

Design of Flat Plate Structures

API BULLETIN 2V
THIRD EDITION, JUNE 2004

ERRATA, MARCH 2008



Design of Flat Plate Structures

API BULLETIN 2V
THIRD EDITION, JUNE 2004

ERRATA, MARCH 2008



SPECIAL NOTES

API publications necessarily address problems of a general nature. With respect to particular circumstances, local, state, and federal laws and regulations should be reviewed.

API is not undertaking to meet the duties of employers, manufacturers, or suppliers to warn and properly train and equip their employees, and others exposed, concerning health and safety risks and precautions, nor undertaking their obligations under local, state, or federal laws.

Information concerning safety and health risks and proper precautions with respect to particular materials and conditions should be obtained from the employer, the manufacturer or supplier of that material, or the material safety data sheet.

Nothing contained in any API publication is to be construed as granting any right, by implication or otherwise, for the manufacture, sale, or use of any method, apparatus, or product covered by letters patent. Neither should anything contained in the publication be construed as insuring anyone against liability for infringement of letters patent.

Generally, API standards are reviewed and revised, reaffirmed, or withdrawn at least every five years. Sometimes a one-time extension of up to two years will be added to this review cycle. This publication will no longer be in effect five years after its publication date as an operative API standard or, where an extension has been granted, upon republication. Status of the publication can be ascertained from the API Standards department telephone (202) 682-8000. A catalog of API publications, programs and services is published annually and updated biannually by API, and available through Global Engineering Documents, 15 Inverness Way East, M/S C303B, Englewood, CO 80112-5776.

This document was produced under API standardization procedures that ensure appropriate notification and participation in the developmental process and is designated as an API standard. Questions concerning the interpretation of the content of this standard or comments and questions concerning the procedures under which this standard was developed should be directed in writing to the Director of the Standards department, American Petroleum Institute, 1220 L Street, N.W., Washington, D.C. 20005. Requests for permission to reproduce or translate all or any part of the material published herein should be addressed to the Director, Business Services.

API standards are published to facilitate the broad availability of proven, sound engineering and operating practices. These standards are not intended to obviate the need for applying sound engineering judgment regarding when and where these standards should be utilized. The formulation and publication of API standards is not intended in any way to inhibit anyone from using any other practices.

Any manufacturer marking equipment or materials in conformance with the marking requirements of an API standard is solely responsible for complying with all the applicable requirements of that standard. API does not represent, warrant, or guarantee that such products do in fact conform to the applicable API standard.

All rights reserved. No part of this work may be reproduced, stored in a retrieval system, or transmitted by any means, electronic, mechanical, photocopying, recording, or otherwise, without prior written permission from the publisher. Contact the Publisher, API Publishing Services, 1220 L Street, N.W., Washington, D.C. 20005.

Copyright © 2004 American Petroleum Institute

FOREWORD

This Bulletin is under jurisdiction of the API Subcommittee on Offshore Structures.

This Bulletin provides guidance for the design of steel flat plate structures. Used in conjunction with API RP 2T or other applicable codes and standards, this Bulletin will be helpful to engineers involved in the design of offshore structures which include flat plate structural components.

The buckling formulations and design considerations contained herein are based on the latest available information. As experience with the use of the Bulletin develops, and additional research results become available, it is anticipated that the Bulletin will be updated periodically to reflect the latest technology.

API publications may be used by anyone desiring to do so. Every effort has been made by the Institute to assure the accuracy and reliability of the data contained in them; however, the Institute makes no representation, warranty, or guarantee in connection with this publication and hereby expressly disclaims any liability or responsibility for loss or damage resulting from its use or for the violation of any federal, state, or municipal regulation with which this publication may conflict.

Suggested revisions are invited and should be submitted to API, Standards Department, 1220 L Street, NW, Washington, DC 20005

CONTENTS

	Page
SECTION 1—Nomenclature and Glossary	1
1.1 Nomenclature	1
1.2 Glossary.....	5
SECTION 2—General	7
2.1 Scope	7
2.2 References	7
2.3 Range of Validity and Limitations	7
2.4 Limit States.....	9
2.5 Verification of Structural Adequacy.....	10
2.6 Structural Component Loads and Load Combinations.....	14
2.7 General Approach to Structural Analysis	15
2.8 General Approach to Structural Design.....	18
SECTION 3—Plates	20
3.1 General	20
3.2 Uniaxial Compression and In-plane Bending.....	23
3.3 Edge Shear.....	26
3.4 Uniform Lateral Pressure	27
3.5 Biaxial Compression With or Without Edge Shear	29
3.6 Combined In-plane and Lateral Loads	30
SECTION 4—Stiffeners.....	33
4.1 General	33
4.2 Column Buckling	35
4.3 Beam-column Buckling.....	35
4.4 Torsional/Flexural Buckling.....	36
4.5 Plastic Bending.....	40
4.6 Design Considerations.....	41
SECTION 5—Stiffened Panels	42
5.1 General	42
5.2 Uniaxially Stiffened Panels in End Compression.....	44
5.3 Orthogonally Stiffened Panels.....	45
5.4 Stiffener Proportions	51
5.5 Trpping Brackets	51
5.6 Effective Flange	51
5.7 Stiffener Requirement for In-plane Shear.....	56
5.8 Other Design Requirements	56
5.9 Design Considerations.....	56
SECTION 6—Deep Plate Girders.....	58
6.1 General	58
6.2 Limit States.....	63
6.3 Design Considerations.....	64
APPENDIX A—COMMENTARY	74
REFERENCES.....	123
APPENDIX B—GUIDELINES FOR FINITE ELEMENT ANALYSIS USE	129
Figures	
2.7-1 Global, Panel, and Plate Stresses	16
3.1-1 Primary Loads Acting on a Rectangular Plate	22
3.2-1 Long Rectangular Plate.....	22
3.2-2 Wide Rectangular Plate.....	22
3.2-3 Buckling Coefficients for Plates in Uniaxial Compression1	25
3.4-1 Coefficients for Computing Plate Deflections	25
3.4-2 Stresses in Plates Under Uniform Lateral Pressure.....	25

	Page
3.5-1 Rectangular Plate Under Biaxial Compression	25
4.4-1 Design Lateral Load for Tripping Bracket	37
5.1-1 Flat Stiffened Panel	43
5.2-1 Uniaxially Stiffened Panel in End Compression	43
5.3-1 Deflection Coefficient for Orthogonally Stiffened Panels	46
5.3-2 Coefficients for Computing Stresses for Orthogonally Stiffened Panels	47
5.6-1 Cases for Effective Flange Calculations	52
5.6-2 Effective Breadth Ratio for Case I (Single Web)	54
5.6-3 Effective Breadth Ratio for Case II (Double Web)	54
5.6-4 Effective Breadth Ratio for Case III (Multiple Webs)	54
5.6-5 Stress Distribution Across Flange	55
5.7-1 Geometry of Stiffened Panels Subjected to In-Plane Shear	55
6.1-1 Typical Deep Plate Girder Structural Arrangement	59
6.1-2 Primary Loads Acting on Plate Girder	59
6.1-3 Stress Distribution Across Section Due to Concentrated Load Applied at the Flange Level	59
6.1-4 Transverse Stresses in Webs Due to Flanges Curved in Elevation	61
6.3-1 Web with Small Openings	65
6.3-2 Web with Large Openings	65
6.3-3 Vertical Stiffener Termination	65
6.3-4 Coefficient for Computing Axial Force Assumed in Preventing Web Buckling	72
6.3-5 Longitudinal Stress in Webs with Transverse Stiffeners	72
C3-1 Rectangular Plate Under Uniaxial Compression	77
C3-2 Comparison of Inelastic Buckling Formulations for Rectangular Plates Under Uniaxial Compression	77
C3-3 Wide Rectangular Plate	84
C3-4 Comparison of Formulations for the Ultimate Strength of Wide Plates with $a/b = 3$	84
C3-5 Comparison of Formulations for the Inelastic Buckling of Rectangular Plates Under Edge Shear	89
C3-6 Model for the Ultimate Strength of Rectangular Plates in Shear	89
C3-7 Comparison of Formulations for the Ultimate Strength of Rectangular Plates in Shear	90
C3-8 Comparison of Formulations for the Ultimate Strength of Rectangular Plates Under Lateral Pressure	91
C3-9 Rectangular Plate Under Biaxial Compression	91
C3-10 Combined In-Plane and Lateral Loads ($b/t = 40$)	93
C3-11 Combined In-Plane and Lateral Loads ($b/t = 20$)	94
C6-1 Comparison of Minimum Longitudinal Stiffener Stiffness Requirements	120
B-1 Panel Weak Axis Bending Stress Evaluation at Center of Panel	135
B-2 Panel Weak Axis Bending Stress Evaluation at Center of Longitudinal Edge	136
B-3 Design Guideline Plate and Stiffened Panel Applied Stress Locations	137
 Tables	
4.4-1 Properties of Thin-Walled Open Cross Sections	37
B-1 Minimum FEA Requirements for Stiffened Plate Structure	138
B-2 FEA Design Guideline for Applied Stresses	139

Section 1-Nomenclature and Glossary

1.1 Nomenclature

Note: The terms not defined here are uniquely defined in the sections in which they are used.

1.1.1 Material Properties

E	=	modulus of elasticity, [ksi].
G	=	shear modulus, [ksi].
ν	=	Poisson's ratio.
F_y	=	minimum specified yield stress of material, [ksi].
τ_y	=	$F_y / \sqrt{3}$ yield stress in shear, [ksi].
F_p	=	proportional limit stress in compression, [ksi].
p_r	=	F_p / F_y stress ratio defining the beginning of nonlinear effects in compression.

1.1.2 Plate Geometry and Related Parameters

a	=	plate length or larger dimension, [in.]
b	=	plate width or shorter dimension, [in.]
D	=	$Et^3/[12(1 - \nu^2)]$ plate flexural rigidity, [kips-in].
t	=	plate thickness, [in.]
α	=	$a/b \geq 1$ aspect ratio
β	=	$(b/t)\sqrt{F_y/E}$ slenderness ratio

1.1.3 Stiffener Geometry and Related Parameters

A	=	cross sectional area, [in. ²]
A_w	=	web area, [in. ²]
b	=	spacing between stiffeners, [in.]
b_e	=	effective width of attached plating, [in.]
b_f	=	flange total width, [in.]
C_w	=	warping constant (see formulas in Table 4.4-1), [in. ⁶]
d	=	web depth, [in].
I	=	minimum moment of inertia, [in. ⁴]
I_c	=	polar moment of inertia about centroid, [in. ⁴]
I_s	=	polar moment of inertia about shear center, [in. ⁴]
I_1	=	moment of inertia of symmetric I-section in the plane of minimum stiffness, [in. ⁴]
I_2	=	moment of inertia of symmetric I-section in the plane of maximum stiffness, [in. ⁴]
J	=	torsion constant (see formulas in Table 4.4-1), [in. ⁴]
K	=	effective length ratio, normally taken as unity.
L	=	unsupported length, [in.]
L_b	=	bracing distance, [in.]

L_y	=	length at which there is a transition between elastic and plastic limit state moments for lateral buckling, [in.]
r	=	$\sqrt{I/A}$ radius of gyration, [in.]
S	=	section modulus for bending of symmetric I-section in the plane of maximum stiffness, [in. ³]
s	=	spacing between tripping brackets, [in.]
t	=	attached plate thickness, [in.]
t_f	=	flange thickness, [in.]
t_w	=	web thickness, [in.]
λ	=	$[KL/(r\pi)]\sqrt{F_y/E}$ stiffener slenderness ratio.

1.1.4 Stiffened Panel Geometry and Related Parameters

A	=	entire panel length, [in.]
A_2	=	area of flange in stiffened plating (zero in the case of flat bar stiffeners), in. ²
A_s	=	stiffener area, [in. ²]
B	=	entire stiffened panel width in the case of a stiffened panel (see Figure 5.1-1), or distance between webs for effective flange breadth calculations (see Figure 5.2-1), [in.]
$2b$	=	plate breadth, or distance between webs, [in.] (See Figure 5.6-1)
b_{ef}	=	effective breadth, [in.]
d	=	spacing between stiffeners = $2b$, [in.]
h	=	one half web depth, [in.]
I_s	=	moment of inertia of one stiffener about an axis parallel to the plate surface at the base of the stiffener, [in. ⁴]
L	=	length, [in.]
cL	=	distance between points of zero bending moment, [in.]
n	=	number of sub-panels (individual plates).
t	=	plate thickness, [in.]
t_f	=	flange thickness, [in.]
t_w	=	web thickness, [in.]
α	=	aspect ratio of whole panel
γ	=	$12(1-\nu^2)I_s/(t^3d)$
δ	=	$A_s/(Bt)$
$\bar{\lambda}$	=	$(B/t)\sqrt{F_y 12(1-\nu^2)/(E\pi^2k)}$, modified slenderness ratio for uniaxially stiffened panels, where k is the buckling coefficient.
I_x, I_y	=	moment of inertia of the stiffeners with effective plating extending in the x- or y-direction, respectively, [in. ⁴]
I_{px}, I_{py}	=	moment of inertia of the effective plating alone associated with stiffeners extending in the x- or y-direction, respectively, about the neutral axis of the entire section, [in. ⁴]
s_x, s_y	=	spacing of the stiffeners extending in the y- or x-direction, respectively, [in.]

t_x, t_y	=	equivalent thickness of the plate and the stiffeners (diffused) extending in the x -direction or y -direction, respectively, [in.]
M_x, M_y	=	moment per unit length that produces a stress f_x or f_y , respectively, [kips]
r_a, r_b	=	bending lever arm associated with f_x or f_y , respectively, i.e., distance from the neutral axis of the stiffener with the effective breadth of plate to the outer fiber of the flange (for the flange stress) or of the plate (for the plate field stress), [in.]

1.1.5 Deep Plate Girder Geometry and Related Parameters

A_f	=	flange cross-sectional area, [in. ²]
a	=	spacing between transverse web stiffeners, [in.]
a_h	=	web opening height, [in.]
B_f	=	width of unstiffened flange in a beam with only one web, or half the distance between successive longitudinal stiffeners or webs, together with any adjacent outstand, [in.] (See Fig. 6.1-4.)
b	=	spacing between longitudinal web stiffeners, [in.] (See Fig. 6.3-1.)
b_e	=	effective plate flange width attached to web stiffeners, [in.]
b_h	=	web opening length, [in.] (See Fig. 6.3-1)
d_s	=	spacing between web longitudinal stiffeners, [in.]
d_w	=	web depth, [in.]
R_f	=	flange radius of curvature, [in.]
s_h	=	clear distance along the longitudinal direction between web openings, [in.]
t_f	=	flange thickness, [in.]
t_w	=	web thickness, [in.]
θ	=	slope of web to horizontal.

1.1.6 Stresses

1.1.6.1 Normal Stresses:

f	=	normal stress, [ksi].
f_x, f_y	=	normal stress directed along the x and y axis, [ksi].
f_{xy}	=	in-plane shear stress, [ksi]
f_{se}	=	elastic serviceability limit state stress, [ksi].
f_{sp}	=	plastic serviceability limit state stress, [ksi].
f_u	=	ultimate limit state stress, [ksi].
f_{xse}	=	normal stress f_{se} when the plate is compressed in the x direction alone, [ksi]
f_{yse}	=	normal stress f_{se} when the plate is compressed in the y direction alone, [ksi].
f_{xyse}	=	edge shear stress f_{se} when the plate is loaded in pure shear, [ksi].
f_{xysp}	=	edge shear stress f_{sp} when the plate is loaded in pure shear, [ksi].
f_{xyu}	=	edge shear stress f_u when the plate is loaded in pure shear, [ksi].
f_{xl}	=	limit state normal stress in the x direction when the plate is compressed in the x direction, [ksi].

f_{yl}	=	limit state normal stress in the y direction when the plate is compressed in the y direction, [ksi].
f_{xyl}	=	limit state shear stress when the plate is loaded in pure shear, [ksi].

1.1.6.2 Shear Stresses:

f_{xy}	=	in-plane shear stress, [ksi].
f_{xyse}	=	elastic serviceability limit state stress, [ksi].
f_{xysp}	=	plastic serviceability limit state stress, [ksi].
f_{xyu}	=	ultimate limit state stress, [ksi].

1.1.7 Plate Lateral Deflections

W_a	=	maximum allowable deflection, [in.]
W_e	=	maximum elastic deflection, [in.]
W_p	=	plastic set (maximum permanent plastic deflection), [in.]

1.1.8 Plate Lateral Pressures

p	=	uniform lateral pressure, [ksi].
p_u	=	ultimate limit state pressure, [ksi].

1.1.9 Stiffener Axial Loads

P	=	applied axial force, [kips].
P_y	=	fully plastic axial force = $A F_y$, [kips].
P_{Ee}	=	column elastic ultimate state axial force, [kips].
P_{Ep}	=	column plastic ultimate state axial force, [kips].
P_{Te}	=	column torsional elastic ultimate state axial force, [kips].
P_{T_p}	=	column torsional plastic ultimate state axial force, [kips].
P_{TFe}	=	column torsional/flexural elastic ultimate state axial force, [kips].
P_{TF_p}	=	column torsional/flexural plastic ultimate state axial force, [kips].

1.1.10 Stiffener Lateral Distributed Loads

q	=	uniform lateral load per unit length, kips per [in.]
q_a	=	load q per unit length on stiffener of length a , kips per [in.]
q_b	=	load q per unit length on stiffener of length b , [kips per in.]
q_u	=	ultimate load, [kips per in.]

1.1.11 Stiffener Bending Moments

M	=	applied bending moment, [in-kips].
M_o	=	fully plastic bending moment, [in-kips].
M_1	=	smaller end moment in the plane of bending, [in-kips].
M_2	=	larger end moment in the plane of bending, [in-kips].
M_{fy}	=	moment at which the flanges are fully plastic, [in-kips].
M_y	=	moment at which yield first occurs in the flanges, [in-kips].
M_u	=	ultimate limit state M , [in-kips].
M_{ue}	=	elastic ultimate limit state M , [in-kips].
M_{up}	=	plastic ultimate limit state M , [in-kips].

1.1.12 SI Metric Conversion Factors

$$\begin{array}{lcl} \text{in} \times 25.4 & = & \text{mm} \\ \text{ksi} \times 6.894757 & = & \text{MPa} \end{array}$$

1.2 GLOSSARY

1.2.1 chord: Deep plate girder flange.

1.2.2 deep plate girder: Deep plate girder with the web stiffened in both the longitudinal and transverse directions and satisfying the requirements of 6.1.1. See also 6.1.2.

1.2.3 design variables: Quantities that define for the purpose of structural design or analysis a structural component and material, its state of stress, and the applied loads.

1.2.4 distortion energy theory: Failure theory defined by the following equation, where the applied stresses are positive for tension and negative for compression:

$$f_x^2 - f_x f_y + f_y^2 + 3f_{xy}^2 = F_y^2$$

1.2.5 effective flange breadth: The reduced breadth of a plate subjected to bending and/or tensile load, which, with an assumed uniform stress distribution, produces the same effect on the behavior of a structural member as the actual breadth of the plate with its non-uniform stress distribution. While the effective flange width applies to a member under compression, the effective flange breadth applies to a member under bending and/or tensile loading, and is associated with shear lag effects. See 5.6.

1.2.6 effective flange width: The reduced width of a plate subjected to compressive load, which, with an assumed uniform stress distribution produces the same effect on the behavior of a structural member as the actual width of the plate with its non-uniform stress distribution. See 4.1.2.

1.2.7 panel: See stiffened panel.

1.2.8 plate: In Bulletin 2V this term refers to a flat thin rectangular plate, see 3.1.2.

1.2.9 global stresses: Stresses resulting from global deformation of the structure.

1.2.10 proportional limit stress (F_p): Stress above which the stress-strain curve is no longer linear and which represents the onset of plastic behavior. If no specific value for the steel being used is available F_p can be taken as $0.60 F_y$, where F_y is the yield stress.

1.2.11 residual stresses: The stresses that remain in an unloaded member after it has been formed and installed in a structure. Some typical causes are forming, welding and corrections for misalignment during installation in the structure.

1.2.12 panel stresses: Stresses on stiffened panels resulting from local applied pressures or transverse loads.

1.2.13 serviceability limit state: Function of design variables which defines a condition at which a member no longer satisfies functional requirements, although it is still capable of carrying additional loads before reaching an ultimate limit state. See 2.4.3.

1.2.14 shear lag: Shear effects on beams that cause a non-uniform distribution of longitudinal bending stresses across the flange.

1.2.15 stiffened panel: Structural component comprising one or two sets of equally spaced uniform stiffeners of equal cross section supporting a thin plate. If there is only one set of stiffeners the panel is uniaxially stiffened, and if there are two the panel is orthogonally stiffened. See 5.1.2.

1.2.16 stiffener: Straight and slender thin-walled member of uniform cross which serves as a stiffening element for a flat plate structure. See 4.1.2.

1.2.17 plate stresses: Stresses on a thin rectangular plate resulting from lateral pressure.

1.2.18 tripping: Torsional buckling of stiffener.

1.2.19 ultimate limit state: Function of design variables that defines the resistance of a member to failure (i.e., its maximum load carrying capacity at failure), see 2.4.2.

1.2.20 yield stress: The yield stress of the material determined in accordance with ASTM A307.

Section 2-General

2.1 SCOPE

2.1.1 Bulletin 2V provides guidance for the design of steel flat plate structures. These often constitute main components of offshore structures. When applied to Tension Leg Platforms (TLPs) this Bulletin should be viewed as a complement to API RP 2T. The Bulletin combines good practice considerations with specific design guidelines and information on structural behavior. As such it provides a basis for taking a “design by analysis” approach to structural design of offshore structures.

2.1.2 Flat plate structures include thin plates, stiffened panels and deep plate girders, and they can constitute the main component of decks, bulkheads, web frames and flats. The external shell of pontoons or columns can also be made of flat stiffened panels if their cross section is, for example, square or rectangular, rather than circular.

2.1.3 Bulletin 2V is not a comprehensive document, and users have to recognize the need to exercise engineering judgment in actual applications, particularly in the areas that are not specifically covered.

2.1.4 Plates are discussed in Section 3, stiffeners in Section 4, stiffened panels in Section 5, and deep plate girders in Section 6. Limit states are given for each relevant load and load combination, and design requirements are also defined. Figure 2.1-1 summarizes the structural components and the limit states covered in Bulletin 2V.

2.2 REFERENCES

Background and references on the contents of Bulletin 2V are included in a Commentary given in the Appendix. Reference is made to API RP 2T, *Recommended Practice for Design of Tension Leg Platforms*, and API RP 2A, *Recommended Practice for Planning, Designing, and Constructing Fixed Offshore Platforms*, American Petroleum Institute, and to the *American Institute of Steel Construction, Specification for the Design, Fabrication and Erection of Structural Steel for Buildings*, latest edition.

2.3 RANGE OF VALIDITY AND LIMITATIONS

2.3.1 The formulations given apply only to members made of structural steel used for offshore structures, as defined in API RP 2T.

2.3.2 Structural components must comply with the dimensional tolerance limits defined in API RP 2T. Members not complying with these requirements should be given special consideration, given the potential negative impact dimensional imperfections can have on structural performance.

2.3.3 The formulations for the limit states given may be replaced by more refined analyses, or model tests, taking into account the real boundary conditions, the actual load distribution, geometrical imperfections, material properties, and residual stresses.

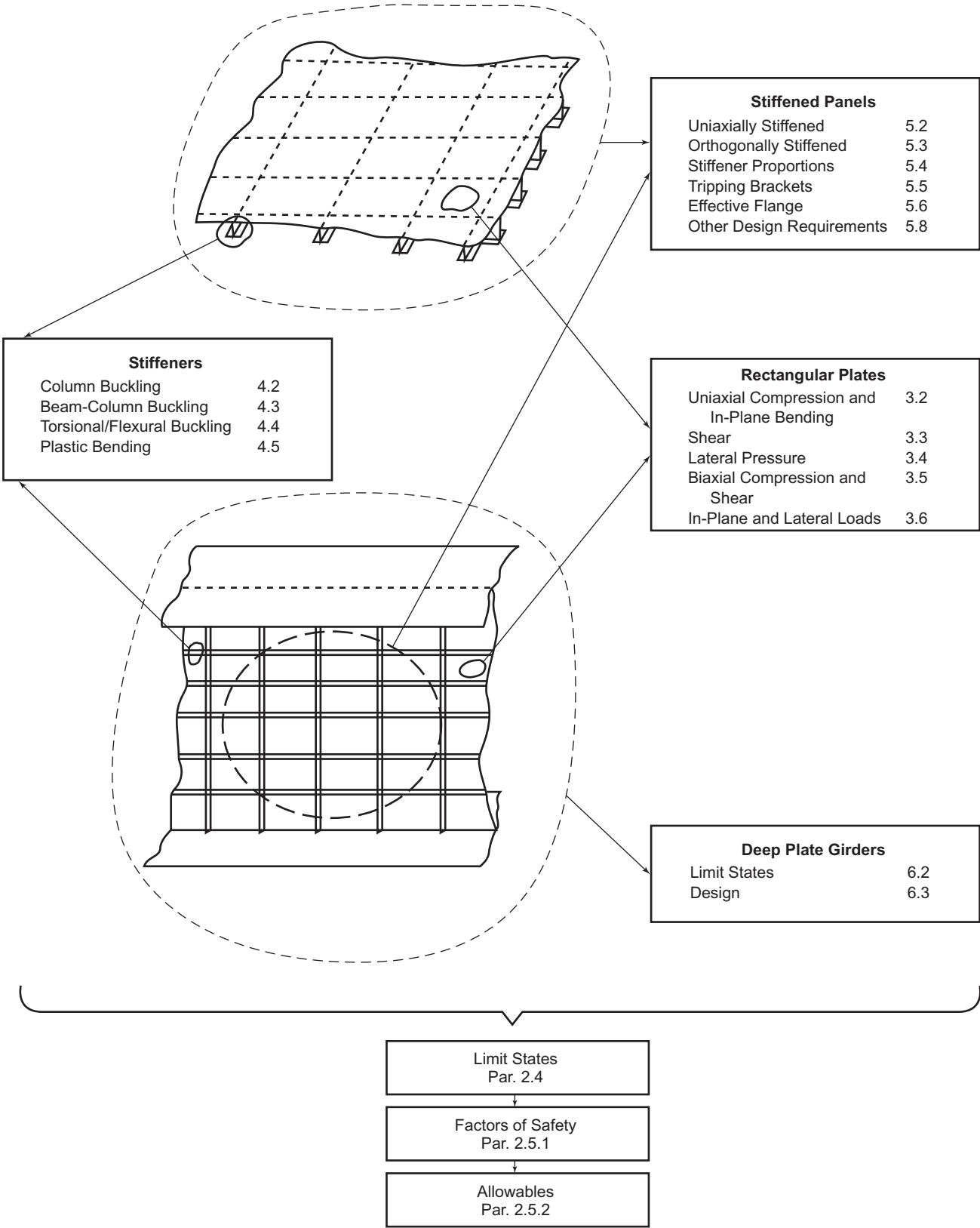


Figure 2.1-1—Structural Components and Limit States Covered in this Bulletin

2.3.4 Ultimate limit states associated with failure due to material fracture are not considered. Provisions have to be made to ensure that this type of failure is properly addressed in the design.

2.3.5 Ultimate limit states associated with accidental loads such as collisions, dropped objects, fire, explosion, or flooding are not considered. Design criteria for these loads have to be established, and provisions have to be made to ensure structural adequacy under such conditions.

2.4 LIMIT STATES

2.4.1 Working Stress Design

2.4.1.1 The design basis adopted in this Bulletin is the working stress design method, whereby stresses in all components of the structure cannot exceed specified allowable values. Allowable stresses are associated with two basic structural requirements: resistance to failure (ultimate limit states); and stiffness and strength criteria (serviceability limit states).

2.4.1.2 In addition to specifying allowable stress values, certain limits on non-dimensional parameters can be defined. Examples are upper limits on web depth to thickness ratio, or flange width to thickness ratio for I-section stiffening elements, which are in general defined to limit the possibility of buckling of the web or flange. These limits on cross sectional proportions are normally associated with good design practice.

2.4.2 Ultimate Limit States

2.4.2.1 Ultimate limit states correspond to the maximum load carrying capacity of a member at failure. Thus, if an ultimate limit state is reached, the structure collapses and loses its load carrying capacity. Failure may be due to:

1. Material plastic flow,
2. Material fracture,
3. Collapse due to local or general instability.

2.4.2.2 The ultimate limit states considered here include only failure due to material plasticity, and collapse due to local or general instability.

2.4.2.3 In identifying material plastic failure as an ultimate limit state it is necessary to distinguish those cases where the material yields, but there is no plastic mechanism and as such no collapse, and those cases where a plastic mechanism leads to structural instability. If material yielding does not lead to collapse, failure is not an ultimate limit state but a serviceability limit state. This distinction is important, since by designing for limited and controlled material yield a more weight efficient design can possibly be achieved. The designer must use critical judgment in identifying those areas and components where plastic design can be adopted.

2.4.2.4 Local instability refers to the type of failure whereby only a localized portion or subcomponent of the structure fails. In a rectangular panel stiffened by two sets of stiffeners intersecting at right angles, such as a transverse bulkhead or flat, the buckling of a single rectangular plate spanning between consecutive stiffeners is an example of local instability. The tripping of a single stiffener over a single span is another example of local instability. If the complete panel buckles as a whole, the mode of failure is general instability.

2.4.3 Serviceability Limit States

2.4.3.1 Serviceability limit states correspond to loads at which a member no longer satisfies functional requirements, although it is capable of carrying additional loads before reaching an ultimate limit state. Serviceability limit states include:

1. Material yield;
2. Local instability;
3. Deformation;
4. Vibration.

2.4.3.2 Material plastic flow should not adversely affect the structure's appearance or efficiency, and should not lead to excessive deformations. The same applies to local instability, such as the buckling of an individual plate, or the local tripping of a secondary stiffener in a stiffened panel.

2.4.3.3 The deformation of the structure or any of its parts resulting from the normal operating conditions or from damage should not adversely affect its appearance or efficiency, violate minimum specified clearances, or cause drainage difficulties. Damage occurring in specific parts of the structure which might entail excessive maintenance or lead to excessive deformation or corrosion, and hence adversely affect the structure's appearance or efficiency, should be limited.

2.4.3.4 Where there is a likelihood of the structure being subjected to vibration from causes such as wind forces, equipment or other transient loads, measures should be taken to prevent discomfort or alarm, or impairment of a proper function.

2.4.3.5 Serviceability limit states associated with local damage or vibration are not considered in Bulletin 2V. Provisions have to be made by the designer to ensure that these are properly accounted for in the design process.

2.5 VERIFICATION OF STRUCTURAL ADEQUACY

2.5.1 Factors of Safety

2.5.1.1 A design is considered satisfactory if the structure has an adequate margin against failure, or reserve strength, for all applicable limit states. The margin against failure to be adopted in the design is defined in terms of allowable values for the stresses, or other relevant design variables (e.g., pressure, axial load, etc.). The allowables are obtained by dividing limit state values by factors of safety, as described in more detail in 2.5.2. The

factors of safety recommended for design are as follows:

F.S. = 1.67 for serviceability limit states

F.S. = 1.67ψ for ultimate limit states

2.5.1.2 The effects of imperfections are very significant in the elastic range but have little effect in the yield and strain hardening ranges of the material. Therefore, a partial factor of safety, ψ , dependent on the buckling stress is recommended for ultimate limit states. The value of ψ is 1.20 when the buckling stress is elastic, 1.00 when the buckling stress equals the yield stress and varies linearly between these limits.

2.5.1.3 A 1/3 increase in allowable stresses may be used where appropriate. The structure should be designed so that all components are proportioned for basic allowable stresses specified by API RP 2A, API RP 2T, or by the AISC *Specification for the Design, Fabrication and Erection of Structural Steel for Buildings*, latest edition. Where the structural element or type is not covered by the above, a rational analysis should be used to determine the basic allowable stresses, with factors of safety equivalent to those defined. Alternative methods for verifying structural adequacy may also be acceptable, as defined in 2.5.6.

2.5.1.4 In determining structural adequacy two types of load conditions have to be considered: a single load acting on the structure and multiple loads (or load combinations).

2.5.2 Single Load Limit States

2.5.2.1 Each limit is defined in terms of a design variable Q_i . Depending on the particular limit state, this design variable can be, for example, a stress component, a pressure, or a deflection. When a limit state is satisfied:

$$Q_i = Q_i^u \quad (2.5-1)$$

where

Q_i = actual value of the relevant design variable (stress, pressure, deflection, etc.),

Q_i^u = limit state value of Q_i , as defined by the formulas in this Bulletin.

2.5.2.2 Given a particular limit state, a design is considered satisfactory if the associated design variable does not exceed an allowable value given by:

$$\frac{Q_i^u}{F.S.}$$

where F.S. is the appropriate factor of safety.

2.5.3 Combined Load Limit States

When n loads $Q_1, \dots, Q_n$ act on a structure a limit state is defined in this Bulletin in terms of an interaction equation:

$$\left(\frac{Q_1}{Q_1^u}\right)^{m_1} + \left(\frac{Q_2}{Q_2^u}\right)^{m_2} + \dots + \left(\frac{Q_n}{Q_n^u}\right)^{m_n} = 1 \quad (2.5-2)$$

where Q_i^u , is the limit state value of Q_i when Q_i is the only load acting on the structure.

Interaction equations are in most cases of an empirical nature, with the exponents m_i being determined on the basis of a best fit of experimental data.

2.5.3.1 Given a particular limit state, a design is considered satisfactory if the relevant design variables do not exceed allowable values given by $Q_1^u / \text{F.S.}$, $Q_2^u / \text{F.S.}$, ... $Q_n^u / \text{F.S.}$, where Q_1^u ... Q_n^u are the limit state design variables satisfying the interaction equation above, and F.S. is the appropriate factor of safety.

2.5.3.2 The interaction equations and the formulations for the limit state values of the relevant design variables given in this Bulletin reflect serviceability and ultimate limit states. In using them for specific applications the designer must ensure that the appropriate factors of safety (F.S.'s) are adopted, as prescribed in 2.5.1, 2.5.2, and 2.5.3.

2.5.4 Governing Limit State

In general, both serviceability and ultimate limit states are defined for each mode of failure. Either of these limit states can govern the design by imposing a lower allowable value on the design variable Q_i . However, the allowable values for Q_i resulting from serviceability and ultimate limit state considerations should be close for an efficient design. A design is considered satisfactory if the design variables do not exceed their allowable values for all the applicable limit states.

Note: formulations given in this Bulletin for the ultimate limit state sometimes yield lower values than the serviceability limit state. This is a function of the plate geometry and material properties.

2.5.5 Other Limit States

To ensure that a structure is adequate, it is necessary to consider other modes of failure not treated in Bulletin 2V. These include failure due to material fracture or fatigue, and failure caused by accidental loads.

2.5.6 Alternative Methods for Verifying Structural Adequacy

2.5.6.a General. The formulations for the limit states included in Bulletin 2V may be replaced by more refined analyses, or model tests, taking into account the real boundary conditions, the actual load distribution, geometrical imperfections, material properties and residual stresses. In adopting these alternative methods it is necessary to ensure that the

structure is correctly modeled, and that all relevant limit states are considered. In particular if weight savings and increased structural efficiency are necessary, more refined methods of analysis should be explored.

2.5.6.b Methods of Analysis. The methods of analysis that are adequate for considering the ultimate limit states include elastic methods, and plastic or yield-line methods. Elastic methods (in which P-delta effect is included and all failure modes are accounted for by appropriate stress limits, but plastic load redistribution does not occur) are acceptable as lower bound collapse solutions, and they will also lead to solutions less likely to violate serviceability criteria. Elastic methods imply that a valid yield criterion is adopted to ensure, together with equilibrium, the static admissibility of the solution.

Plastic or yield-line methods may be adopted when appropriate to the structural configuration. Plastic methods or other procedures for permitting redistribution of moments and shears may be used only when:

- a. The structural configuration and the materials have an adequate plateau of resistance under the appropriate ultimate conditions, and are not prone to deterioration of strength due to shakedown under repeated loading;
- b. The development of bending plasticity does not cause an indeterminate deterioration in shear, torsional or axial strength, when relevant;
- c. The supports or supporting structures are capable of withstanding reactions calculated by elastic methods.

The methods of analysis that are adequate for considering the serviceability limit states are in general elastic methods. Linear methods may be used when changes in geometry do not significantly influence the structure's performance. Nonlinear methods may be adopted with appropriate allowances for loss of stiffness, and should be used where geometric changes significantly modify the structure's performance. The method used should at all times satisfy equilibrium requirements and compatibility of deformations.

The mathematical idealization of the structure should reflect the nature of its response. The boundaries assumed in such an idealization should either calculate accurately the stiffness of adjacent parts, or be sufficiently remote from the part under consideration, for the stresses to be insensitive to the boundary assumptions.

2.5.6.c Model Analysis and Testing. Model analysis and testing may be used either to define the load effects in a structure, or to verify a proposed theoretical analysis. The models used should be capable of simulating the response of the structure appropriately, and the interpretation of the results should be carried out by engineers having the relevant experience. Model tests are particularly important in those cases where the geometry being proposed is novel, or not proven for the specific application under consideration.

The reliability of the test results depends upon the accuracy or knowledge of several factors, such as:

- a. Material properties (model and prototype);

- b. Methods of measurement;
- c. Methods used to derive load effects from measurements;
- d. Loading and reactions.

In interpreting results, the load effects to be used in design should exceed those derived from the test data by a margin dependent upon:

- e. Number of tests;
- f. Method of testing;
- g. An assessment of a., b., and c. above.

In all cases the interpreted results should satisfy equilibrium and compatibility.

Where prototype testing is adopted as a basis for proving the resistance of a component, the test loading should adequately reproduce the range of stress combinations to be sustained in service. A sufficient number of prototypes should be tested to enable a mean value and standard deviation of resistance to be calculated for each critical stress condition. A particular aspect of structural behavior that may not be modeled correctly in small scale testing is residual stresses. It is important that this factor be accounted for in interpreting results, and in extrapolating to full scale.

The material strengths to be specified for construction of the model should have mean values and coefficients of variation compatible with those in the prototypes. Tolerances and dimensions should be similarly prescribed so that the models are compatible with the prototypes.

2.6 STRUCTURAL COMPONENT LOADS AND LOAD COMBINATIONS

2.6.1 General

The loads and load combinations that are to serve as a basis of design are defined in appropriate documents such as API RP 2T, API RP2FPS, etc.

2.6.2 Primary Loads

2.6.2.1 Primary loads and load combinations for structural component design, such as stiffened panels or deep girders, result in general from global platform analysis, to be discussed in 2.7. These primary loads can typically be classified as follows:

- axial tension or compression;
- shear;
- bending;
- twisting;
- lateral loading (distributed or concentrated).

Typical load combinations that are relevant for design include, for example:

- axial compression and shear;
- axial compression and bending;
- biaxial bending;
- bending and torsion.

2.6.2.2 The most relevant loads and load combinations for structural component analysis are treated in Bulletin 2V. The structural components considered are thin rectangular plates, stiffeners, stiffened panels and deep plate girders. However, the treatment is not comprehensive, and the designer should use other methods to ensure structural adequacy for those loads or load combinations not treated in the Bulletin. In particular, no consideration is given to concentrated loads on plates.

2.6.3 Secondary Loads

2.6.3.1 For most commonly encountered load cases, secondary loads do not directly affect the limit states, but the designer should ensure that they are included, when appropriate.

2.6.3.2 Examples of secondary loads include:

- shrinkage forces due to welding;
- stresses due to construction tolerances;
- thermal loads.

2.6.3.3 In cases controlled by fire considerations, thermal loads should be treated as primary loads.

2.6.4 Accidental Loads

As indicated in 2.3, accidental loads, such as those caused by collisions, dropped objects, fire, explosion, or flooding, are not considered. Some of these loads can lead to the rapid loss of strength of the primary structure and bring about an ultimate limit state. The designer should use acceptable methods to assess the adequacy of the structure to withstand such loads.

2.7 GENERAL APPROACH TO STRUCTURAL ANALYSIS

2.7.1 General

General principles regarding analysis methods, modeling, stress analysis and fatigue analysis for structures are covered in API RP 2T.

2.7.2 Global, Panel, and Plate Stresses

2.7.2.1 The structural analysis of a stiffened plate structure requires the consideration of several models. Global behavior can be represented through the use of a 3-D finite element model describing the whole structure. A more precise definition of stress distribution requires the consideration of smaller models, representing main structural components, or more localized areas of the structure, such as stiffened panels. Finally, main structural components can be further subdivided into the most basic elements, which are thin plates and stiffeners.

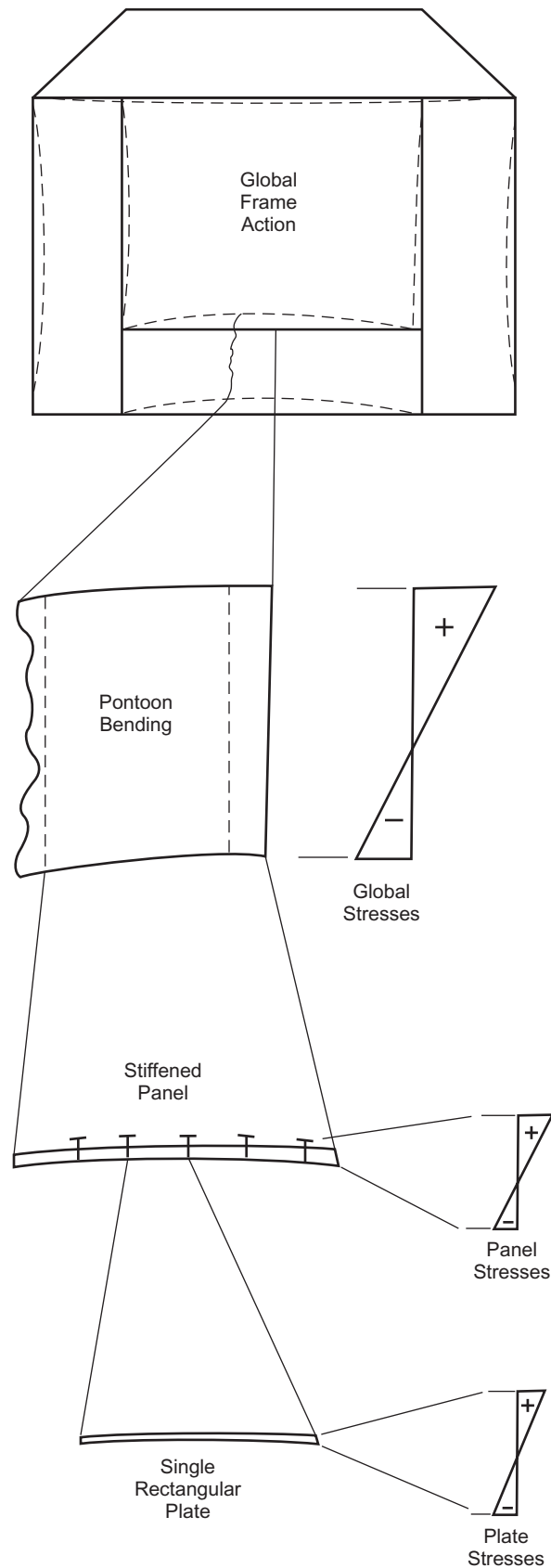


Figure 2.7-1—Global, Panel, and Plate Stresses

2.7.2.2 The 3-D finite elements model leads to stress distributions over gross cross sections of the structure, such as the columns or pontoons. These stresses resulting from deformation of the structure are global stresses. In the case of a pontoon of rectangular cross section, for example, the global stresses result from axial load, shear, biaxial bending and torsion. Assuming that the members in the space frame model are slender the global stresses can be obtained from simple beam theory, with corrections for shear, if necessary.

2.7.2.3 The next main structural component is the stiffened panel. The main stresses are generally due to bending and transverse shear, and are a result of local applied pressures or transverse loads. These stresses can be called panel stresses, and can be derived on the basis of orthotropic plate or grillage theory.

2.7.2.4 A single rectangular plate is the most basic component of flat plate structures. If the plate behavior between stiffeners under lateral pressure is considered, the resulting stresses are the plate stresses. These can be derived on the basis of thin plate theory.

2.7.2.5 Typical global longitudinal bending stress distributions for a pontoon cross section are sketched in Figure 2.7-1. They vary linearly across the depth of the cross section. Typical panel stresses for the pontoon bottom are also shown. They vary linearly across the depth of the stiffened panel, reaching maximum values at the extreme fiber of the stiffener flange, or at the shell plate. Plate bending stresses vary linearly across the plate thickness and are zero at its middle surface.

2.7.2.6 Given this breakdown of stresses into the three main categories, global, panel and plate, it becomes possible to use linear superposition to assess the resulting stress in different components of the structure, assuming elastic material properties and small deformations.

2.7.2.7 This classification of stresses is practical in those areas where the structure can easily be subdivided into global (space frame), panel (stiffened panel), and plate (plate) functions. In areas such as the nodes (where the columns and pontoons intersect), more refined stress analysis methods become necessary, such as the finite element method (Ref. APPENDIX B).

2.7.3 Dimensional Imperfections

Dimensional imperfections, such as out-of-straightness of stiffeners or out-of-flatness of plates, can have a strong impact on structural performance. Structural analysis has to account for dimensional imperfections in case these are beyond the tolerances established in 10.2.3 of API RP 2T. Numerical methods, such as the finite element method, are usually required to study the implications of imperfections on performance.

2.7.4 Residual Stresses and Weld Shrinkage Forces

2.7.4.1 Residual stresses can have some impact on structural performance. There are no simple analytical ways of determining how they affect the structure. Weld shrinkage forces can only be estimated on the basis of empirical equations, but they depend on many factors that cannot be controlled by the designer.

2.7.4.2 Examples of factors that can affect residual stresses and weld shrinkage forces are the assembly sequence, the welding procedure and the use of temporary bracing.

2.7.4.3 The designer should use engineering judgment in deciding how relevant residual stresses and weld shrinkage forces can be for a particular application.

2.8 GENERAL APPROACH TO STRUCTURAL DESIGN

2.8.1 General

Structural design is an iterative process through which the layout and scantlings for a structure are determined, such that it meets all the requirements of structural adequacy. The overall configuration results from a synthesis of all design requirements, which are in general dictated by non-structural considerations, such as volume and space requirements, global stability, safety, etc. Thus, structural design is assumed here to concentrate on the choice of an appropriate structural layout and scantlings, or cross-sectional dimensions, of structural components.

2.8.2 Major Structural Design Steps

2.8.2.1 There is no unique way of designing a structure, but in general terms the major steps that are involved can be summarized as follows:

- a. Identify loads and load combinations acting on the structure as a whole, or on its main subcomponents.
- b. Select initial structural layout and scantlings. In general this is based on past experience with similar structures. In those cases where some limits on proportions are specified, these should be respected in the initial configuration. Examples are stiffener proportions, such as maximum web depth to thickness ratio. Absolute minimum or maximum scantlings result in general from practical considerations related to constructability, weldability, etc.
- c. Identify structure's main components, and determine through structural analysis the loads and load combinations acting on each component. Structural analysis would normally start with a global space frame analysis and would then move into specific components, such as stiffened panels and single plates. For selected areas of the structure, global, panel and plate stresses can be computed and combined using linear superposition. In those areas where the structural arrangement is complex, a numerical method of analysis, such as the finite element method, may have to be adopted in order to obtain an accurate picture of the stress distributions.
- d. Identify relevant limit states and associated factors of safety.
- e. Check structural adequacy. If any limit state is violated, adjust scantlings and repeat the analysis and the structural checks. Perform the iterations required to converge to a structurally adequate design. Exercise engineering judgment in

those cases where the design is governed by serviceability, see 2.5.4. Investigate structural adequacy with alternative acceptable methods, in case limit state checks are perceived to lead to structural inefficiency.

- f. Check other limit states, such as fatigue, which requires the selection of main structural detail configurations. Also check the adequacy of the design against accidental loads. If the structure is found to be inadequate, then new design iterations have to be conducted.
- g. “Optimize” structural design. Once an adequate design has been achieved it is in general possible to “optimize” it for a given objective. The objective depends on the structure’s intended use, and can be, for example, the structural weight or the cost of fabrication and installation. Thus, once a new configuration and set of scantlings are derived, structural adequacy (Step e) has to be checked again, in an iterative fashion.

2.8.2.2 Structural “optimization” as a tool of structural design has to be considered with some caution, since proper balance between all desirable features, such as weight efficiency and cost, is in general very difficult to attain. However, it is important that the iterative nature of the design be recognized, and that possible and practical improvements be explored at the design stage. It is also important to note that special attention should be given to a weight engineering function.

2.8.3 Structural Details

2.8.3.1 The importance of good structural details must be emphasized. These have a great impact on structural efficiency and ensure that the structure will perform adequately.

2.8.3.2 The design of structural details requires a coordinated effort between designer, fabricator and installer to ensure constructability. Whenever possible, details should be made uniform, and advantage should be taken of repeatability.

2.8.3.3 Considerations regarding the design of structural details are not provided herein. However, the designer must ensure that good engineering practice is followed in designing details.

Section 3-Plates

3.1 GENERAL

3.1.1 Scope

3.1.1.1 Flat thin rectangular plates, where the thickness is very small as compared to the other plate dimensions, are considered. It is assumed that normal stress in the direction transverse to the plate surface can be disregarded.

3.1.1.2 The provisions in this Bulletin are not valid when the plate thickness is not small, in which case more refined analyses have to be conducted.

3.1.2 Definitions

3.1.2.1 Thin rectangular plates are the simplest component of flat stiffened plate structures. Each plate is usually supported around the four edges by stiffeners. When considering an individual rectangular plate the edge stiffeners are assumed to be sufficiently strong to remain essentially straight under loading.

3.1.2.2 If the plate slope at the edges is fixed, as happens with plating under uniform lateral pressure over continuous supports, the edges can be taken as perfectly clamped. If the edges rotate freely about the supports simply supported conditions govern the plate behavior. The plate edges should in general be assumed simply supported, unless it can be shown that other conditions apply. In particular partial fixity (degree of restraint between fully clamped and simply supported) should be examined, if engineering judgment indicates it is a better representation of the actual structural arrangement.

3.1.2.3 In the case of plate deflections that are not small in comparison with the thickness it is necessary to distinguish between immovable edges and edges free to move in the plane of the plate. This distinction can have a considerable impact on the magnitude of deflections and stresses. If the plate edges are fully prevented from moving in the plane of the plate, membrane effects can significantly affect its carrying capacity, and could be included provided the deflection limits are not exceeded.

3.1.2.4 The following nomenclature will be adopted here: The long plate dimension or length is parallel to the x -axis or longitudinal direction and is labeled a . The small plate dimension or width is parallel to the y -axis or transverse direction and is labeled b . Thus the plate's aspect ratio, $= a/b$, is always equal to or larger than unity. The plate thickness is t .

3.1.3 Loads and Load Combinations

3.1.3.1 A rectangular plate can be subjected to a variety of primary and secondary loads and load combinations.

3.1.3.2 The following loads can be classified as primary loads, as shown in Figure 3.1-1.

- In-plane longitudinal tension or compression;
- In-plane transverse tension or compression;
- In-plane longitudinal bending;
- In-plane transverse bending;
- In-plane shear;
- Twisting;
- Lateral pressure.

3.1.3.3 In addition to these primary loads the plate can also be subjected to secondary loads as follows:

- Shrinkage forces due to welding;
- Stresses due to construction tolerances;
- Loads due to thermal effects.

3.1.3.4 The following loads and load combinations are considered in Bulletin 2V:

- Uniaxial (longitudinal or transverse) compression;
- In-plane bending;
- In-plane edge shear;
- Uniform lateral pressure;
- Biaxial compression with or without edge shear;
- Uniform lateral pressure and in-plane biaxial loading.

3.1.3.5 If other load types or load combinations are known to be acting on the plate, special consideration will have to be given to their treatment, since they are not covered by the provisions in this Bulletin. This applies in particular to the case of concentrated loads.

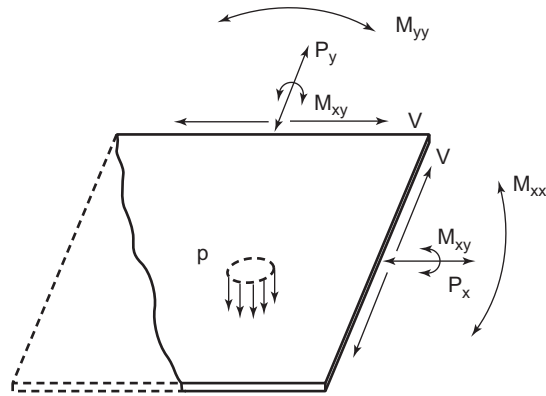
3.1.4 Stress Analysis

3.1.4.1 The stresses in a thin plate can be calculated on the assumption that plane sections remain plane, following the approach adopted in classical thin plate theory.

3.1.4.2 Finite element or other type of numerical analysis can be used in those cases where the applied loads and/or boundary conditions require a more refined treatment, or when the thin plate assumptions are no longer acceptable.

3.1.5 Stress Distributions

3.1.5.1 For an in-plane load P applied uniformly across the plate's edges the corresponding stress is $f = P/A_e$, where A_e is the edge area. Similarly, for an in-plane shear load V the average shear stress is $f_{xy} = V/A_e$.



P_x, P_y : in-plane tension or compression
 M_{xx}, M_{yy} : in-plane bending
 M_{xy} : Twisting
 V : in-plane shear
 p : lateral pressure

Figure 3.1-1—Primary Loads Acting on a Rectangular Plate

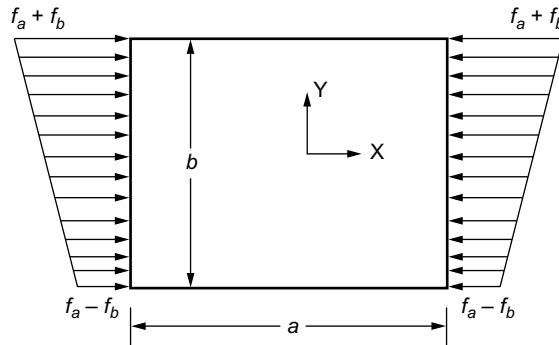


Figure 3.2-1—Long Rectangular Plate

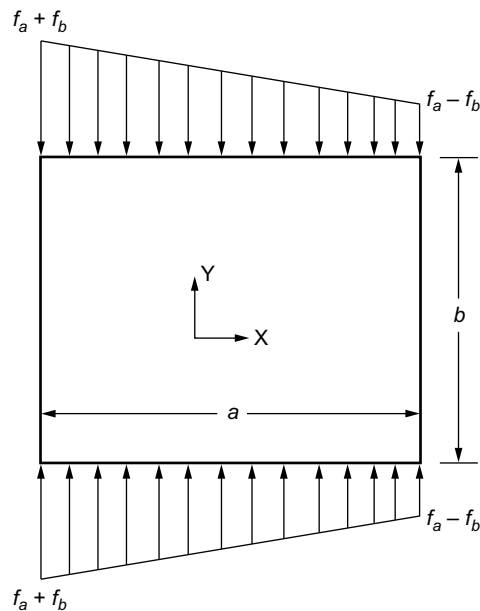


Figure 3.2-2—Wide Rectangular Plate

3.1.5.2 In the case of lateral loads the bending stresses are zero at the mid-surface and vary linearly across the thickness of the plate, with a maximum at the surface given by:

$$f_{x \max} = \frac{6M_x}{t^2} \quad (3.1-1)$$

$$f_{y \max} = \frac{6M_y}{t^2} \quad (3.1-2)$$

where M_x is the bending moment per unit length for bending about the y axis, and M_y is the bending moment per unit length for bending about the x -axis. The shear stress resulting from a twisting moment per unit length M_{xy} is also zero at the plate's mid-surface and varies linearly across the thickness, with a maximum at the surface given by:

$$f_{xy \max} = \frac{6M_{xy}}{t^2} \quad (3.1-3)$$

3.1.5.3 The shear stresses f_{xz} and f_{yz} can be determined by assuming that they are distributed across the plate thickness according to a parabolic law, as in simple beam theory. Thus the maximum values are:

$$f_{xz \max} = \frac{3}{2} \frac{Q_x}{t} \quad (3.1-4)$$

$$f_{yz \max} = \frac{3}{2} \frac{Q_y}{t} \quad (3.1-5)$$

where Q_x and Q_y are the transverse shear force per unit length along the edges parallel to the y and x axis, respectively.

3.2 UNIAXIAL COMPRESSION AND IN-PLANE BENDING

3.2.1 Definitions

Two types of plates are considered. Figure 3.2-1 shows long plates under longitudinal compression stress (f_a) and in-plane bending stress (f_b), where the load is applied to the shorter edges. Figure 3.2-2 shows wide plates, or plates under transverse compression stress (f_a) and in-plane bending stress (f_b), where the load is applied to the larger edges.

The serviceability limit state is reached when the applied in-plane compressive stress, f , equals the appropriate limiting stress. The limit stress is f_{se} when f is in the elastic range, or f_{sp} when f is in the inelastic or plastic range. Specifically, elastic serviceability limit f_{xse} applies for long plates, and f_{yse} applies for wide plates. Likewise, the plastic serviceability limit f_{xsp} applies for long plates, and f_{ysp} for wide plates. The ultimate limit state is reached

when f equals f_{xu} for long plates, or f_{yu} for wide plates, respectively. The allowable stress is obtained by dividing the limit state stress f_{se} , f_{sp} , or f_u by the appropriate factor of safety F.S. The wide plate formulas should be used for square plates.

3.2.2 Serviceability Limit State

a. Long Plates (Figure 3.2-1)

$$f_{xse} = k \frac{\pi^2 E}{12(1-\nu^2)} \left(\frac{t}{b} \right)^2 \quad (3.2-1)$$

$$k = 8.4/(1.1+r), \quad 0 \leq r \leq 1$$

$$= 7.6 - 6.4r + 10r^2, \quad -1 \leq r < 0$$

where

$$r = (f_a - f_b)/(f_a + f_b), \quad f_b \geq 0$$

The expression above is based on the assumption that the plate edges are simply supported. If other boundary conditions apply the buckling coefficient k can be determined from Figure 3.2-3.

Elastic range ($f_{xse} < F_p$):

$$f_{xsp} = f_{xse} \quad (3.2-2)$$

Plastic range ($f_{xse} \geq F_p$):

$$f_{xsp} = \frac{F_y f_{xse}^2}{F_p (F_y - F_p) + f_{xse}^2} \quad (3.2-3)$$

b. Wide Plates (Figure 3.2-2)

$$f_{yse} = k \frac{\pi^2 E}{12(1-\nu^2)} \left(\frac{t}{b} \right)^2 \quad (3.2-4)$$

$$k = (1 + (b/a)^2)^2 2.1/(1.1+r) \quad 0 \leq r \leq 1$$

$$= (1 + (b/a)^2)^2 (2.1/1.1)(1+r) \quad -1 \leq r < 0 \text{ and } a/b \leq 1.5$$

$$+ 10r(1+r)(b/a)^2 - 24r(b/a)^2$$

$$= (1 + (b/a)^2)^2 (2.1/1.1)(1+r)$$

$$+ 10r(1+r)(b/a)^2 \quad -1 \leq r < 0 \text{ and } a/b > 1.5$$

$$- r[2 + 16(b/a)^2 + 8(b/a)^4]$$

where

$$r = (f_a - f_b)/(f_a + f_b), \quad f_b \geq 0$$

Elastic Range ($f_{yse} < F_p$):

$$f_{ysp} = f_{yse} \quad (3.2-5)$$

Plastic Range ($f_{yse} \geq F_p$):

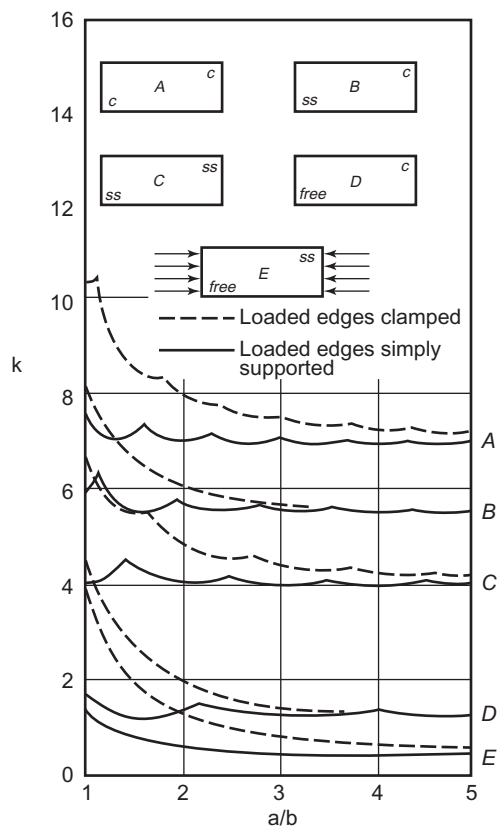


Figure 3.2-3—Buckling Coefficients for Plates in Uniaxial Compression¹

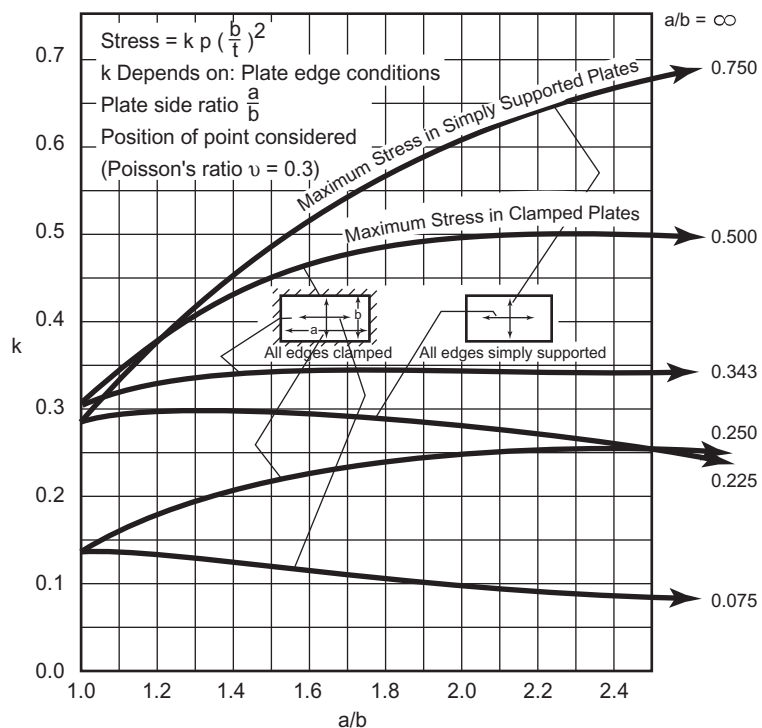


Figure 3.4-2—Stresses in Plates Under Uniform Lateral Pressure³

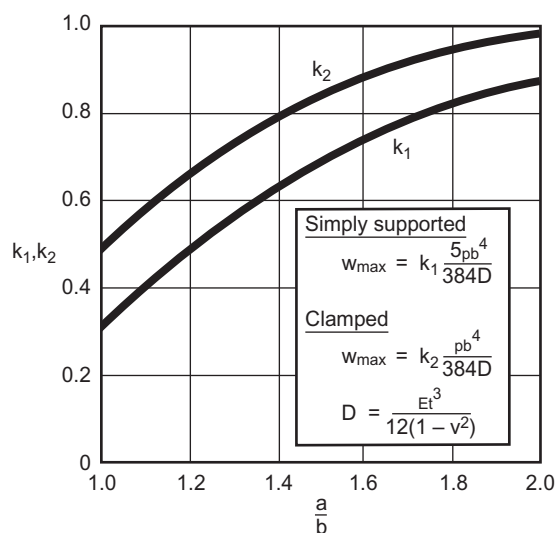


Figure 3.4-1—Coefficients for Computing Plate Deflections²

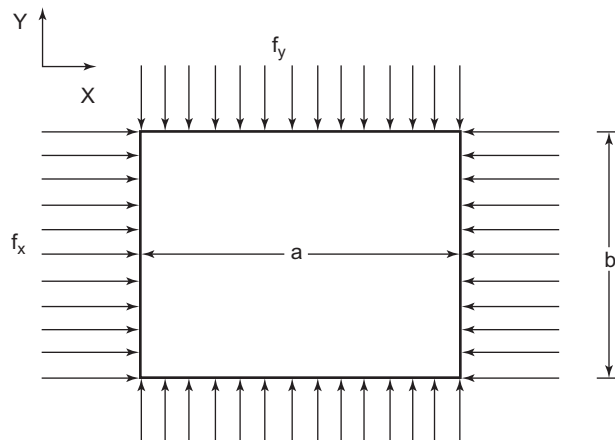


Figure 3.5-1—Rectangular Plate Under Biaxial Compression

¹From D.O. Brush and B.O. Almroth, "Buckling of Bars, Plates and Shells," McGraw-Hill, 1975.

²From O. Hughes, "Ship Structural Design: A Rationally Based Computer-Aided, Optimization Approach," Wiley Interscience, 1988.

³From S.P. Timoshenko and S. Woinowski-Krieger, "Theory of Plates and Shells," McGraw-Hill, 1959.

$$f_{ysp} = F_y - \frac{F_p(F_y - F_p)}{f_{yse}} \quad (3.2-6)$$

3.2.3 Ultimate Limit State

a. Long Plates (Figure 3.2-1)

$$f_{xu} = F_y \left(\frac{2}{\beta} - \frac{1}{\beta^2} \right), \beta \geq 1 \quad (3.2-7)$$

$$f_{xu} = F_y, \beta < 1 \quad (3.2-8)$$

where

$$\beta = \frac{b}{t} \sqrt{\frac{F_y}{E}}$$

These apply when the plate edge stress reaches yield before the stiffener fails. Otherwise, the following formulas should be used:

$$f_{xu} = F_y \frac{1}{\beta}, \beta \geq 1 \quad (3.2-9)$$

$$f_{xu} = F_y, \beta < 1 \quad (3.2-10)$$

b. Wide Plates (Figure 3.2-2)

$$f_{yu} = F_y \left[\frac{1}{\alpha} C + 0.10 \left(1 - \frac{1}{\alpha} \right) \left(1 + \frac{1}{\beta^2} \right)^2 \right] \leq F_y \quad (3.2-11)$$

$$C = \frac{2}{\beta} - \frac{1}{\beta^2}, \beta \geq 1$$

$$C = 1, \beta < 1$$

where

$$\alpha = a/b \geq 1$$

3.3 EDGE SHEAR

3.3.1 Definitions

The serviceability limit state is reached when the applied edge shear stress f_{xy} equals f_{xyse} or f_{xysp} . The limit f_{xyse} applies in the elastic range, while f_{xysp} applies in the plastic range. The ultimate limit state is reached when f_{xy} equals f_{syu} . The allowable stress is obtained by dividing the limit state stress (f_{xyse} , f_{xysp} or f_{syu}) by the appropriate factor of safety F.S.

3.3.2 Serviceability Limit State

$$f_{xyse} = k \frac{\pi^2 E}{12(1-\nu^2)} \left(\frac{t}{b} \right)^2 \quad (3.3-1)$$

$$k = 5.34 + \frac{4}{\alpha^2}$$

The result given is based on the assumption that the plate edges are simply supported. If the plate edges can be considered clamped the buckling coefficient k takes the form:

$$k = 8.98 + \frac{5.60}{\alpha^2}$$

Elastic range ($f_{xyse} < F_p / \sqrt{3}$):

$$f_{xyse} = f_{xyse} \quad (3.3-2)$$

Plastic range ($f_{xyse} \geq F_p / \sqrt{3}$):

$$f_{xyse} = \frac{3\tau_y f_{xyse}^2}{F_p (F_y - F_p) + 3f_{xyse}^2} \quad (3.3-3)$$

3.3.3 Ultimate Limit State

$$f_{xyu} = f_{xyse} + \frac{\sqrt{3}}{2\sqrt{1+\alpha+\alpha^2}} (\tau_y - f_{xyse}) \quad (3.3-4)$$

where f_{xyse} is the serviceability limit state shear stress defined in 3.3.2.

$$\text{where } \tau_y = \frac{1}{\sqrt{3}} (F_y)$$

3.4 UNIFORM LATERAL PRESSURE

3.4.1 Definitions

a. Serviceability Limit State. The serviceability limit state is based on a deflection criterion and a stress criterion.

b. Deflection Criterion. The deflection criterion is associated with a maximum allowable deflection W_a . Two cases have to be considered: (1) no permanent plastic deformations allowed, so that W_a is an elastic deflection; (2) permanent plastic deformation or plastic set allowed, so that W_a is a plastic deformation. No specific guidelines can be given on the allowable deflection, and whether it should remain purely elastic or become a permanent plastic set, since it depends on the type of service intended for the structure. In general the deflection should not be such as to adversely affect the structure's appearance or its performance requirements. In those cases where in-plane compressive loads are not present, and where specific operational requirements do not rule against it, a permanent plastic set W_p can be allowed. If as a result of a permanent set membrane effects are induced in the plate its capacity to carry in-plane tensile loads and structural efficiency are improved.

The designer has to use engineering judgment in establishing a maximum allowable deflection, and deciding if a permanent plastic set is acceptable.

If an absolute value cannot be specified, a criterion based on the maximum span and/or thickness can be adopted, such as the maximum of $W_a = C_1 \times (\text{span})$ and $W_a = C_2 \times (\text{thickness})$, where C_1 and C_2 are non-dimensional parameters (such as $C_1 = 1/360$ and $C_2 = 1$). If a permanent plastic set is allowed a criterion for determining its magnitude is given in 3.4.2.

Expressions for estimating the maximum elastic deflection in a rectangular plate subjected to uniform lateral pressure are given in 3.4.2.

c. Stress Criterion. The serviceability limit state stress criterion implies that the plate's material must remain in the elastic range, and it is expressed in the form of a yield criterion, defined in 3.4.2. In cases where a permanent plastic set is allowed this stress criterion does not apply.

d. Ultimate Limit State. The ultimate limit state is reached when the lateral pressure equals ρ_u , as defined in 3.4.3.

3.4.2 Serviceability Limit State

a. Deflection Criterion. If no permanent plastic set is allowed a maximum allowable elastic deflection W_a must be selected by the designer, given the particular application being considered (see discussion in 3.4.1). The computed maximum elastic deflection W_e must satisfy:

$$W_e \leq W_a \quad (3.4-1)$$

W_e can be estimated from the following expressions:

$$W_e = k_1 \frac{5pb^4}{384D}, \text{ simply supported edges} \quad (3.4-2)$$

$$W_e = k_2 \frac{pb^4}{384D}, \text{ clamped edges} \quad (3.4-3)$$

where D is the plate's flexural rigidity

$$D = \frac{Et^3}{12(1-\nu^2)}$$

and the coefficients k_1 and k_2 can be found from the graphs in Figure 3.4-1.

If a permanent plastic set is allowed (again, the designer has to take into consideration all aspects of performance requirements, as discussed in 3.4.1), it should be limited to:

$$W_p \leq 0.2b \sqrt{\frac{F_y}{E}} \quad (3.4-4)$$

b. Stress Criterion. If no permanent plastic set is allowed the plate's material must remain in the elastic regime, so that the maximum stresses f_x and f_y must satisfy the following relation:

$$f_x^2 + f_y^2 - f_x f_y \leq F_p^2 \quad (3.4-5)$$

where tensile stresses are taken as positive and compressive stresses as negative.

The maximum stresses f_x and f_y can be estimated from the following expression:

$$f_x \text{ or } f_y = k p \left(\frac{b}{t} \right)^2 \quad (3.4-6)$$

where the coefficient k can be found from the graphs in Figure 3.4-2 for simply supported and clamped edge conditions.

If a permanent plastic set is allowed the stress criterion is not applicable.

3.4.3 Ultimate Limit State

$$p_u = F_y \left(\frac{t}{b} \right)^2 \frac{6}{\sqrt{\alpha}} \left(1 + \frac{2}{\alpha} \frac{W_p}{t} \right) \quad (3.4-7)$$

where W_p is the permanent set (see 3.4.2). If no permanent set is allowed $W_p = 0$. These formulas are restricted to plates with aspect ratio $1 \leq \alpha \leq 5$. The allowable pressure is obtained by dividing the limit state pressure p_u by the appropriate factor of safety F.S.

3.5 BIAXIAL COMPRESSION WITH OR WITHOUT EDGE SHEAR

3.5.1 Definitions

The limit state (serviceability or ultimate) is reached if the combination of the applied compressive stresses due to axial compression only, in the x and y directions, or f_x and f_y respectively, Figure 3.5-1, and the edge shear stress f_{xy} are equal to the limit state stresses, f_{xl} , f_{yl} and f_{xyl} , respectively, that satisfy the interaction formulas defined in 3.5.2 and 3.5.3.

3.5.2 Serviceability Limit State

Elastic range:

$$\left[\left(f_{xl} / f_{xse} \right)^c + \left(f_{yl} / f_{yse} \right)^c + \left(f_{xyl} / f_{xyse} \right)^c \right]^{2/c} + \left(f_e / F_y \right)^2 = 1.0 \quad (3.5-1)$$

where

$$\begin{aligned} c &= 2 - 1/\alpha, \alpha \geq 1.0 \\ f_e &= \text{limit state von Mises stress} \\ &= \left(f_{xl}^2 + f_{yl}^2 - f_{xl} f_{yl} + 3 f_{xyl}^2 \right)^{1/2} \end{aligned}$$

and f_{xse} is given in 3.2.2a considering axial compression only, f_{yse} is given in 3.2.2b considering axial compression only, and f_{xyse} is given in 3.3.2.

The allowable stresses are obtained by dividing these limit state stresses, f_{xl} , f_{yl} and f_{xyl} , by the appropriate factor of safety F.S.

3.5.3 Ultimate Limit State

$$(f_{xl} / f_{xu})^A - \bar{\eta}(f_{xl} / f_{xu})(f_{yl} / f_{yu}) + (f_{yl} / f_{yu})^2 + (f_{xyl} / f_{xyu})^2 = 1 \quad (3.5-2)$$

where f_{xu} is given in Par. 3.2.3a, f_{yu} is given in Par. 3.2.3b, f_{xyu} is given in Par. 3.3.3 and

$A = 1$, $\bar{\eta} = 0.25$, for $\alpha \geq 3$

$A = 2$, $\bar{\eta} = 3.2e^{-0.35\beta} - 2$, for $\alpha = 1$

For $1 < \alpha < 3$ and for a given value of the ratio f_{yl} / f_{yu} , the corresponding values of f_{xl} / f_{xu} and f_{xyl} / f_{xyu} can be found by linear interpolation between the values of A and $\bar{\eta}$ obtained for $\alpha = 3$ and for $\alpha = 1$.

The allowable stresses are obtained by dividing these limit state stresses, f_{xl} , f_{yl} , and f_{xyl} , by the appropriate factor of safety F.S.

3.6 COMBINED IN-PLANE AND LATERAL LOADS

3.6.1 Definitions

The serviceability or ultimate limit state is reached if the combination of applied axial stresses in the x and y directions, or f_x and f_y , respectively, edge shear stress f_{xy} , and pressure p , satisfy the interaction formulas defined in 3.6.2 and 3.6.3.

3.6.2 Serviceability Limit State

The serviceability limit state shall be checked if a permanent set is not allowed.

- f_x compression, f_y compression:

$$(f_x / f_{xsp})^2 + (f_y / f_{ysp})^2 + (f_{xy} / f_{xysp})^2 + p / p_s = 1 \quad (3.6-1)$$

where p = applied pressure, p_s = collapse pressure calculated assuming zero permanent plastic set and f_{xsp} , f_{ysp} , and f_{xysp} are the serviceability limit state stresses defined in 3.2.2 and 3.3.2.

- f_x tension, f_y compression:

$$(f_x / F_y)^2 + (f_y / f_{ysp})^2 + (f_{xy} / f_{xysp})^2 + p / p_s = 1 \quad (3.6-2)$$

- f_x tension, f_y tension:

$$-\left[\left(f_x / F_y\right)^2 + \left(f_y / F_y\right)^2\right]^{1/2} + f_{xy} / f_{xysp} = 1 \quad (3.6-3)$$

- f_x compression, f_y tension:

$$\left(f_x / f_{xsp}\right)^2 + \left(f_{xy} / f_{xysp}\right)^2 + p / p_s = 1 \quad (3.6-4)$$

A von Mises based yield criterion is also applied in all quadrants but does not control for compression-compression:

$$\left(f_x / f_{xcr}\right)^2 - \left(f_x / f_{xcr}\right)\left(f_y / f_{ycr}\right) + \left(f_y / f_{ycr}\right)^2 + 3\left(f_{xy} / f_{xycr}\right)^2 = 1 \quad (3.6-5)$$

3.6.3 Ultimate Limit State:

- f_x compression, f_y compression:

$$\left[\left(f_x / \sigma'_{xu}\right)^2 + \left(f_y / \sigma'_{yu}\right)^2\right]^{1/2} + \left(f_{xy} / f_{xyu}\right)^2 = 1 \quad (3.6-6)$$

In this case, σ'_{xu} and σ'_{yu} are reduced from the API Bulletin 2V values due to the presence of lateral pressure:

$$\sigma'_u / F_y = \left(f_u / F_y\right)^{(0.8Q^2 + 0.84Q + 1)}$$

where $Q = pE / F_y^2$, p = applied pressure.

- f_x tension, f_y compression:

$$\left(f_x / F_y\right)^2 + \left(f_y / f_{yu}\right)^2 + \left(f_{xy} / f_{xyu}\right)^2 + p / p_u = 1 \quad (3.6-7)$$

where p = applied pressure, p_u = ultimate pressure under pressure loading only.

- f_x tension, f_y tension:

$$-\left[\left(f_x / F_y\right)^2 + \left(f_y / F_y\right)^2\right]^{1/2} + f_{xy} / f_{xyu} = 1 \quad (3.6-8)$$

- f_x compression, f_y tension:

$$\left(f_x / f_{xu}\right)^2 + \left(f_{xy} / f_{xyu}\right)^2 + p / p_u = 1 \quad (3.6-9)$$

A von Mises based yield criterion is also applied in all quadrants but does not control for compression-compression:

$$\left(f_x / \sigma'_x\right)^2 - \left(f_x / \sigma'_x\right)\left(f_y / \sigma'_y\right) + \left(f_y / \sigma'_y\right)^2 + 3\left(f_{xy} / f_{xyu}\right)^2 = 1$$

where:

$$\sigma'_x = F_y \left(1 - (Q / Q_u) \bar{\lambda}\right)^{1/2}$$

$$\sigma'_y = F_y \left(1 - (Q / Q_u) \bar{\lambda} \lambda\right)^{1/2}$$

$\lambda = f_{yb} / f_{xb}$, bending stress ratio

$$\bar{\lambda} = (1 - \lambda + \lambda^2)^{-1/2}$$

$$Q = pE / F_y^2$$

$$Q_u = p_u E / F_y^2$$

3.7 DESIGN CONSIDERATIONS

When a thin rectangular plate of a given material is subjected to compressive stresses it can fail by instability, and the strength depends primarily on the type of loads and/or load combinations, the boundary conditions and the geometry (dimensions, aspect ratio).

The plate is in general part of a stiffened panel, such as in a deck or bulkhead, and it is supported by stiffeners. The stiffener spacing should be selected so as to limit the plate geometry and aspect ratio to dimensions and proportions that can provide the necessary strength. The designer must change the plate proportions and thickness until all applicable limit states are satisfied. If necessary, additional stiffeners might have to be introduced in the design. The minimum stiffener spacing should be based on fabrication considerations.

When the plate is primarily subjected to lateral loading, the tensile membrane effects substantially improve its carrying capacity. In designing the supports, full in-plane fixity should be provided whenever possible in order to take advantage of membrane effects.

Section 4-Stiffeners

4.1 GENERAL

4.1.1 Scope

Straight and slender thin-walled members of uniform cross section containing at least one plane of symmetry and thin-walled angles that serve as stiffening elements for flat plate structures are considered.

4.1.2 Definitions

Stiffeners are used to strengthen plates and to increase their load carrying capacity. In most cases they are made of a thin-walled web welded to the plate and a flange. Thus, when determining the cross sectional properties, account should be given to the attached plating acting with the stiffener as an effective flange. When the stiffeners are subjected to axial compressive loads the effective plate flange width b_e , when the maximum edge stress reaches the yield stress, is

$$b_e = b \frac{f_u}{F_y} \quad (4.1-1)$$

where f_u (f_{xu} or f_{yu}) is determined from 3.2.3. When the stiffeners are subjected to lateral or tensile loading alone, the effective plate flange is governed by shear lag effects and should be determined from 5.6.

The following ultimate limit states will be considered:

- column buckling;
- beam-column buckling;
- torsional/flexural buckling;
- plastic bending.

4.1.3 Loads and Load Combinations

A plate stiffener can be subjected to a variety of primary and secondary loads and load combinations.

The following loads can be classified as primary loads:

- axial tension or compression;
- bending about the axis of maximum moment of inertia;
- bending about the axis of minimum moment of inertia;
- lateral distributed load;
- lateral concentrated loads.

In addition to these primary loads the plate can also be subjected to secondary loads as follows:

- shrinkage forces due to welding;
- stresses due to construction tolerances;
- loads due to thermal effects.

The following loads and load combinations are considered here:

- axial compression;
- axial compression and lateral load;
- lateral load.

If other load types or load combinations are known to be acting on the plate special consideration will have to be given to their treatment, since they are not covered by the provisions of this Bulletin.

4.1.4 Stress Analysis

The stresses in a slender thin-walled stiffener can be calculated on the assumption that plane sections remain plane, following the approach adopted in classical beam theory.

Finite element or other type of numerical analysis can be used in those cases where the applied loads and/or boundary conditions require a more refined treatment, or when the classical beam theory assumptions are no longer acceptable.

4.1.5 Stress Distributions

As a result of conventional beam theory, the longitudinal bending stresses in a stiffener vary linearly across the depth. If the section is subjected to both compression and bending, the stress distribution is given by:

$$f = \frac{P}{A} + \frac{M\bar{y}}{I_{ef}} \quad (4.1-2)$$

where P is the compressive load, A is the cross sectional area, M is the applied bending moment, $\bar{y}$ is the distance to the neutral axis and I_{ef} is the effective moment of inertia about the neutral axis. In computing I_{ef} the effective flange should be used, as prescribed in 4.1.2. If the stiffener is subjected to lateral or tensile load alone, the effective flange is governed by shear lag effects and should be determined from 5.6.

The shear stress distribution can be obtained from:

$$f_{xy} = \frac{VQ}{It} \quad (4.1-3)$$

where V is the shear force, Q is the moment of the area above the point where shear stress is being determined about the neutral axis, I is the moment of inertia about the neutral axis, and t is the thickness at the point under investigation.

For webs with constant thickness the shear stress can be approximated by:

$$f_{xy} = \frac{V}{A_w} \quad (4.1-4)$$

where A_w is the web area.

4.2 COLUMN BUCKLING

4.2.1 Definitions

The ultimate limit state is reached when the applied axial load P equals P_{Ee} or P_{Ep} . The limit P_{Ee} applies in the elastic range, while P_{Ep} applies in the plastic range. The allowable axial load is obtained by dividing the limit state axial load (P_{Ee} or P_{Ep}) by the appropriate factor of safety F.S.

4.2.2 Ultimate Limit State

$$P = \frac{P_y}{\lambda^2}$$

where $\lambda = \frac{1}{\pi} \left(\frac{KL}{r} \right) \sqrt{\frac{F_y}{E}}$ (4.2-1)

Elastic range ($P < p_r P_y$):

$$P_{Ee} = P \quad (4.2-2)$$

Plastic range ($P \geq p_r P_y$):

$$P_{Ep} = P_y \left[1 - \frac{p_r (1 - p_r)}{P / P_y} \right]$$

where $p_r = \frac{F_p}{F_y}$; $P_y = A F_y$ (4.2-3)

4.3 BEAM-COLUMN BUCKLING

4.3.1 Definitions

The ultimate limit state is reached when the applied axial load P and bending moment M satisfy the interaction curve specified in 4.3.2.

4.3.2 Ultimate Limit State

$$\frac{P}{P_u} + B_1 \frac{M}{M_u} = 1 \quad (4.3-1)$$

where P_u is equal to P_{Ee} or P_{Ep} , as given in 4.2.2, depending on whether the material is in the elastic or inelastic range, respectively, and $M_u = M_{fy}$, and where the amplification reduction factor C_m is defined in AISC.

$$B_1 = \frac{C_m}{1 - P/P_{Ee}} \geq 1.0$$

The allowable axial load and bending moment are obtained by dividing the limit state axial load P and bending moment M by the appropriate factor of safety F.S.

4.4 TORSIONAL/FLEXURAL BUCKLING

4.4.1 Definitions

The following properties of a cross section are particularly related to stiffener torsional/flexural buckling, as well as lateral buckling: the location of the shear center, the torsion constant J , and the warping constant C_w . Expressions for determining the value of these parameters for a number of thin-walled open cross sections are listed in Table 4.4-1.

Two cases have to be considered when dealing with stiffener torsional/flexural buckling. If the stiffener shear center and centroid coincide (as happens with doubly symmetric sections such as equal flanged I-sections), buckling by twisting, with the longitudinal axis through the centroid remaining straight, can occur. In such cases twisting and flexure are decoupled, and the ultimate limit state discussed here is determined by torsional buckling only. If the shear center and the centroid do not coincide (as happens with sections containing only one plane of symmetry such as unequal flanged I-sections), the ultimate limit state is governed by a combination of twisting and bending, since these two actions cannot be decoupled.

In the case of doubly symmetric sections the limit state is reached when the applied axial compressive load P equals P_{Te} or P_{Tp} . The limit P_{Te} applies in the elastic range, while P_{Tp} applies in the plastic range.

In the case of sections containing only one plane of symmetry the limit state is reached when the applied axial compressive load equals P_{TFe} or P_{TFp} , which correspond to the elastic and inelastic ranges, respectively. The allowable axial compressive load is obtained by dividing the limit state load (P_{Te} , P_{Tp} , P_{TFe} , or P_{TFp}) by the appropriate factor of safety F.S.

Sections containing no plane of symmetry, such as angle stiffeners, shall meet the compact section criteria of 4.4.4a.

4.4.2 Ultimate Limit State for Doubly Symmetric Sections

$$P_T = \frac{A}{I_s} \left(GJ + \frac{\pi^2 E C_w}{L^2} \right) \quad (4.4-1)$$

where I_s is the polar moment of inertia about shear center

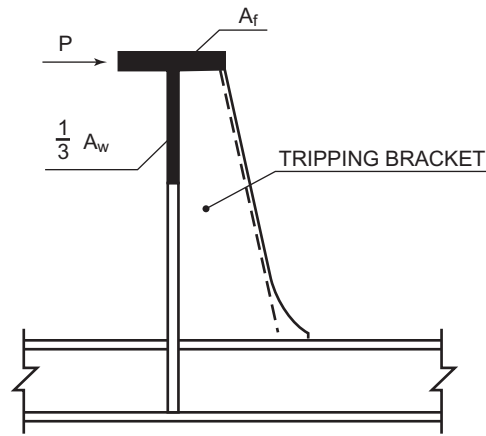


Figure 4.4-1—Design Lateral Load for Tripping Bracket

Table 4.4-1—Properties of Thin-Walled Open Cross Sections¹

0 = shear center	J = torsion constant	C_w = warping constant
	$J = \frac{2bt_f^3 + ht_w^3}{3}$ $C_w = \frac{t_f h^2 b^3}{24}$	If $t_f = t_w = t$: $J = \frac{t^3}{3} (2b + h)$
	$e = h \frac{b_1^3}{b_1^3 + b_2^3}$ $J = \frac{(b_1 + b_2) t_f^3 + ht_w^3}{3}$ $C_w = \frac{t_f h^2}{12} \frac{b_1^3 b_2^3}{b_1^3 + b_2^3}$	If $t_f = t_w = t$: $J = \frac{t^3}{3} (b_1 + b_2 + h)$
	$e = \frac{3b^2 t_f}{6bt_f + ht_w}$ $J = \frac{2bt_f^3 + ht_w^3}{3}$ $C_w = \frac{t_f b^3 h^2}{12} \frac{3bt_f + 2ht_w}{6bt_f + ht_w}$	If $t_f = t_w = t$: $e = \frac{3b^2}{6b + h}$ $J = \frac{t^3}{3} (2b + h)$ $C_w = \frac{tb^3 h^2}{12} \frac{3b + 2h}{6b + h}$
	$J = \frac{2bt_f^3 + ht_w^3}{3}$ $C_w = \frac{b^3 h^2}{12 (2b + h)^2} \times [2t_f (b^2 + bh + h^2) + 3t_w bh]$	If $t_f = t_w = t$: $J = \frac{t^3}{3} (2b + h)$ $C_w = \frac{tb^3 h^2}{12} \frac{b + 2h}{2b + h}$
	$e = 2\alpha \frac{\sin \alpha - \alpha \cos \alpha}{\alpha - \sin \alpha \cos \alpha}$ $J = \frac{2\alpha a t^2}{3}$ $C_w = \frac{2ta^5}{3} \times \left[\alpha^3 - \frac{6(\sin \alpha - \alpha \cos \alpha)^2}{\alpha - \sin \alpha \cos \alpha} \right]$	If $2\alpha = \pi$: $e = \frac{4a}{\pi} \quad J = \frac{\pi \alpha t^3}{3}$ $C_w = \frac{2ta^5}{3} \left(\frac{\pi^3}{8} - \frac{12}{\pi} \right) = 0.0374ta^5$

¹From S.P. Timoshenko, "Theory of Elastic Stability," McGraw-Hill, 1961