)^2$$

$$A_{22} = E_\theta \left(\frac{n}{R} \right)^2 + G_{x\theta} \left(\frac{m\pi}{L_j} \right)^2$$

$$A_{33} = D_x \left(\frac{m\pi}{L_j} \right)^4 + D_{x\theta} \left(\frac{m\pi}{L_j} \right)^2 \left(\frac{n}{R} \right)^2 + D_\theta \left(\frac{n}{R} \right)^4 + \frac{E_\theta}{R^2} + \frac{2C_\theta}{R} \left(\frac{n}{R} \right)^2$$

$$A_{12} = (E_{x\theta} + G_{x\theta}) \left(\frac{m\pi}{L_j} \right) \left(\frac{n}{R} \right)$$

$$A_{23} = \frac{E_\theta}{R} \left(\frac{n}{R} \right) + C_\theta \left(\frac{n}{R} \right)^3$$

$$A_{13} = \frac{E_{x\theta}}{R} \left(\frac{m\pi}{L_j} \right) + C_x \left(\frac{m\pi}{L_j} \right)^3$$

$$E_x = \frac{Et}{1-\nu^2} \left(\frac{b_e}{b} \right) + \frac{EA_s}{b}$$

$$E_{x\theta} = \frac{\nu Et}{1-\nu^2}$$

$$\begin{aligned}
 E_{\theta} &= \frac{Et}{1-\nu^2} \left(\frac{L_e}{L_r} \right) + \frac{EA_r}{L_r} \\
 G_{x\theta} &= \frac{Gt}{2} \left(\frac{L_e}{L_r} + \frac{b_e}{b} \right) \\
 D_x &= \frac{Et^3}{12(1-\nu^2)} \left(\frac{b_e}{b} \right) + \frac{EI_s}{b} + \frac{EA_s Z_s^2}{b} \\
 D_{\theta} &= \frac{Et^3}{12(1-\nu^2)} \left(\frac{L_e}{L_r} \right) + \frac{EI_r}{L_r} + \frac{EA_r Z_r^2}{L_r} \\
 D_{x\theta} &= \frac{\nu Et^3}{6(1-\nu^2)} + \frac{Gt^3}{6} \left(\frac{L_e}{L_r} + \frac{b_e}{b} \right) + \frac{GJ_s}{b} + \frac{GJ_r}{L_r} \\
 C_{\theta} &= \frac{EA_r Z_r}{L_r} \\
 C_x &= \frac{EA_s Z_s}{b}
 \end{aligned}$$

The term Y in Equation 4.4-1 is dependent upon the loading condition and is defined in the sections below.

4.4.1 Axial Compression or Bending ($N_{\theta} = 0$)

The elastic buckling stresses in the longitudinal direction for the bay instability and general instability modes of failure are given by Equations 4.4-3 and 4.4-5 with N_{xej} determined from Equation 4.4-1. The following relationships for Y , t_x , and L_e are to be used for both bay and general instability stresses. When $b_e < b$, the values for F_{xej} must be determined by iteration since the effective width is a function of the buckling stress.

$$Y = \left(\frac{m\pi}{L_j} \right)^2$$

$$t_x = \frac{A_s + b_e t}{b}$$

$$L_e = L_r$$

a. For Bay Instability

- Elastic Buckling Stresses.** Use the following relationships together with those above for Y , t_x , and L_e to determine the bay instability buckling stresses:

$$j = B, A_r = I_r = J_r = 0, L_j = L_r$$

$$b_e = 1.9t\sqrt{E / F_{xeB}} \leq b \quad (4.4-2)$$

$$F_{xeB} = \alpha_{xB} \frac{N_{xeB}}{t_x} \quad (4.4-3)$$

F_y is to be substituted for F_{xeB} when $F_{xeB} > F_y$ so that $b_e = 1.9t(E / F_y)^{1/2} \leq b$.

Equations 4.4-2 and 4.4-3 may require an iterative solution

2. Imperfection Factors

$$\alpha_{xB} = \begin{cases} 0.65 & \text{if } \bar{A}_s \geq 0.06 \\ \alpha_{xL} & \text{if } \bar{A}_s < 0.06 \end{cases}$$

$$\bar{A}_s = \frac{A_s}{bt} \quad \text{and } \alpha_{xL} \text{ is given by Equation 4.1-3}$$

3. **Inelastic Buckling Stresses:** Inelastic instability stresses are determined by applying plasticity reduction factors to elastic buckling stresses as recommended in Section 5.

b. For General Instability

1. **Elastic Buckling Stresses.** Use the following relationships together with those above for Y , t_x , and L_e to determine the general instability stresses. The local buckling stress, F_{xcL} , obtained from Equations 4.3-1 and 5-1 and the general instability stress, F_{xcG} , obtained from Equations 4.4-5 and 5-1 should be substituted into Equation 4.4-4.

$$j = G, L_j = L_b$$

$$b_e = b\sqrt{F_{xcL} / F_{xcG}} \leq b \quad (4.4-4)$$

$$F_{xcG} = \alpha_{xG} \frac{N_{xcG}}{t_x} \quad (4.4-5)$$

Equations 4.4-4 and 4.4-5 may require an iterative solution.

2. Imperfection Factors

α_{xG} is given by Equation 4.2-2.

3. **Inelastic Buckling Stresses :** Inelastic instability stresses are determined by applying plasticity reduction factors to elastic buckling stresses as recommended in Section 5.

4.4.2 External Pressure ($N_\phi / N_\theta = 0$ or 0.5)

The elastic buckling stresses in the hoop direction for the bay instability and general instability modes of failure are given by Equations 4.4-6 and 4.4-7 with $N_{\theta ej}$ determined from Equation 4.4-1. The following relationships for Y and t_r are to be used for both bay and general instability stresses.

$$Y = k \left(\frac{m\pi}{L_j} \right)^2 + \left(\frac{n}{R} \right)^2$$

$$t_r = \frac{A_r + L_e t}{L_e}$$

where $k = 0$ for radial pressure and $k = 0.5$ for hydrostatic pressure.

a. For Bay Instability

- Elastic Buckling Stresses.** Use the following relationships together with that given above for Y to determine the bay instability stresses.

$$j = B, A_r = I_r = J_r = 0, L_j = L_r$$

$$L_e = L_r, b_e = b$$

$$F_{reB} \text{ or } F_{heB} = \alpha_{\theta B} \frac{N_{\theta eB}}{t} K_{\theta L} \quad (4.4-6)$$

See Equation 11. 3-3b for $K_{\theta L}$.

- Imperfection Factors**

$$\alpha_{\theta B} = 1.0$$

- Inelastic Buckling Stresses:** Inelastic instability stresses are determined by applying plasticity reduction factors to elastic buckling stresses as recommended in Section 5.

b. For General Instability

- Elastic Buckling Stresses.** Use the following relationships together with that above for Y to determine the general instability stresses.

$$j = G, L_j = L_b$$

$$L_e = 1.56\sqrt{Rt} \leq L_r, b_e = b$$

$$F_{reG} \text{ or } F_{heG} = \alpha_{\theta G} \frac{N_{\theta G}}{t} K_{\theta G} \quad (4.4-7)$$

Use the larger of the $K_{\theta G}$ values obtained from Equations 11.3-12 and 11.3-16.

2. Imperfection Factors

$$\alpha_{\theta G} = 0.8$$

3. **Inelastic Buckling Stresses:** Inelastic instability stresses are determined by applying plasticity reduction factors to elastic buckling stresses as recommended in Section 5.

4.5 BAY INSTABILITY OF STRINGER STIFFENED AND RING AND STRINGER STIFFENED CYLINDERS- ALTERNATE METHOD

The method of determining the bay instability stresses for stringer stiffened and ring and stringer stiffened cylinders given in Section 4.4 is based upon a modified orthotropic shell equation. This equation is not valid if the minimum number of stringers is less than about three times the number of circumferential waves for the bay instability mode. The following equations can be used when these restrictions are not met. The rings are to be sized using the equations in Section 4.2 for ring stiffened cylinders.

4.5.1 Axial Compression or Bending ($N_{\theta} = 0$)

The following method for determining the bay instability loads and stresses for axial compression and bending is quite lengthy but gives the best correlation between test and predicted loads of those methods considered. The method is based on the procedure proposed by Faulkner, et al. in Ref. 3.

- a. **Elastic Buckling Stresses.** The elastic bay instability stress F_{xeB} is approximated by summing the buckling stress of a shell panel and the column buckling stress of a stringer plus effective width of shell.

$$F_{xeB} = \frac{\alpha_{xL} C_x E 2t / D}{1 + A_s / bt} + \frac{\pi^2 EI'_{es}}{(b_{eu} t + A_s) L_r^2} \quad (4.5-1)$$

where $\alpha_{xL} C_x$ is obtained from Equations 4.5-12. The other parameters are defined as follows:

$$I'_{es} = I_s + A_s Z_s^2 \frac{b'_e t}{A_s + b'_e t} + \frac{b'_e t^3}{12} \quad (4.5-2)$$

$$b_{eu} = \begin{cases} b \left(\frac{1.05}{\lambda_o} - \frac{0.28}{\lambda_o^2} \right) R_r & \text{for } \lambda_o > 0.53 \\ b & \text{for } \lambda_o \leq 0.53 \end{cases} \quad (4.5-3)$$

$$b'_e = \begin{cases} b \left(\frac{0.53}{\lambda_o} \right) R_r & \text{if } \lambda_o \geq 0.53 \\ b & \text{if } \lambda_o < 0.53 \end{cases} \quad (4.5-4)$$

$$\lambda_o = \sqrt{F_y / \sigma_e} \quad (4.5-5)$$

$$\sigma_e = B \rho_\eta \sigma_{xeL} \quad (4.5-6)$$

$$\sigma_{xeL} = \begin{cases} 0.605 E 2t / D & M_\theta \geq 3.46 \\ \left(\frac{3.62}{M_\theta^2} + 0.0253 M_\theta^2 \right) E 2t / D & M_\theta < 3.46 \end{cases} \quad (4.5-7)$$

$$\rho_\eta = \begin{cases} 0.27 + \frac{1.57}{M_\theta^2} + \frac{29.6}{M_\theta^4} + 0.008 \left(1 - \frac{D/t}{600} \right) M_g & 3.46 < M_\theta < 8.57 \\ 1.0 - 0.018 M_\theta^{2.5} + 0.0023 M_\theta^2 \left(1 - \frac{D/t}{600} \right) & M_\theta \leq 3.46 \end{cases} \quad (4.5-8)$$

$$B = \begin{cases} 1.15 & \text{for } \lambda_\eta \geq 1.0 \\ 1 + 0.15 \lambda_\eta & \text{for } \lambda_\eta < 1.0 \end{cases} \quad (4.5-9)$$

$$\lambda_\eta = \sqrt{F_y / (\rho_\eta \sigma_{xeL})} \quad (4.5-10)$$

$$R_r = \begin{cases} 1.0 & \text{if } \lambda_\eta \leq 0.53 \\ 1.0 - \frac{2c}{b/t - 2c} \left(\frac{\lambda_\eta^2}{1 + 0.25 \lambda_\eta^4} \right)^2 \frac{\lambda_\eta^2}{1.05 \lambda_\eta - 0.28} & \text{if } \lambda_\eta > 0.53 \end{cases} \quad (4.5-11)$$

where $c = 4.5$ for continuous structural fillet welds.

The term $\alpha_{xL} C_x$ in Equation 4.5-1 can be computed from Equation 4.5-12:

$$\text{at } M_0 < 3$$

$$\alpha_{xL} C_x = 0.33 + 160(M_x)^{-0.5} / [200 + 0.5(D/t)] \quad (4.5-12)$$

if $M_0 = 15$

$$\alpha_{xL} C_x = 350(M_x)^{-0.5} / [200 + 0.5(D/t)]$$

For values of M_0 between 3 and 15, $\alpha_{xL} C_x$ can be obtained by interpolation.

- b. Inelastic Buckling Stresses.** Inelastic instability stresses are determined by applying plasticity reduction factors to elastic buckling stresses as recommended in Section 5.
- c. Failure Load.** The failure load is the product of the failure stress and the effective area. The effective shell width for determining the failure load or applied stresses (see Equations 11.1-2 and 11.2-2) is given by:

$$b_e = \begin{cases} b \left(\frac{1.05}{\lambda_e} - \frac{0.28}{\lambda_e^2} \right) R_r & \text{if } \lambda_e \geq 0.53 \\ b & \text{if } \lambda_e < 0.53 \end{cases} \quad (4.5-13)$$

where

$$\lambda_e = \lambda_o \sqrt{F_{xcB} / F_y}$$

See Equation 4.5-1 for F_{xcB} , Equation 4.5-5 for λ_o , Equation 4.5-11 for R_r , and the critical buckling stress, F_{xcB} should be obtained from equations in Section 5.

The failure load P_{cB} is computed from:

$$P_{cB} = N_s F_{xcB} (A_s + b_e t) \quad (4.5-14)$$

4.5.2 External Pressure

- a. Elastic Buckling Stresses:** Elastic instability stresses are determined from either inelastic instability or yield stresses as defined in Equation 4.5-15, Section 4.5.2b and the use of equations in Section 5.
- b. Inelastic Buckling Stresses**

Inelastic Bay Instability Stress, $F_{\theta cB}$, is determined from Equation 4.5-15:

$$F_{\theta cB} \text{ or } F_{hcB} = \frac{P_{cB} R_o}{t} K_{\theta L} \quad (4.5-15)$$

NOTE: $K_{\theta L}$ is defined in Section 11.

The failure pressure for bay instability, p_{cB} , is defined as the total capacity of shell plate and stringers in Equation (4.5-16):

$$p_{cB} = (p_{cL} + p_s) K_p \quad (4.5-16)$$

The local shell failure pressure, p_{cL} , is determined by Equation 4.5-17:

$$p_{cL} = F_{rcL} t / R_o \quad (4.5-17)$$

The local shell instability stress, F_{rcL} , is determined from Equation (4.1-5) and the equations in Section 5 by assuming that the cylinder is without stringer stiffeners and has a ring spacing equal to L_r .

The failure pressure associated with the development of plastic hinges in the stringers with effective shell plate, p_s , is determined from Equation 4.5-18:

$$p_s = \left(\frac{16}{bL_r^2} \right) A_s | Z_s | F_y \quad (4.5-18)$$

An effective pressure correction factor, K_p , is applied to the computed bay instability pressure to normalize the test data. Effective pressure correction factor is determined from Equation 4.5-19.

$$K_p = \begin{cases} 0.20 + 0.90(g / 500) & \text{for } g < 500 \\ 1.10 & \text{for } g > 500 \end{cases} \quad (4.5-19)$$

$$\text{where, } g = M_x M_\theta L_r t A_s / I_s$$

SECTION 5—Plasticity Reduction Factors

The predicted elastic buckling stresses given in Section 4 for local shell buckling, bay instability and general instability must be reduced by a plasticity factor when the elastic buckling stresses exceed $0.50F_y$. Inelastic buckling stress definitions given in Equations 5-1 and 5-2 should be used together with Equation 5-3 defining the plasticity reduction factor.

$$F_{icj} = \eta F_{iej} \quad \text{for } F_{iej} > 0.5F_y \quad (5-1)$$

(i.e., $F_{iej} > \text{material proportional limit}$)

$$F_{icj} = F_{iej} \quad \text{for } F_{iej} \leq 0.5F_y \quad (5-2)$$

(i.e., $F_{iej} < \text{material proportional limit}$)

where

$$\eta = (F_y / F_{iej}) \left[\frac{1.0}{1.0 + 3.75(F_y / F_{iej})^2} \right]^{1/4} \quad (5-3)$$

SECTION 6—Predicted Shell Buckling Stresses for Combined Loads

Interaction equations are given for determining the failure stresses for cylinders subjected to combined loads. The stresses due to bending moment are treated as equivalent axial stresses. The interaction equations are applicable to all modes of failure and to both elastic and inelastic buckling stresses. Each mode must be checked independently.

6.1 GENERAL LOAD CASES

In the following equations for N_ϕ and N_θ , P is the total axial load including any pressure load on the end of the cylinder, M is the bending moment and p is the external radial pressure.

a. Axial Compression and Hoop Compression

$$N_\phi = P / (2\pi R)$$

$$N_\theta = pR_o$$

b. Bending and Hoop Compression

$$N_\phi = M / (\pi R^2)$$

$$N_\theta = pR_o$$

c. Axial Compression, Bending and Hoop Compression

$$N_\phi = P / (2\pi R) + M / (\pi R^2)$$

$$N_\theta = pR_o$$

6.2 AXIAL TENSION, BENDING AND HOOP COMPRESSION

The failure stresses are the lower of the values determined from Equations 6.2-1 and 6.2-2. The longitudinal stress $F_{\phi cj}$ is the sum of the axial and bending stresses.

$$F_{\theta cj} = F_{rcj} \left(1 - 0.25 \frac{F_{\phi cj}}{F_y} \right) \quad (6.2-1)$$

$$F_{\phi cj} + F_{\theta cj} = F_y \quad (6.2-2)$$

where $F_{\theta cj}$ is the failure stress in the hoop direction corresponding to $F_{\phi cj}$ which is the coexistent failure stress in the axial direction, and F_{rcj} is the predicted failure stress for radial pressure. F_{rcj} is given by Equation 4.5-15 and the equations in Section 5.

The values of $F_{\phi cj}$ and $F_{\theta cj}$ are determined from Equations 6.2-1 and 6.2-2 by letting $F_{\phi cj} = F_{\theta cj} k K_{\phi j} / K_{\theta j}$ where $k = N_{\phi} / N_{\theta}$ and then solving for $F_{\theta cj}$. See Table 6.2-1 for K_{ij} .

6.3 AXIAL COMPRESSION, BENDING AND HOOP COMPRESSION

The following equation is applicable to any combination of axial compression, bending and external pressure. The axial buckling stress, $F_{\phi cj}$, is the sum of the longitudinal stresses due to axial compression and bending. If the bending stress is greater than the axial stress, the failure stresses are determined from Section 6.2. $F_{\theta cj}$ is the failure stress in the hoop direction.

$$R_a^2 - c R_a R_h + R_h^2 = 1.0 \quad (6.3-1)$$

where

$$R_a = F_{\phi cj} / F_{xcj}$$

$$R_h = F_{\theta cj} / F_{rcj}$$

The equations for c are dependent upon the cylinder geometry.

a. Unstiffened and Ring Stiffened Cylinders (all buckling modes)

$$c = \frac{F_{xcj} + F_{rcj}}{F_y} - 1.0 \quad (6.3-2)$$

b. Stringer Stiffened and Ring and Stringer Stiffened Cylinders

1. Local Buckling ($j = L$)

$$c = \frac{0.4(F_{xcj} + F_{rcj})}{F_y} - 0.8 \quad (6.3-3)$$

2. Bay Instability and General Instability ($j = B, j = G$)

$$c = \frac{1.5(F_{xcj} + F_{rcj})}{F_y} - 2.0 \quad (6.3-4)$$

The buckling stresses for any combination of longitudinal compression and hoop compression, N_{ϕ} / N_{θ} , are determined by the following procedure for each of the buckling modes.

Step 1. Calculate F_{xcj} and F_{rcj} from the equations in Section 4.

Step 2. Solve for $F_{\theta cj}$ in Equation 6.3-1 by letting $F_{\phi cj} = F_{\theta cj} k K_{\phi j} / K_{\theta j}$.

where

$$k = N_{\phi} / N_{\theta},$$

$K_{\phi j}$ = axial stress modifier (see Table 6.2-1),

$K_{\theta j}$ = hoop stress modifier (see Table 6.2-1).

Table 6.2-1: Stress Distribution Factors, K_{ij}

K_{ij}	Unstiffened	Ring Stiffened	Stringer Stiffened	Ring and Stringer Stiffened
$K_{\phi j}$	1.0	1.0	1.0	1.0
$K_{\phi B}$			t/t_x Section 4.4.1	t/t_x Section 4.4.1
$K_{\phi G}$		1.0		t/t_x Section 4.4.1
$K_{\theta L}$	1.0	Equation 11.3-3a	Equation 11.3-3b	Equation 11.3-3b
$K_{\theta B}$			Same as $K_{\theta L}$	Same as $K_{\theta L}$
$K_{\theta G}$		Equation 11.3-12a		Larger of Eqn. 11.3-12a or 11.3-16

SECTION 7—Stiffener Requirements

The flow chart in Figure 3.1 shows the procedure for determining the allowable stresses for all buckling modes. The sizes of the stringers are determined from the bay instability mode equations and the sizes of the rings are determined from the general instability mode equations.

The shell buckling stress equations were developed on the basis of no interaction between the buckling modes. The buckling stresses for local buckling, however, may be reduced if the predicted buckling stress for either bay instability or general instability is approximately equal to the predicted local buckling stress. Similarly, if the predicted general instability stress is approximately equal to the bay instability stress, the actual stress for either of these modes may be less than predicted.

Mode interaction can be avoided by applying a factor β to the strains corresponding to the buckling stresses. It is desirable to provide a hierarchy for failure with general instability preceded by bay instability and bay instability preceded by local shell buckling. A minimum factor of $\beta = 1.2$ is recommended for both the bay and general instability modes. The designer may elect to select a higher β value for the general instability mode.

7.1 HIERARCHY CHECKS

The buckling stresses which include the factor β are called the “design buckling stresses” and defined by the following equations:

$$\bar{F}_{iej} = F_{iej} / \beta \quad (7.1-1)$$

$$\bar{F}_{icj} = F_{icj} / \beta \quad (7.2-2)$$

where $\beta = 1.0$ for local buckling and at least 1.2 for bay instability and general instability. Critical buckling stresses five by Equation 7.1-2 are determined through the use of equations given in Section 5.

It should be noted that β factor is intended to be applied to the failure strain and not the failure stress. Thus, while the bay and general instability-to-local stress ratios will be equal to β in the material elastic range (i.e., below material proportional limit), these ratios will be less than β in the material elasto-plastic range (i.e., above material proportional limit).

Cylindrical shell design should meet the desired hierarchy checks (i.e., $F_{ieG} > F_{ieB} > 1.2F_{ieL}$). If the hierarchy is not achieved, the design should be modified by either raising F_{ieG} and F_{ieB} stresses or by lowering F_{ieL} stress. These objectives can be achieved by changing:

- (1) ring and stringer stiffener spacing
- (2) ring and stringer stiffener sizes
- (3) shell plate thickness

7.2 STIFFENER STRESSES AND BUCKLING

Both the ring and the stringer are subjected to localized stresses that need to be combined with global stresses. The stiffeners also need to be checked against local web or flange buckling.

7.2.1 Local Stiffener Buckling

To preclude stiffener buckling prior to shell buckling, the local stiffener buckling stress must be greater than the shell buckling stress given by the foregoing equations. The local stiffener buckling stress can be assumed to be equal to the yield stress for stiffeners which satisfy the following compact section requirements. For stiffeners not meeting these requirements, the local stiffener buckling stress can be determined from Equation C7.2-1 in the Commentary.

- a. Flat Bar Stiffener, Flange of a Tee Stiffener and Outstanding Leg of an Angle Stiffener**

$$\frac{h_s}{t_s} \leq 0.375 \sqrt{E / F_y} \quad (7.2-1)$$

where h_s is the full width of a flat bar stiffener or outstanding leg of an angle stiffener and one-half of the full width of the flange of a tee stiffener and t_s is the thickness of the bar, leg of angle or flange of tee.

- b. Web of Tee Stiffener or Leg of Angle Stiffener Attached to Shell**

$$\frac{h_s}{t_s} \leq 1.0 \sqrt{E / F_y} \quad (7.2-2)$$

where h_s is the full depth of a tee section or full width of an angle leg and t_s is the thickness of the web or angle leg.

7.2.2 Stiffener Global and Local Stresses

Some of the external pressure on an orthogonally stiffened cylindrical shell will be directly transferred to the rings through the stringers and the resulting bending stresses in the stringers may be appreciable. In addition to meeting API requirements on bay instability, it is recommended that an effective stringer column instability check be performed for an appropriate beam column element subjected to combined global axial and local bending stresses.

7.2.3 Tripping Brackets

The ring stiffeners supporting the stringers may be susceptible to tripping and the ring tripping can be minimized by introducing tripping brackets. The spacing, s , between the tripping brackets should not exceed:

$$s < 0.44 b_t [E / F_y]^{0.5} \quad (7.2-3)$$

The design lateral load on the flange for tripping bracket sizing can be taken as the compressive stress in the flange multiplied by 2% of the combined area of the flange plus one-third of the web area, see Figure 7.2-1.

7.3 STIFFENER ARRANGEMENT AND SIZES

An optimum design provides a natural hierarchical order of failure modes, minimizes steel requirements and simplifies fabrication. Ring spacing and shell thickness are primarily controlled by external pressure and the stringer spacing and size are primarily controlled by axial and bending loads.

Stiffener arrangement and sizes should meet both the applied combined loads as discussed in Section 6 and the axial and bending loads and external pressure separately. For cylinders subjected to loads in one direction alone, $F_{\phi ej} = F_{xej}$ and $F_{\theta ej} = F_{hej}$. The ring and stringer stiffeners, together with the effective shell area, should yield general, bay and local instability stresses that meet the following requirements:

a. Stringers

$$\bar{F}_{\phi eB} = \bar{F}_{\phi eB} / 1.2 \geq \bar{F}_{\phi eL} \quad (7.3-1)$$

$$\bar{F}_{\theta eB} = \bar{F}_{\theta eB} / 1.2 \geq \bar{F}_{\theta eL} \quad (7.3-2)$$

b. Rings

$$\bar{F}_{\phi eG} = \bar{F}_{\phi eG} / 1.2 \geq \bar{F}_{\phi eL} \quad (7.3-3)$$

$$\bar{F}_{\theta eG} = \bar{F}_{\theta eG} / 1.2 \geq \bar{F}_{\theta eL} \quad (7.3-4)$$

A general procedure that can be used to meet both design safety factors and the hierarchical failure mode requirements is presented in Section C7.3 of the Commentary.