

Evaluation of Design Criteria for Storage Tanks With Frangible Roof Joints

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EVALUATION OF DESIGN CRITERIA FOR STORAGE TANKS WITH FRANGIBLE ROOF JOINTS

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FOREWORD

American Petroleum Institute (API) provides standards for the design, construction and inspection of storage tanks. One of the family of design standards, API 650—Welded Steel Tanks for Oil Storage—is the target of the research addressed by this Bulletin. API 650 includes, among other important design provisions, criteria governing the design of frangible roof joints for welded steel, above-ground, cylindrical, storage tanks.

The frangible joint is the primary design feature which assures the functional integrity of the cylindrical shell and bottom joint to contain the liquid in the event of an unexpected over-pressurization of a tank. Because of the importance of the frangible joint design feature from a safety and environmental implication, API decided additional research was warranted. In May 1989, Guido Karcher, acting in behalf of API, defined specific areas where research is needed to evaluate the frangible roof joint provisions of API 650 standard and initiated a request to PVRC Design Division to address these research needs. The request was assigned to the Committee on Dynamic Analysis and Testing for administrative and technical direction. The main goals of the Committee included initiating the research program, monitoring the progress, assuring the results are responsive to the API request and making sure that the results are clearly and concisely documented. Liaison with the Committee on Shells and Ligaments was maintained throughout the research activities.

The Committee on Dynamic Analysis and Testing solicited proposals from qualified organizations to address the needs defined in Guido Karcher's May 1989 letter. The solicitation resulted in four proposals which were thoroughly and objectively evaluated by the Committee. The proposal by Kansas State University, Professors Daniel Swenson and Don Fenton, was selected as the most responsive to the defined needs. A PVRC grant (91-02) was issued in October 1990 for the first phase of the three-phase research program.

Thanks to Dr. Martin Prager for his technical and administrative skills and the dedication and aggressive nature of the research team at Kansas State University under the direction of Professor Swenson, the comprehensive three-phase research program, including a final report of the analytical and experimental evaluations, an interactive Windows based PC program for the analysis of storage tanks with frangible roof joints and user manual for the PC program was completed on schedule in April 1994—a duration of approximately 3½ years from the time the grant was issued. The thoroughness and quality of the products are commendable.

The research team at Kansas State University which consisted of:

- co-investigators Professors Daniel Swenson and Don Fenton
- graduate research assistants Zhi Lu, Asif Ghori and Joe Baalman
- an undergraduate honor student Mark Devries

deserves commendable recognition for a job well done.

In addition special thanks are extended to:

- Mr. Richard Basile for his invaluable liaison efforts with the responsible API Committees, feedback to PVRC and project directors, and review of research reports and the PC-based computer program

- Dr. Greg Hollinger for his liaison efforts with the Committee on Shells and Ligaments

- Mr. J. C. Thompson, a veteran expert in the field, for his valuable comments from an independent consultant's viewpoint.

The members of the Committee on Dynamic Analysis also provided significant contributions by direct interaction with the project directors during numerous progress reports provided at regular Committee meetings and the review of draft reports for each phase of the research.

Although specific acknowledgements to others that assisted in the research are given in Section 9.0 of this Bulletin, it is appropriate to note the cost support provided by Texaco, El Dorado Plant in constructing the tanks utilizing Ace Construction.

The project was jointly funded by PVRC and API with the majority of the funds provided by API. PVRC thanks API for the support of this extremely important project which not only provides an analytical and experimental bases for the current API 650 design rules for frangible roof joints, but also provides the technical bases for potential improvements to the current rules.

Jerry L. Bitner
Committee on Dynamic Analysis and Testing
Pressure Vessel Research Council

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Evaluation of Design Criteria for Storage Tanks With Frangible Roof Joints

Dr. Daniel Swenson, Dr. Don Fenton, Dr. Zhi Lu, Asif Ghori, and Joe Baalman

1.0 Introduction

API 650 (API, 1993) provides design criteria for fluid storage tanks with frangible roof joints. The rules are intended for fluid storage tanks used to store flammable liquids which could be subjected to sudden over-pressurization. Over-pressurization can occur due to the ignition of flammable vapors and can exceed the capability of the pressure relief vents specified in storage tank design.

Sudden over-pressurization can lead to the catastrophic loss of tank integrity. One undesirable mode of failure is the loss of the shell-to-bottom joint, which results in loss of containment and spillage of the contents. One such example is shown in Figure 1.1 for a tank not designed to API 650 rules (Gugan, 1978). A similar failure occurred at the Monsanto Company (Morgenegg, 1978), when lightning ignited the vapor above the liquid surface of a 20 foot diameter by 20 foot tall tank, resulting in a tear of the shell-to-bottom joint. To prevent such a failure, the rules in API 650 are intended to ensure the frangible roof-to-shell joint fails before failure occurs in the tank shell or the shell-to-bottom joint. Failure of the frangible roof-to-shell joint provides a large venting area and reduces the pressure in the tank.

This report describes research that evaluated the ability of the present design criteria to ensure the desired frangible joint behavior. Particular questions include:

- Evaluation of the area inequality as a method to predict the buckling response of the compression ring.
- Effect of roof slope, tank diameter, and weld size on the frangible joint.
- Effect of the relative strength of the roof-to-shell joint compared to the shell-to-bottom joint.

All objectives of the research have been completed. The research was performed in three phases.

Phase I (Swenson, et al., 1992) of this project included:

- Initial calculations of pressure loads due to deflagration.

- Linear structural analyses of storage tanks.
- Static testing of joints and scale models.

Phase II (Swenson, et al., 1993) tasks included:

- Completing development of the capability to predict pressure loads due to deflagration. These loads were used as input to the structural analyses of tanks and in the design of scale model tests.
- Performing simplified analysis of tanks to evaluate yielding of the compression ring and the appropriate W_h and W_c values for calculating compression area.
- Dynamically testing scale models (the major focus of the Phase II effort). Two model tanks (7.5 feet diameter and 8 feet high) were tested under deflagration loads using a combustible gas mixture ignited in the tanks. This provided data on pressure loads and the response of the tanks.

Phase III of the project synthesized what had been learned in the previous phases into a computer program for the design and analysis of storage tanks. Tasks included:

- Developing a PC based computer program, API-Tank, to design and analyze storage tanks with frangible roof joints. API-Tank includes design, analysis, and post-processing modules. The user can input tank parameters and API-Tank will develop a design following API 650 guidelines. The user can then analyze the stresses and displacements in the tank at pressures corresponding to selected tank failure modes. The pressures at each failure mode can be used to help evaluate safety of the tank due to overload pressures.
- Using API-Tank, develop failure pressure charts for a range of tank sizes.

This report is a description of the entire research project. Additional information is provided on a video of the testing that has been submitted to the PVRC and API.



Fig. 1.1—An undesired tank failure (Gugan, 1978)

2.0 Background Information

2.1 Description of Typical Tank Design

(Note: Drawings used in the following description come from more than one source, and so do not represent any particular tank. However, they do represent typical construction.)

The storage tanks addressed in this research include the typical tanks seen in oil refineries and chemical plants. The tanks are cylindrical with conical roofs. The roof slope is quite small, limited by API 650 to a rise not greater than 2 inches in 12 inches. The tanks typically have a flat floor that rests on a sand foundation, with a hard concrete ringwall at the periphery of the bottom. The tanks are manufactured in the field using fillet or seam welded plates. Overlapping joints (with fillet welds) are typical on the roof and floor. Butt welds are used for the shell. Diameters of the tanks of interest range from 10 to over 200 feet, with heights of 10 to 60 feet. The material used in construction is steel, with yield strengths ranging from 30,000 to 60,000 psi.

Figure 2.1 shows a photograph of a typical tank. The overall construction of a typical 25 feet diameter tank is shown in Figure 2.2, and includes all the features described above. The roof is constructed using overlapping flat plates, with fillet welds at the joints. The top angle is butt-welded to the top of the shell. The roof is then fillet-welded to the top angle around the entire periphery. The construction of larger tanks is similar.

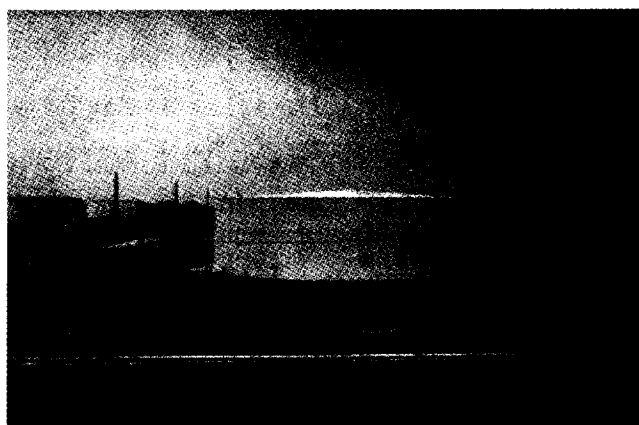


Fig. 2.1—Photograph of typical tank

The relatively flat roof is supported by interior rafters (Fig. 2.3). As can be seen, the rafters extend from a central column to the periphery of the tank. The rafters are welded to the column and bolted to the tank wall. The roof is not attached to the rafters, but rests on them. When walking on the roof, one can feel the deformation of the roof between the rafters. Also, there is typically some noticeable “waviness” in the roof. On larger tanks, there can be additional columns and intermediate supports for the rafters.

2.2 API 650 Design Rules for Frangible Roofs

The design rules given in API 650 cover vertical cylindrical aboveground, welded steel storage tanks for small internal pressures. Several sections of the code address the design of a frangible roof joint for supported cone roofs. Summary descriptions of these sections are given below, followed by a discussion of the apparent intent of the rules, and some difficulties with the present rules. The present description is based on wording in the Ninth Edition of API Standard 650, July 1993 (API, 1993).

Given below are descriptions of the sections of API 650 that impact the design of frangible roof joints.

Section 3.1.5.9 (Item e): This section specifies minimum sizes for the top angles. For tanks with diameters of 35 feet and smaller, the minimum size is 2 by 2 by $\frac{3}{16}$ inches; for tanks with diameters of 35 to 60 feet, 2 by 2 by $\frac{1}{4}$ inches; for larger tanks 3 by 3 by $\frac{3}{8}$ inches.

Section 3.10.2.2: Roof plates shall have a minimum nominal thickness of $\frac{3}{16}$ inch (permissible ordering basis—7.65 pounds per square foot of plate, 0.180-inch plate, or 7-gauge sheet).

Section 3.10.2.3: Roof plates of supported cone roofs shall not be attached to the supporting members.

Section 3.10.2.5: Basic criteria for frangible roof joints are given in this section. Section 3.10.2.5 states that for a frangible joint, the top angle may be smaller than that specified in Section 3.1.5.9.

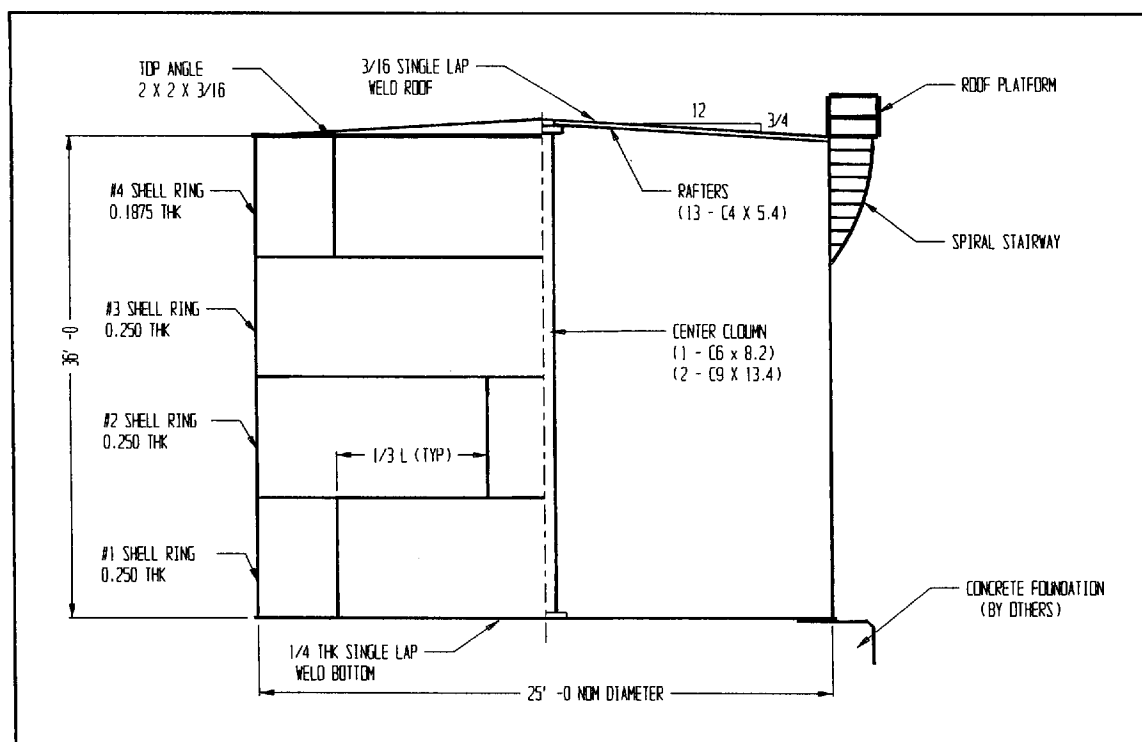


Fig. 2.2—Overall layout of typical tank

Section 3.10.2.5.1: The continuous fillet weld between the roof plates and the top angle can not exceed $\frac{3}{16}$ inch. The slope of the roof at the top-angle attachment can not exceed 2 inches in 12 inches.

Section 3.10.2.5.3: The cross-sectional area at the roof-to-shell junction should not exceed:

$$A = \frac{0.153 W}{30,800 \tan \theta} \quad (2.2.1)$$

where,

A = Area resisting the compressive force, in square inches,

W = Total weight of the shell and any framing (but not roof plates) supported by the shell and roof, in pounds,

θ = Angle between the roof and a horizontal plane at the roof-to-shell junction, in degrees.

Section 3.10.4.1: Roof plates shall be welded on the top side with continuous full-fillet welds on all seams. The size of the roof-to-top angle weld shall be $\frac{3}{16}$ inch or smaller if so specified on the purchase order.

Section 3.10.4.2: The slope of the roof shall be $\frac{3}{4}$ inch in 12 inches, or greater if specified by purchaser. (Note: Section 3.10.2.5 limits maximum slope to 2 inches in 12 inches.)

Section 3.10.4.5: Rafters shall be spaced so that at the outer edges their center shall be not more than 6.28 feet apart. Spacing on inner rings shall not be less than $5\frac{1}{2}$ feet.

Appendix F.2.2.1: When the construction of the compression ring conforms to but does not exceed the minimum requirements of Item e of 3.1.5.9, 3.10.2, and 3.10.4, the frangible characteristic of the ring is retained, and additional emergency venting devices are not required. (Note: The Appendix F requirement to conform to 3.1.5.9 (Item e) is not consistent with Section 10.3.2.5 which allows compression ring area to be reduced for frangible joints.)

Appendix F.4.1: Maximum design pressure is given by:

$$P = \frac{(30,800)(A)(\tan \theta)}{D^2} + 8t_h \quad (2.2.2)$$

where,

P = internal design pressure, in inches of water,

D = tank diameter, in feet,

t_h = nominal roof thickness, in inches.

Appendix F.4.2: The maximum design pressure, limited by uplift at the base of the shell, shall not exceed:

$$P_{\max} = \frac{0.245W}{D^2} + 8t_h - \frac{0.735M}{D^3} \quad (2.2.3)$$

where, M = wind moment, in foot-pounds. Note: the wind moment term was not in the Eighth Edition of API 650 (API, 1988).

Appendix F.5.1: Where the maximum design pressure has already been established (not higher than

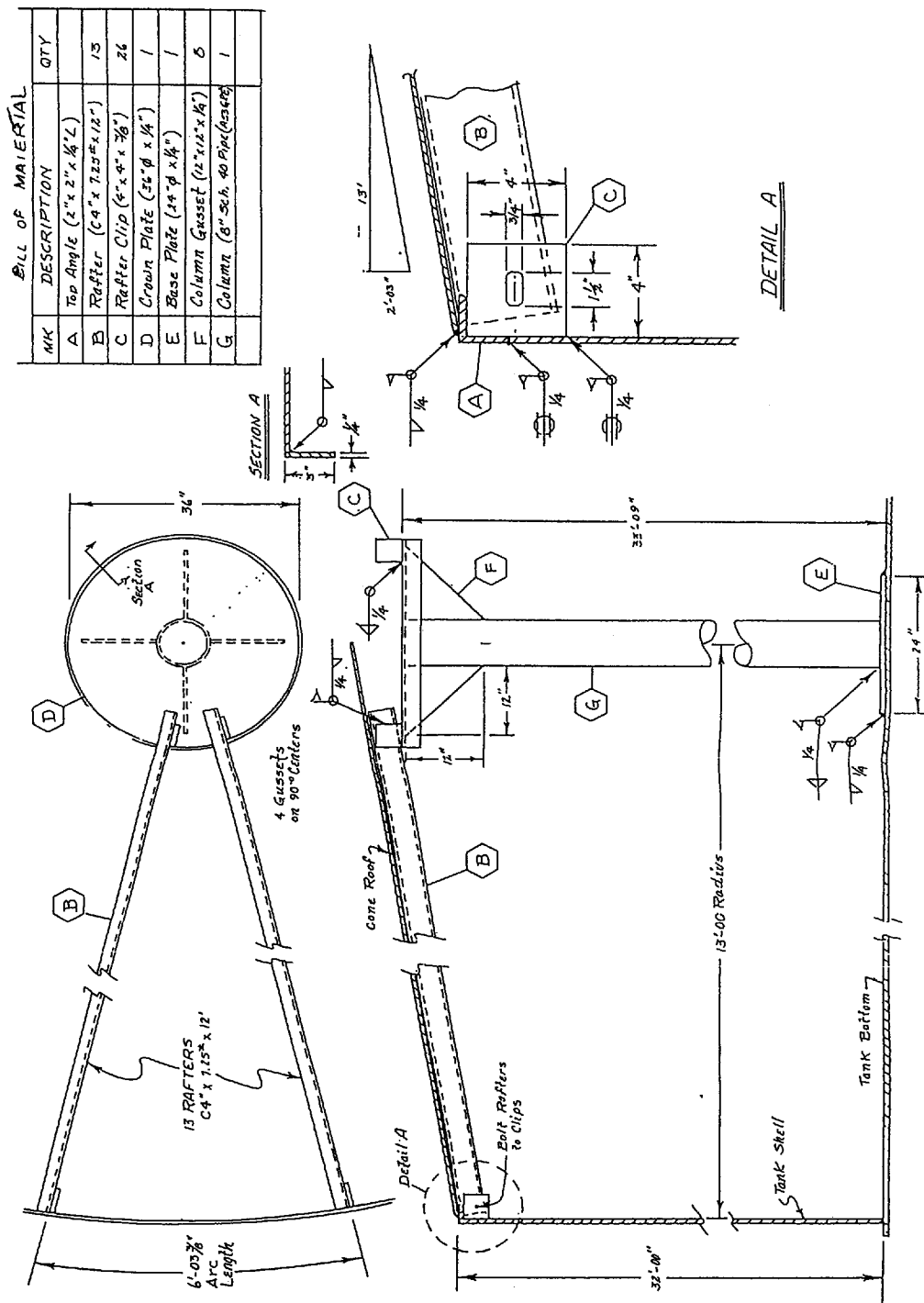


Fig. 2.3—Roof detail of a typical tank

that permitted by F.4.2 or F.4.3), the total required compression area at the roof-to-shell junction may be calculated from the following:

$$A = \frac{D^2(P - 8t_h)}{30,800(\tan \theta)} \quad (2.2.4)$$

Appendix F.5.6: Failure can be expected to occur when the stress in the compression ring area reaches the yield point, as given by:

$$P_f = 1.6P - 4.8t_h \quad (2.2.5)$$

where P_f = calculated failure pressure, in inches of water.

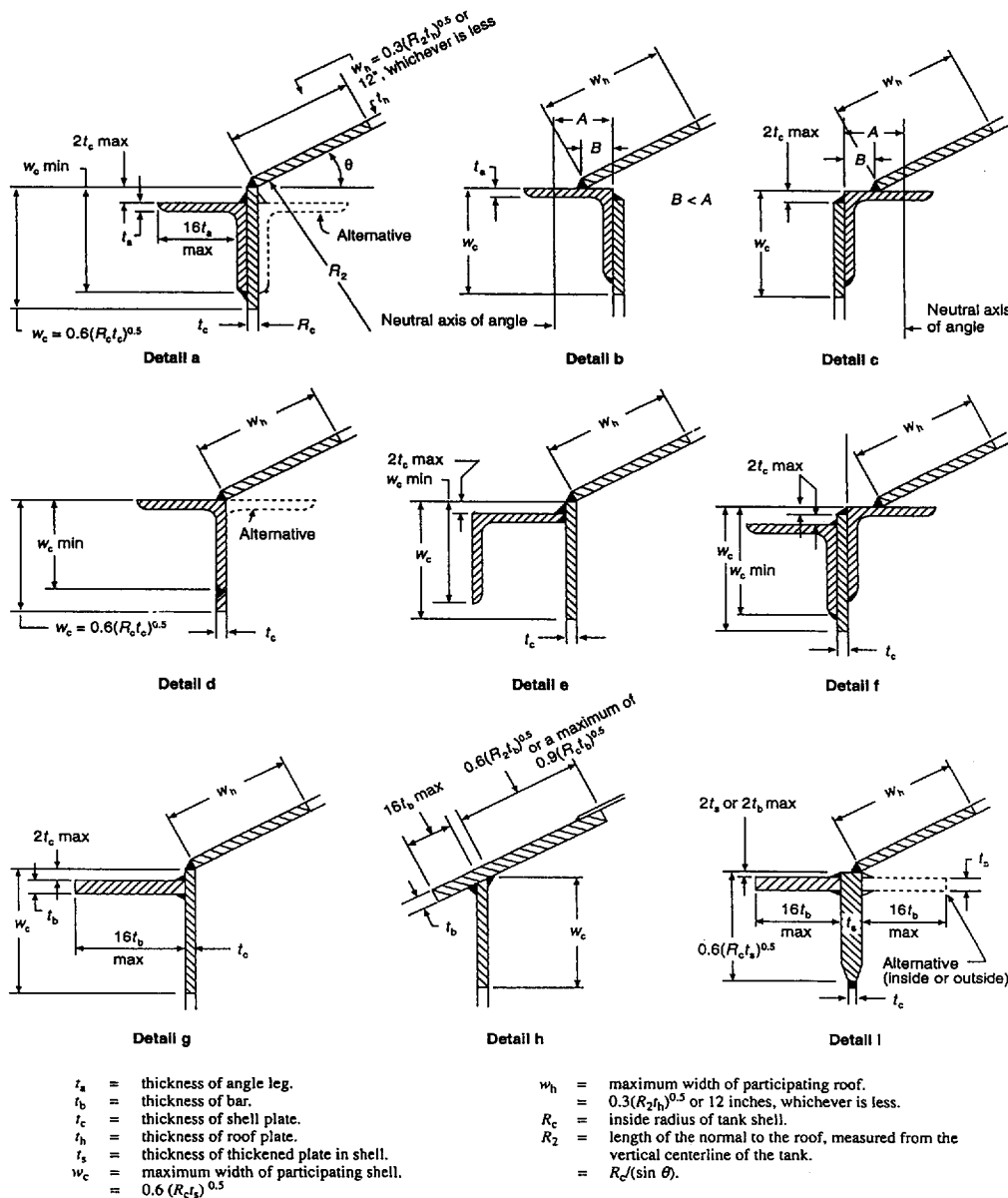
In addition to the rules given above, only certain configurations of joint design are allowed. These are

given in Appendix F and are shown in Figure 2.4. As the roof lifts due to internal pressure, it applies an inward radial force on the compression area. The compression area calculations in Figure 2.4 allow the designer to calculate the area available to resist this compressive force.

2.3 Derivation of API 650 Rules

In the above section, the rules that govern frangible joint design were listed as facts. To truly understand the rules, it is necessary to understand the intent and derivation of the rules. In this section, we re-derived the API 650 rules to make clear the assumptions on which they are based.

The intent is stated in Section 3.10.2.5.1, where, if



Note: All dimensions and thicknesses shown are in inches.

Fig. 2.4—Permissible details of compression rings (from API 650)

Frangible Roof Joints

the frangible joint rules are met "... the joint may be considered to be frangible and, in case of excessive internal pressure, will fail before failure occurs in the tank shell joints or the shell-to-bottom joint. Failure of the roof-to-shell joint is usually initiated by buckling of the top angle and followed by tearing of the $\frac{3}{16}$ inch continuous weld of the periphery of the roof plates." Although it is true that buckling initiates the actual failure of the joint, buckling follows yielding of the compression ring, and it is the yielding of the compression ring that is used as the basis for API 650 rules.

Figure 2.5 shows the equilibrium forces (per unit circumference) on the compression ring. The weight of the plates, the pressure acting on this region, and the bending moments are assumed small and neglected. The forces consist of a downward force, caused by stresses in the shell, and an inward force, caused by stresses in the roof. The roof force can be separated into vertical and radial components.

Equilibrium in the vertical direction gives:

$$V = F_{\text{shell}} = \frac{RP}{2} \quad (2.3.1)$$

$$H = \frac{V}{\tan \theta} = \frac{RP}{2 \tan \theta} \quad (2.3.2)$$

where,

V = Vertical component of roof force per unit circumference,

F_{shell} = Force in shell per unit circumference,

H = Horizontal component of roof force per unit circumference,

P = Internal pressure in the tank,

R = radius of the tank.

Equilibrium of the compression force in the ring and the horizontal force, as shown in Figure 2.6, gives:

$$2RH = 2F_{\text{comp}} \quad (2.3.3)$$

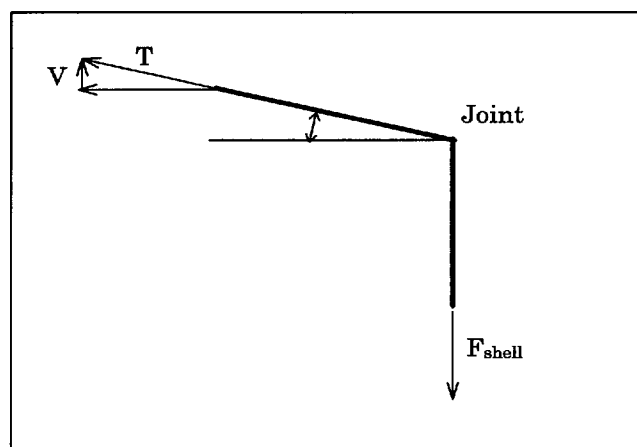


Fig. 2.5—Side view of equilibrium forces on compression ring

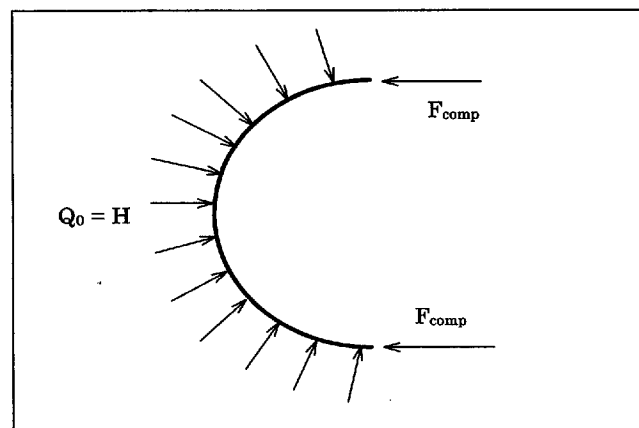


Fig. 2.6—Top view of equilibrium forces on compression ring

The force in the compression ring F_{comp} is equal to the compression stress σ_{comp} times the compression area A :

$$F_{\text{comp}} = \sigma_{\text{comp}} A. \quad (2.3.4)$$

Substituting equations (2.3.2) and (2.3.3) into (2.3.4), gives:

$$A = \frac{RH}{\sigma_{\text{comp}}} = \frac{R^2 P}{2 \sigma_{\text{comp}} \tan \theta}. \quad (2.3.5)$$

Assuming yielding occurs when the bottom is about to uplift, then equation (2.3.5) can be written as:

$$A = \frac{W}{2 \pi \sigma_{\text{yield}} \tan \theta} \quad (2.3.6)$$

where W is the total weight of the shell.

Equation (2.3.6) is the basis of the area inequality rule. From this equation follow the remaining equations given in API 650 for frangible roofs.

To avoid uplift of the bottom, considering a factor of safety $n = 1.6$ and neglecting wind moment, the maximum design pressure and the weight of shell are related by (Note: in API 650, the unit of P is inches of water, and the density of the roof plates is about 8 times of that of water):

$$(P - 8 \rho_{\text{water}} t_h) \frac{\pi D^2}{4} = \frac{W}{n}. \quad (2.3.7)$$

Substituting W in equations (2.3.6) into (2.3.7) we have:

$$P = \frac{8 A \sigma_{\text{yield}} \tan \theta}{n D^2} + 8 \rho_{\text{water}} t_h. \quad (2.3.8)$$

Assume a compressive yield stress of 32,000 psi (as described in Appendix F, Section F.6), substitute into equation (2.3.8), and convert units by 1 inch of water = 0.03606 psi, gives the equation for calculating maximum design pressure as stated in API 650 F.4.1

(neglecting the wind moment term):

$$P = \frac{30,800A \tan \theta}{D^2} + 8t_h \quad (2.3.9)$$

If the design pressure (P) has already been established, equation (2.3.9) can be inverted to obtain the total required area expression as in API 650 F.5.1:

$$A = \frac{(P - 8t_h)D^2}{30,800 \tan \theta} \quad (2.3.10)$$

Modifying (2.3.9) by equating the maximum uplift force by pressure $P_{\max}$ to the weight of the shell gives:

$$0.25\pi D^2(P_{\max} - 8\rho_{\text{water}}t_h) = W \quad (2.3.11)$$

Rearranging equation (2.3.11) and converting units to inches of water gives the equation in API F.4.2 less the wind moment term which is new in the API 650 ninth edition:

$$P_{\max} = \frac{0.245W}{D^2} + 8t_h \quad (2.3.12)$$

If the area A satisfies equation (2.3.6), then the uplift pressure $P_{\max}$ is also the calculated failure pressure, P_f . From equation (2.3.7), the relation between P_f and the design pressure P is:

$$0.25\pi D^2(P_f - 8\rho_{\text{water}}t_h) = W = 0.25\pi D^2n(P - 8\rho_{\text{water}}t_h), \quad (2.3.13)$$

or, in terms of inches of water:

$$P_f = 1.6P - 4.8t_h \quad (2.3.14)$$

This is identical to equation API 650 F.6.

The above derivations are based on using static equilibrium and the original geometry to calculate the inward force on the compression ring due to roof lifting. The resulting equations demonstrate that the basis of the inequality rule is to ensure that yielding of the compression ring will occur before uplift of the bottom. Yielding and subsequent loss of stiffness are expected to cause buckling of the compression ring, followed by gross deformation of the roof, shell, and associated failure of the roof-to-shell weld. As discussed in Section 5.2 and 5.3, our testing confirms that this is the mode in which failure occurs.

2.4 Derivation of W_c and W_h in API 650 Design Rules

Section 2.3 demonstrated that the objective of the inequality rule is to ensure that yielding of the compression ring will occur before uplift of the bottom. The area of the compression ring is determined using W_c , the maximum width of participating shell, and W_h , the maximum width of the participating roof. To verify the validity of the rules, it is necessary to know the basis for the derivations of W_c and W_h .

As shown below, W_c and W_h can be derived from linear approximations of cylindrical and conical shells

loaded by a shearing force at the edge. Internal pressure, bending moments, and the effect of large displacement are all neglected in the analysis.

2.4.1 Derivation of W_c . For a long cylindrical shell submitted to the action of a shearing force Q_0 , as shown in Figure 2.7, the governing differential equation is:

$$\frac{d^4w}{dx^4} + 4\beta^4w = \frac{P}{D} \quad (2.4.1)$$

where,

$$\beta^4 = \frac{3(1 - \nu^2)}{R^2h^2}$$

w = lateral displacement,
 R = radius of the shell,
 h = thickness of the shell,
 P = internal pressure,
 D = flexural rigidity.

The general solution can be written as (Timoshenko, and Woinowsky-Krieger, 1959):

$$w = e^{-\beta x}(c_3 \cos \beta x + c_4 \sin \beta x). \quad (2.4.2)$$

Using the boundary condition of zero moment at the end of the shell:

$$(M_x)_{x=0} = -D \left(\frac{d^2w}{dx^2} \right)_{x=0} = 0$$

$$\frac{d^2w}{dx^2} = \beta e^{-\beta x}(c_3 \cos \beta x + c_4 \sin \beta x)$$

$$- 2\beta^2 e^{-\beta x}(-c_3 \sin \beta x + c_4 \cos \beta x)$$

$$+ \beta^2 e^{-\beta x}(-c_3 \cos \beta x - c_4 \sin \beta x)$$

$$\left(\frac{d^2w}{dx^2} \right)_{x=0} = 0 = -2\beta^2 e^{-\beta x} c_4$$

$$c_4 = 0. \quad (2.4.3)$$

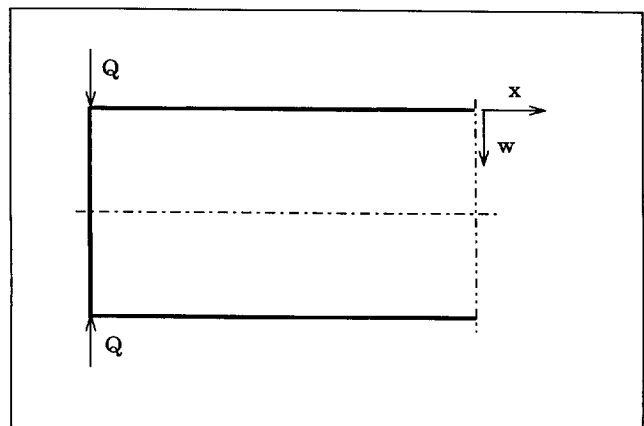


Fig. 2.7—A long shell with edge shear loading

Thus:

$$w = e^{-\beta x} c_3 \cos \beta x \quad (2.4.4)$$

$$\begin{aligned} \frac{d^2 w}{dx^2} &= \beta^2 e^{-\beta x} c_3 \cos \beta x + 2\beta^2 e^{-\beta x} c_3 \sin \beta x \\ &- \beta^2 e^{-\beta x} c_3 \cos \beta x = 2 c_3 \beta^2 e^{-\beta x} \sin \beta x \\ \frac{d^3 w}{dx^3} &= 2 c_3 \beta^3 e^{-\beta x} (\cos \beta x - \sin \beta x). \end{aligned} \quad (2.4.5)$$

Under the shearing force Q_0 at the edge where $x = 0$:

$$\begin{aligned} (Q_0)_{x=0} &= -Q_0 \\ &= -D \left(\frac{d^3 w}{dx^3} \right)_{x=0} \\ &= -2D\beta^3 c_3 \\ c_3 &= \frac{Q_0}{2\beta^3 D}. \end{aligned} \quad (2.4.6)$$

The solution for the given problem is then:

$$w = \frac{Q_0}{2\beta^3 D} e^{-\beta x} \cos \beta x. \quad (2.4.7)$$

The circumferential force per unit length of meridian (compression) is:

$$\begin{aligned} N_\theta &= -\frac{E h w}{R} \\ &= -\frac{E h Q_0}{2R\beta^3 D} e^{-\beta x} \cos \beta x \end{aligned} \quad (2.4.8)$$

where E is the Young's modulus. The circumferential stress can be written as:

$$\begin{aligned} \sigma_\theta &= \frac{N_\theta}{h} \\ &= -\frac{E Q_0}{2R\beta^3 D} e^{-\beta x} \cos \beta x \end{aligned} \quad (2.4.9)$$

The maximum value of compression stress is located at the edge of the shell where $x = 0$, and is given by:

$$\sigma_{\theta \max} = -\frac{E Q_0}{2R\beta^3 D} \quad (2.4.10)$$

Then:

$$\sigma_\theta = \sigma_{\theta \max} e^{-\beta x} \cos \beta x \quad (2.4.11)$$

A plot of normalized force as a function of βx is plotted in Figure 2.8. If one considers W_c to be the region where the circumferential stress is equal or greater than $\frac{1}{3}$ of the maximum value, its length can be evaluated:

$$\begin{aligned} e^{-\beta W_c} \cos \beta W_c &= \frac{1}{3} \\ \beta W_c &= 0.76865 \end{aligned} \quad (2.4.12)$$

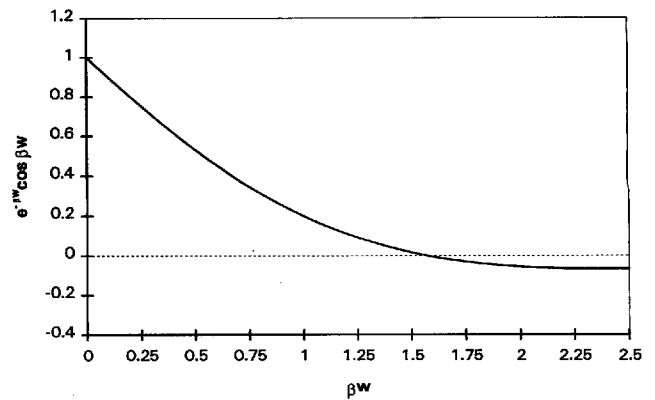


Fig. 2.8—Change of circumferential force along meridian

Assuming Poisson's ratio = 0.3 gives:

$$\begin{aligned} W_c &= \frac{0.76865}{\beta} \\ &= \frac{0.76865}{[3(1 - 0.3^2)]^{1/4}} \sqrt{R h} \\ &= 0.59798 \sqrt{R h} \\ &\approx 0.6 \sqrt{R h} \end{aligned} \quad (2.4.13)$$

This is the formula given by API 650. Thus, the value 0.6 in API 650 arises from considering the shear force acting on a shell and finding the distance for the resulting circumferential stress to drop to $\frac{1}{3}$ of the maximum value.

2.4.2 Derivation of W_h . Consider a complete cone shown in Figure 2.9. Define:

$$\begin{aligned} m^4 &= 12(1 - \nu^2) \\ \lambda^4 &= \frac{m^4}{t_h} \tan^2 \alpha \\ \xi &= 2\lambda \sqrt{y} \end{aligned}$$

where t_h is the thickness of the cone and ν is Poisson's ratio.

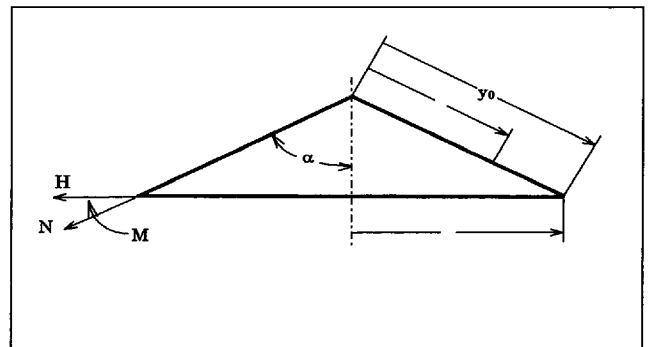


Fig. 2.9—A complete cone loaded by edge moment and forces

The linear governing differential equation for the cone can be written as (Timoshenko and Woinowsky-Krieger, 1959):

$$y \frac{d^2(yQ_y)}{dy^2} + \frac{d(yQ_y)}{dy} - Q_y \pm i\lambda^2 y Q_y = 0 \quad (2.4.14)$$

where the Q_y is the shearing force per unit circumference. The solutions in terms of Kelvin's functions have the form:

$$H = -\frac{\csc\alpha}{y} (c_1 \text{ber}_2 \xi + c_2 \text{bei}_2 \xi) \quad (2.4.15)$$

$$M_y = \frac{h}{2m^2 y} [c_1 (\text{bei}_2' \xi + 2\nu \text{bei}_2 \xi) - c_2 (\xi \text{ber}_2' \xi + 2\nu \text{ber}_2 \xi)] \quad (2.4.16)$$

$$N_\theta = \frac{1}{2} (c_1 \text{ber}_2' \xi + c_2 \text{bei}_2' \xi) \quad (2.4.17)$$

where ber_2 and bei_2 are second order Kelvin's functions and the prime denotes the derivative with respect to ξ . For a cone acted by unit radial stress resultant only, the boundary conditions are:

$$(H)_{\xi=\xi_0} = -1; (M_y)_{\xi=\xi_0} = 0 \quad (2.4.18)$$

Substitute (2.5.18) into (2.5.16):

$$c_1 = \frac{\xi_0 \text{ber}_2' \xi_0 + 2\nu \text{ber}_2 \xi_0}{\xi_0 \text{bei}_2' \xi_0 + 2\nu \text{bei}_2 \xi_0} c_2 \quad (2.4.19)$$

Substituting (2.4.18) and (2.4.19) into (2.4.15), the constants can be determined as:

$$c_1 = \frac{y(\xi_0 \text{ber}_2' \xi_0 + 2\nu \text{ber}_2 \xi_0)}{\csc\alpha [(\xi_0 \text{ber}_2' \xi_0 + 2\nu \text{ber}_2 \xi_0) \text{ber}_2 \xi_0 + (\xi_0 \text{bei}_2' \xi_0 + 2\nu \text{bei}_2 \xi_0) \text{bei}_2 \xi_0]}$$

$$c_2 = \frac{y(\xi_0 \text{bei}_2' \xi_0 + 2\nu \text{bei}_2 \xi_0)}{\csc\alpha [(\xi_0 \text{ber}_2' \xi_0 + 2\nu \text{ber}_2 \xi_0) \text{ber}_2 \xi_0 + (\xi_0 \text{bei}_2' \xi_0 + 2\nu \text{bei}_2 \xi_0) \text{bei}_2 \xi_0]} \quad (2.4.20)$$

Using an asymptotic expansion, for a real, positive and large x , the Kelvin's functions of zero order can be written as:

$$\text{ber}(x) = \frac{e^{\alpha_1}}{\sqrt{2\pi x}} \cos \beta_1$$

$$\text{bei}(x) = \frac{e^{\alpha_1}}{\sqrt{2\pi x}} \sin \beta_1 \quad (2.4.21)$$

where,

$$\alpha_1 \sim \frac{x}{\sqrt{2}} + \frac{1}{8\sqrt{2}x} - \frac{25}{384\sqrt{2}x^3} + \dots$$

$$\beta_1 \sim \frac{x}{\sqrt{2}} - \frac{\pi}{8} - \frac{1}{8\sqrt{2}x} - \frac{1}{16x^2} - \frac{25}{384\sqrt{2}x^3} - \dots$$

Assume x is large enough and neglecting $0(1/x)$ and all higher order terms, gives the approximation:

$$\text{ber}(x) \sim \frac{e^{x/\sqrt{2}}}{\sqrt{2\pi x}} \cos \left(\frac{x}{\sqrt{2}} - \frac{\pi}{8} \right)$$

$$\text{bei}(x) \sim \frac{e^{x/\sqrt{2}}}{\sqrt{2\pi x}} \sin \left(\frac{x}{\sqrt{2}} - \frac{\pi}{8} \right)$$

The second and zero order Kelvin's functions have the following relations:

$$\text{ber}_2(x) = -\text{ber}(x) + \frac{2}{x} \text{bei}'(x)$$

$$\text{bei}_2(x) = -\text{bei}(x) + \frac{2}{x} \text{ber}'(x) \quad (2.4.22)$$

Neglecting all the $0(1/x)$ terms, the second order Kelvin's functions and their derivatives are:

$$\text{ber}_2(x) \approx -\text{ber}(x)$$

$$\approx -\frac{e^{x/\sqrt{2}}}{\sqrt{2\pi x}} \cos \left(\frac{x}{\sqrt{2}} - \frac{\pi}{8} \right)$$

$$\text{ber}_2'(x) \approx \frac{e^{x/\sqrt{2}}}{\sqrt{2\pi x}} \left[-\cos \left(\frac{x}{\sqrt{2}} + \frac{\pi}{8} \right) + \frac{1}{2x} \cos \left(\frac{x}{\sqrt{2}} - \frac{\pi}{8} \right) \right]$$

$$\approx -\frac{e^{x/\sqrt{2}}}{\sqrt{2\pi x}} \cos \left(\frac{x}{\sqrt{2}} + \frac{\pi}{8} \right) \quad (2.4.23)$$

$$\text{bei}_2(x) \approx -\text{bei}(x)$$

$$\approx -\frac{e^{x/\sqrt{2}}}{\sqrt{2\pi x}} \sin \left(\frac{x}{\sqrt{2}} - \frac{\pi}{8} \right) \quad (2.4.24)$$

$$\text{bei}_2'(x) \approx \frac{e^{x/\sqrt{2}}}{\sqrt{2\pi x}} \left[-\sin \left(\frac{x}{\sqrt{2}} - \frac{\pi}{8} \right) + \frac{1}{2x} \sin \left(\frac{x}{\sqrt{2}} - \frac{\pi}{8} \right) \right]$$

$$\approx -\frac{e^{x/\sqrt{2}}}{\sqrt{2\pi x}} \sin \left(\frac{x}{\sqrt{2}} + \frac{\pi}{8} \right) \quad (2.4.25)$$

Substituting (2.4.22) through (2.4.25) into (2.4.20), the coefficients are:

$$c_1 \approx \frac{y\sqrt{2\pi\xi_0}e^{-\xi_0/\sqrt{2}} \cdot \left[\xi_0 \cos \left(\frac{\xi_0}{\sqrt{2}} + \frac{\pi}{8} \right) + 2\nu \cos \left(\frac{\xi_0}{\sqrt{2}} - \frac{\pi}{8} \right) \right]}{\csc\alpha \left[\xi_0 \cos \left(\frac{\pi}{4} \right) + 2\nu \right]}$$

$$c_2 \approx \frac{y\sqrt{2\pi\xi_0}e^{-\xi_0/\sqrt{2}} \cdot \left[\xi_0 \sin \left(\frac{\xi_0}{\sqrt{2}} + \frac{\pi}{8} \right) + 2\nu \sin \left(\frac{\xi_0}{\sqrt{2}} - \frac{\pi}{8} \right) \right]}{\csc\alpha \left[\xi_0 \cos \left(\frac{\pi}{4} \right) + 2\nu \right]} \quad (2.4.26)$$

Substituting c_1 and c_2 into (2.4.17), the circumferential force per unit meridional length is:

$$N_\theta \approx - \frac{\sqrt{\xi\xi_0}}{\csc\alpha(\sqrt{2}\xi_0 + 2\sqrt{2}\nu)} e^{\xi_0 - \xi/\sqrt{2}} \left[\xi_0 \cos\left(\frac{\xi_0 - \xi}{\sqrt{2}}\right) + 2\nu \cos\left(\frac{\xi_0 - \xi}{\sqrt{2}} + \frac{\pi}{4}\right) \right] \quad (2.4.27)$$

Since in our case $\xi_0 \gg \nu$ and $(\xi_0 - \xi)/\sqrt{2}$ is small, the ν terms can be neglected with less than 0.5% error. Then (2.4.27) becomes:

$$N_\theta \approx - \frac{\sqrt{\xi\xi_0}}{2\csc\alpha} e^{(\xi_0 - \xi)/\sqrt{2}} \cos\left(\frac{\xi_0 - \xi}{\sqrt{2}}\right) \quad (2.4.28)$$

At the edge of the shell where $\xi = \xi_0$, there exists the maximum compression:

$$(N_\theta)_{\xi=\xi_0} = - \frac{\xi_0}{\sqrt{2}\csc\alpha} \quad (2.4.29)$$

The ratio of force at any place to the edge force is:

$$\begin{aligned} \frac{N_\theta}{(N_\theta)_{\xi=\xi_0}} &= \sqrt{\frac{\xi}{\xi_0}} e^{(\xi_0 - \xi)/\sqrt{2}} \cos\left(\frac{\xi_0 - \xi}{\sqrt{2}}\right) \\ &\approx e^{(\xi_0 - \xi)/\sqrt{2}} \cos\left(\frac{\xi_0 - \xi}{\sqrt{2}}\right) \end{aligned} \quad (2.4.30)$$

Notice (2.4.30) is exactly the same curve as (2.4.11) shown in Figure 2.8.

If one considers W_h the region where the stress is greater or equal to $1/3$ of the edge compression stress, then the above ratio will equal $1/3$. This gives:

$$\begin{aligned} \frac{\xi_0 - \xi}{\sqrt{2}} &= 0.76865 \\ \xi_0 - \xi &= 0.76865\sqrt{2} \\ &= 2\lambda(\sqrt{y_0} - \sqrt{y}) \\ &= 2\lambda\left(\frac{y_0 - y}{\sqrt{y_0} + \sqrt{y}}\right) \\ &\approx \lambda \frac{y_0 - y}{\sqrt{y_0}} \end{aligned} \quad (2.4.31)$$

The width of participating roof, W_h , will be:

$$\begin{aligned} W_h &= y_0 - y \\ &= 0.76865 \frac{\sqrt{2y_0}}{\lambda} \\ &= 1.087 \frac{\sqrt{y_0 t_h \tan \alpha}}{[12(1 - \nu^2)]^{1/4}} \end{aligned} \quad (2.4.32)$$

Taking $\nu = 0.3$, then:

$$\begin{aligned} W_h &= 1.087 \frac{\sqrt{y_0 t_h \tan \alpha}}{1.8178} \\ &= 0.598 \sqrt{\frac{R \sin \alpha}{\sin \alpha \cos \alpha}} t_h \\ &\approx 0.6\sqrt{R_2 t_h} \end{aligned} \quad (2.4.33)$$

In API 650 W_h is half the above value, but in BS 2654 (the corresponding British standard) W_h is exactly as given in (2.4.33). Since α is close to $\pi/2$, W_h is typically much larger than W_c . This is the likely reason that API 650 uses half of the calculated length to the point where the stress drops to $1/3$ of the maximum.

At this point, we understand the origin of the W_h and W_c terms. These values will be compared to finite element calculations in the following sections.

2.5 Summary of Frangible Joint Study by British Researchers

The Tank Committee of the Engineering Equipment and Materials Users Association (including all major oil, chemical and gas companies in Britain) studied frangible roof joints and prepared a report in January 1990 (The Tank Committee, 1990). Their main points will be discussed briefly.

The British noted that the sudden high pressure in a tank may be caused by:

- An explosion inside the tank due to mis-operation.
- Welding operations on the tank, causing an internal explosion.
- A lightning strike on the tank, causing an internal explosion.
- Discharge of static electricity, causing an internal explosion.
- A fire in an adjacent tank.
- Water entering a tank storing a product with a temperature over 100°C.
- Pyrophoric iron sulfide deposits inside the tank.
- Carbonaceous deposits (glowing coke deposits) in bitumen tanks.

The report mentioned that bottom-to-shell failures occurred only in a few cases with small tanks having diameters not exceeding 12.5 meter (41 feet). Most failures occurred with dome roof tanks. A number of failures also occurred in tanks with heavy welds at the roof-to-shell connection.

Assuming ideal material properties, critical vapor pressures were calculated. It was shown that pressures up to 1.5 bar (21 psig) might be reached before failure will occur at either the roof-to-shell or bottom-to-shell connections. Analysis details are unknown, but this result could be valid for tanks with large angle slopes.

Both BS 2654 and API 650 specify a maximum strength value for the roof-to-shell connection acting

as a compression ring. When this maximum strength is exceeded, the compression ring can not be considered a frangible roof joint. The British report stated that, for small tanks, the compression ring exceeds this value in almost all cases.

They concluded that the roof slope of 1:5 specified in BS 2654 is a reasonable compromise for the frangible joint and maximum design internal pressure. They also concluded that dome roofs should not be used. It should be noted that increasing the roof slope increases the failure pressure, see Section 4.0.

The British researchers also found that in many cases the seal weld on existing tanks was made as a full fillet weld. They suggested that double fillet welds (not allowed in API 650) shall not be used and a 3 mm ($\frac{1}{8}$ inch) leg length seal weld should be preferred. We have considered the same factor in the joint tests in Phase I. Tests described in Section 5.0 confirmed that a full fillet weld joint may be stronger than the shell plate. However, from the view point of manufacturing, it is difficult to make a small weld joint.

In the British report, the lap-welded bottom plates under the tank were considered a weak spot for the bottom-to-shell connection. The use of butt-welded annular plates under the tank shell was recommended. It was considered effective to require a minimum plate thickness of 8 mm ($\frac{5}{16}$ inch) for the bottom annular plates and the lowest shell course. The current API 650 requires $\frac{1}{4}$ inch thickness bottom plate (6.4 mm).

In the report, concrete foundation ringwalls with anchor bolts are considered only to prevent overturning by wind and not recommended to compensate for the high internal pressure. API 650 requires anchors for some tanks to increase overturning stability and to reduce the uplift of the bottom and hence the stresses at the bottom-to-shell joint.

3.0 Combustion Analysis

It is possible for the combustible vapor above the liquid surface of a tank to be within its flammability limits. Combustion may be initiated when a tank is struck by lightning or heated by surrounding fires. To evaluate how the tank responds after vapor ignition, the pressure time history inside the tank is required. The objective of this section is to calculate the pressure time history for an arbitrarily sized tank.

The available literature provides information concerning flame temperature (Ferguson, 1986) and burning velocity (Lind and Whitson, 1977). There has been research in the area of unconfined and confined explosions (Lee, 1984; Lind and Whitson, 1977), but none was found that considered the effects of different ignition source locations. The location of the ignition source is important because it determines when the flame front will contact the surfaces of the tank. Because the storage tanks can be of different sizes, with liquid fuel levels varying from full to empty, a general solution procedure is needed.

The calculations for the pressure rise inside the

tank are done numerically. Analytic solutions are not possible because of the chemical equilibrium and variable ignition source location assumptions.

3.1 Combustion Wave Geometry

Deflagration and detonation are the two extreme combustion processes. A deflagration is characterized by a relatively low burning velocity and small pressure rise across the combustion wave. In contrast, detonations are identified by supersonic burning velocities and large pressure increases across the combustion wave. Furthermore, detonations require about two orders higher ignition energy than that required to start a deflagration. Because fuel storage tanks consist of steel shells and are electrically grounded, the possibility of a lightning strike initiating a detonation is considered remote. This analysis assumes that the combustion wave will burn as a radial deflagration, having a smooth spherical shaped flame front from a point source ignition.

The pressure rise in the vapor space inside a storage tank is calculated by evaluating the pressure resulting from combustion in small increments of time. The volume of reactants burned in a given time increment is required to determine the pressure after each time increment. To evaluate this pressure, the speed at which the flame front moves is multiplied by a time increment, giving a distance. From this distance, an incremental volume is calculated. This volume is then burned at constant volume combustion and then allowed to expand until the pressure inside the tank is uniform.

The flame front is assumed to grow in a spherical fashion originating from a point source ignition in the tank. If the ignition flame is not in contact with the tank or the fluid, the fireball is a sphere. Otherwise the fireball will be constrained by the tank or liquid in the tank. Numerical integration is necessary to calculate the volume of the fireball with sufficient accuracy.

Consider a tank of radius R with height of vapor space H as shown in Figure 3.1. The ignition source is located at point O , which is distance e away from center line of the tank shell and distance h below the roof of the tank. At a given instant, the radius of the fireball is r . Using a cylindrical coordinate system with the origin at the ignition source and the z axis up, the cross sectional area of the fireball S at height z can be determined analytically. The total volume of the fireball V can be obtained using the following integration:

$$V = \int_{\max(h-H, -r)}^{\min(h, r)} S dz \quad (3.1.1)$$

When the ignition source is on the centerline of the tank, the value of e is zero and the cross section at any z location will be a circle with radius:

$$\rho = \min(\sqrt{r^2 - z^2}, R) \quad (3.1.2)$$

When e is not zero, the calculation of cross sectional

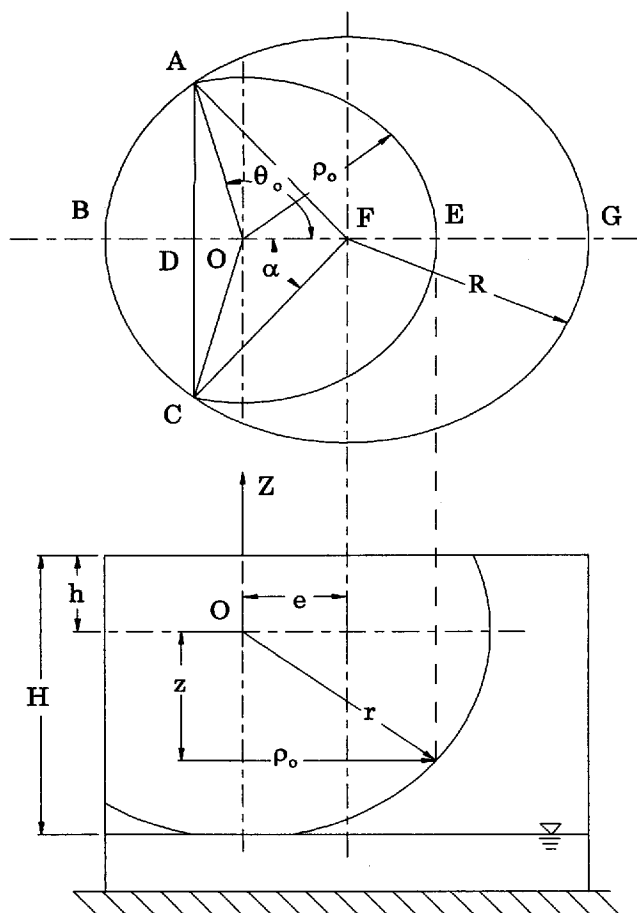


Fig. 3.1—Geometry of the fireball at position z when $(R - e)^2 < r^2 - z^2 < R^2 + e^2$

area can be performed for three cases. When the fireball and the tank shell do not touch each other for the given z coordinate, the cross sectional area is that of a circle with radius ρ , as given in equation (3.1.2). Otherwise, the cross sectional area is the common part of two circles. The fireball in the tank is shown in Figure 3.1. The projection of the radius of the fireball at position z is ρ_0 :

$$\rho_0 = \sqrt{r^2 - z^2} \quad (3.1.3)$$

The tank shell in the given coordinate system can be written as:

$$(\rho \cos \theta - e)^2 + (\rho \sin \theta)^2 = R^2 \quad (3.1.4)$$

Solving for ρ gives:

$$\rho = e \cos \theta \pm \sqrt{e^2 \cos^2 \theta + R^2 - e^2} \quad (3.1.5)$$

Since $\rho > 0$, θ can be obtained from (3.1.5) by taking the positive sign:

$$\theta = \frac{\rho^2 + e^2 - R^2}{2e\rho} \quad (3.1.6)$$

At point A where the surface of the fireball contacts the tank shell, θ_0 can be obtained by substituting (3.1.3) into (3.1.6):

$$\theta_0 = \cos^{-1} \left(\frac{r^2 - z^2 + e^2 - R^2}{2e\sqrt{r^2 - z^2}} \right) \quad (3.1.7)$$

From geometric relations shown in Figure 3.1:

$$\overline{AO} = \sqrt{r^2 - z^2},$$

$$\overline{DO} = -\sqrt{r^2 - z^2} \cos \theta_0 = \frac{R^2 - e^2 - r^2 + z^2}{2e},$$

$$\overline{AD} = \sqrt{r^2 - z^2} \sin \theta_0$$

$$= \sqrt{r^2 - z^2 - \frac{(r^2 - z^2 + e^2 - R^2)^2}{4e^2}},$$

$$\sin \alpha = \frac{\overline{AD}}{\overline{AF}} = \frac{1}{R} \sqrt{r^2 - z^2 - \frac{(r^2 - z^2 + e^2 - R^2)^2}{4e^2}}.$$

The area of AECO is:

$$\begin{aligned} S_{AECO} &= \rho_0^2 \theta_0 \\ &= (r^2 - z^2) \cos^{-1} \left(\frac{R^2 - e^2 - r^2 + z^2}{2e} \right) \end{aligned} \quad (3.1.8)$$

The area of ABCD is:

$$\begin{aligned} S_{ABCD} &= R^2 \alpha - \overline{AD} \sqrt{R^2 - \overline{AD}^2} \\ &= R^2 \sin^{-1} \left[\frac{1}{R} \sqrt{r^2 - z^2 - \frac{(r^2 - z^2 + e^2 - R^2)^2}{4e^2}} \right] \\ &\quad - \sqrt{r^2 - z^2 - \frac{(r^2 - z^2 + e^2 - R^2)^2}{4e^2}} \\ &\quad \times \sqrt{R^2 - r^2 + z^2 + \frac{(r^2 - z^2 + e^2 - R^2)^2}{4e^2}} \end{aligned} \quad (3.1.9)$$

The triangle area AOC is:

$$\begin{aligned} S_{AOC} &= \overline{AD} \times \overline{AO} \\ &= -\rho_0^2 \sin \theta_0 \cos \theta_0 \\ &= \frac{R^2 - e^2 - r^2 + z^2}{2e} \\ &\quad \times \sqrt{r^2 - z^2 - \frac{(r^2 - z^2 + e^2 - R^2)^2}{4e^2}} \end{aligned} \quad (3.1.10)$$

The total cross section area S at height z is:

$$\begin{aligned}
 S &= S_{AECO} + S_{ABCD} + S_{AOC} \\
 &= (r^2 - z^2) \cos^{-1} \left(\frac{R^2 - e^2 - r^2 + z^2}{2e} \right) \\
 &\quad + R^2 \sin^{-1} \left[\frac{1}{R} \sqrt{r^2 - z^2 - \frac{(r^2 - z^2 + e^2 - R^2)^2}{4e^2}} \right] \\
 &\quad - \sqrt{r^2 - z^2 - \frac{(r^2 - z^2 + e^2 - R^2)^2}{4e^2}} \\
 &\quad \times \sqrt{R^2 - r^2 + z^2 + \frac{(r^2 - z^2 + e^2 - R^2)^2}{4e^2}} \\
 &\quad + \frac{R^2 - e^2 - r^2 + z^2}{2e} \\
 &\quad \times \sqrt{r^2 - z^2 - \frac{(r^2 - z^2 + e^2 - R^2)^2}{4e^2}} \quad (3.1.11)
 \end{aligned}$$

Equation (3.1.11) is valid for $\alpha < \pi/2$, that is, the cross section at a z coordinate satisfies the following relations:

$$(R - e)^2 < r^2 - z^2 < R^2 + e^2. \quad (3.1.12)$$

If α is larger than $\pi/2$, the geometry of the cross section is shown in Figure 3.2. Notice that when $\theta_0 < \pi/2$ the area S_{AOC} is negative in value, the total area of the cross section will be:

$$S = S_{OAEC} + S_{AOC} + S_{ABC}. \quad (3.1.13)$$

The angle β in Figure 3.2 can be calculated as:

$$\begin{aligned}
 \beta &= \sin^{-1} \frac{\overline{AD}}{R} \\
 &= \sin^{-1} \left[\frac{1}{R} \sqrt{r^2 - z^2 - \frac{(r^2 - z^2 + e^2 - R^2)^2}{4e^2}} \right] \quad (3.1.14)
 \end{aligned}$$

The area S_{ABC} can be written as:

$$\begin{aligned}
 S_{ABC} &= \pi R^2 - (R^2 \beta - \overline{AD} \sqrt{R^2 - \overline{AD}^2}) \\
 &= \pi R^2 - R^2 \sin^{-1} \\
 &\quad \times \left[\frac{1}{R} \sqrt{r^2 - z^2 - \frac{(r^2 - z^2 + e^2 - R^2)^2}{4e^2}} \right] \\
 &\quad + \sqrt{r^2 - z^2 - \frac{(r^2 - z^2 + e^2 - R^2)^2}{4e^2}} \\
 &\quad \times \sqrt{R^2 - r^2 + z^2 + \frac{(r^2 - z^2 + e^2 - R^2)^2}{4e^2}} \quad (3.1.15)
 \end{aligned}$$

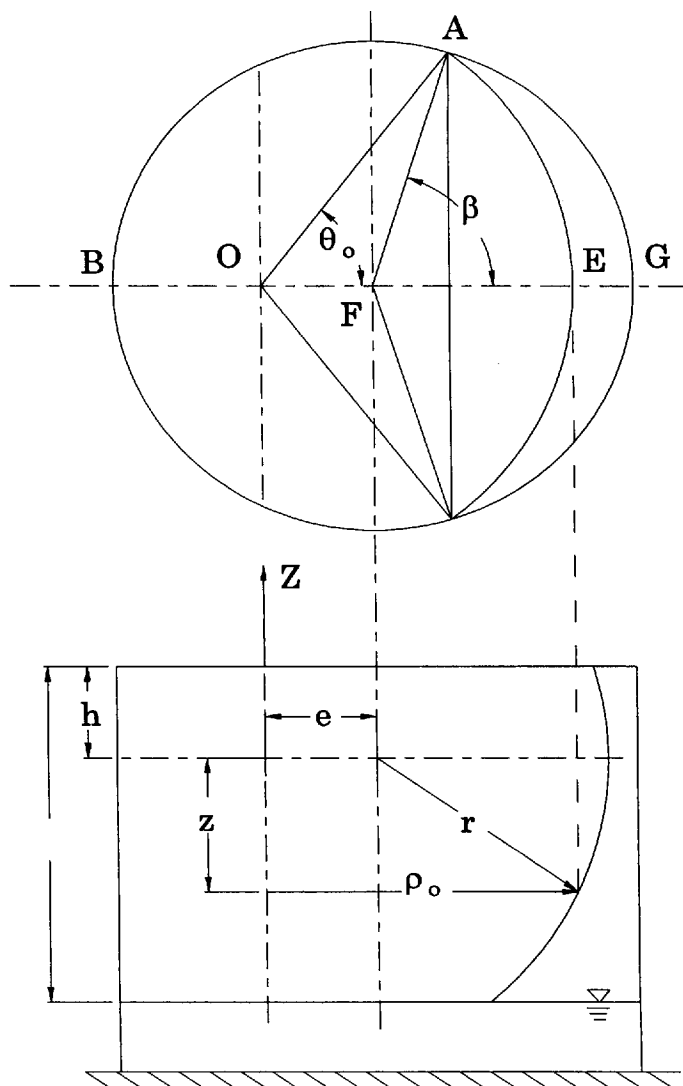


Fig. 3.2—Geometry of the fireball at position z when $R^2 + e^2 < r^2 + z^2 < (R + e)^2$

The total cross sectional area will then be:

$$\begin{aligned}
 S &= (r^2 - z^2) \cos^{-1} \left(\frac{r^2 - z^2 + e^2 - R^2}{2e\sqrt{r^2 - z^2}} \right) \\
 &\quad + \frac{R^2 - e^2 + z^2 - r^2}{2e} \\
 &\quad \times \sqrt{r^2 - z^2 - \frac{(r^2 - z^2 + e^2 - R^2)^2}{4e^2}} \\
 &\quad + \pi R^2 - R^2 \sin^{-1} \\
 &\quad \times \left[\frac{1}{R} \sqrt{r^2 - z^2 - \frac{(r^2 - z^2 + e^2 - R^2)^2}{4e^2}} \right] \\
 &\quad + \sqrt{r^2 - z^2 - \frac{(r^2 - z^2 + e^2 - R^2)^2}{4e^2}} \\
 &\quad \times \sqrt{R^2 - r^2 + z^2 + \frac{(r^2 - z^2 + e^2 - R^2)^2}{4e^2}} \quad (3.1.16)
 \end{aligned}$$

The calculated area S is then integrated with respect to z coordinate using the Runge-Kutta-Gill method. The step length is set to $1/150$ of the z domain, which will give an error equal or less than $0.42 \times 10^{-9} \tau^5$.

3.2 Combustion Calculation

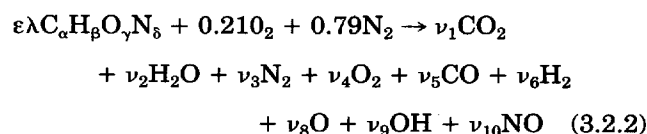
The calculation of the adiabatic constant volume flame temperature is essential to calculating the pressure time history inside a fuel storage tank. The approach taken uses chemical equilibrium and is described in *Internal Combustion Engines* by Colin R. Ferguson (1986). Because the goal is to determine the pressure time history inside the tank, a calculation assuming complete combustion is not sufficient. However, since the purpose of the combustion calculation is to find the tank pressures, and not to determine, in great detail, the amount of each product formed, a detailed chemical kinetics calculation was not necessary.

To determine the pressure-time history inside a storage tank during combustion the tank is assumed to be adiabatic and the structure of the tank rigid. The analysis also assumes no net effect due to radiation heat transfer between the flame front and wall of the tank. The combustion process is continuous, but to calculate the pressure rise numerically, the pressure is calculated at small increments of time. The volume swept out by the flame front is the outer shell of a sphere or partial sphere. The thickness of the shell equals the burning velocity multiplied by a time increment. For the calculation procedure there exist three volumes during combustion: products, which are enclosed by the inner surface of the flame front; the volume swept out by the flame front, which includes the reactants burned during a small increment of time; and the reactants, which is the volume of fuel vapor outside the flame front.

To calculate a pressure rise during a time increment, an adiabatic flame temperature is calculated for the flame front, assuming constant volume combustion using chemical equilibrium. The pressure inside the volume swept out by the flame front, during a single time increment, increases due to the burning of the reactants at constant volume. The volume swept by the flame front is then allowed to expand causing the reactants and products to compress, with no mixing, until the pressure inside the tank is uniform. After the pressure inside the tank is balanced, time is incremented and the process repeats itself until all the reactants are burned.

3.2.1 Flame Temperature Calculation. To determine the adiabatic flame temperature, the vapor is allowed to react with air consisting of 21% oxygen and 79% nitrogen by volume (Ferguson, 1986). A chemical reaction equation describes the fuel, air, and their products. Ferguson suggests equations (3.2.1) and (3.2.2) as an adequate description of the combustion reaction and the products formed:

$$\epsilon = \frac{0.210}{\alpha + 0.25\beta - 0.5\gamma} \quad (3.2.1)$$



where,

- ϵ = molar fuel-air ratio,
- λ = fuel-air ratio: actual fuel-air ratio divided by the stoichiometric fuel-air ratio,
- α = number of Carbon atoms,
- β = number of Hydrogen atoms,
- γ = number of Oxygen atoms,
- δ = number of Nitrogen atoms,
- ν = mole numbers of each respective product.

The fuel-air ratio λ is assumed to be well mixed and can be arbitrarily chosen, or calculated directly from known properties of the fuel and the conditions at which the fuel exists:

$$F = \frac{m_f}{m_a} = \frac{M_a(P_t - P_f)^2}{M_f P_f^2} \quad (3.2.3)$$

$$F_s = \frac{\epsilon(12.01\alpha + 1.008\beta + 16.00\gamma + 14.01\delta)}{28.85} \quad (3.2.4)$$

$$\lambda = \frac{F}{F_s} \quad (3.2.5)$$

where,

- F = actual fuel-air ratio by mass,
- F_s = stoichiometric fuel-air ratio by mass,
- m_f = mass of the fuel vapor (kg),
- m_a = mass of the air (kg),
- M_f = molecular weight of the fuel (kg/kmol),
- M_a = molecular weight of the air (kg/kmol),
- P_t = total pressure of the fuel-air mixture (Pa),
- P_f = vapor pressure of the fuel (Pa).

Following the procedure described by Ferguson for equilibrium combustion, the mole numbers for the products of equation (3.2.2) are calculated using data from Gordon and McBride (1971) to calculate the enthalpy, specific heat at constant pressure, specific volume, and the internal energy of the reactants and products. Two derivatives are calculated numerically,

$$\left(\frac{\partial \ln \nu}{\partial \ln T} \right)_P \text{ and } \left(\frac{\partial \ln \nu}{\partial \ln P} \right)_T.$$

These are used to determine the constant volume adiabatic flame temperature employing the Newton-Raphson iteration in the following procedure:

1. Guess an initial flame temperature and pressure.
2. Calculate the change in internal energy from reactants to products.
3. Calculate the change in specific volume from reactants to products.
4. Solve the following equations to obtain the new guess for the flame temperature and pressure.

Equations (3.2.6) through (3.2.10) are solved during the Newton-Raphson iteration to yield the adiabatic constant volume flame temperature. Define:

$$\text{DET} = \frac{v^2}{2} \left[-\frac{\partial \ln v}{\partial \ln P} \left(\frac{10C_p T}{Pv} + \frac{\partial \ln v}{\partial \ln T} \right) + \frac{\partial \ln v}{\partial \ln T} \left(\frac{\partial \ln v}{\partial \ln T} + \frac{\partial \ln v}{\partial \ln P} \right) \right] \quad (3.2.6)$$

then:

$$\Delta T = \frac{-v}{\text{DET}} \left[\frac{10\Delta U}{P} \frac{\partial \ln v}{\partial \ln P} + \Delta v \left(\frac{\partial \ln v}{\partial \ln T} + \frac{\partial \ln v}{\partial \ln P} \right) \right] \quad (3.2.7)$$

$$\Delta P = \frac{-vP}{T \cdot \text{DET}} \left[\left(\frac{10C_p T}{Pv} - \frac{\partial \ln v}{\partial \ln T} \right) \Delta v - \frac{10\Delta U}{P} \frac{\partial \ln v}{\partial \ln T} \right] \quad (3.2.8)$$

$$T_{\text{new}} = T - \Delta T \quad (3.2.9)$$

$$P_{\text{new}} = P - \Delta P \quad (3.2.10)$$

where,

v = specific volume (cm^3/g),

P = pressure (bars),

T = flame temperature ($^{\circ}\text{K}$),

ΔU = internal energy of the reactants minus that of the products (J/g),

Δv = specific volume of the reactants minus that of the products (cm^3/g),

C_p = specific heat capacity at constant pressure (J/gK).

3.2.2 Burning Velocity. The combustion process can occur at any rate between a deflagration to a detonation. In the event of a detonation, the enclosure will be pressurized before the tank structure can respond and will fragment with the production of a strong air blast wave. This project considers only the deflagration combustion process where the burning velocity starts at the laminar velocity and accelerates according to an increase in temperature, pressure, and the level of turbulence inside the tank.

Confusion can arise due to the different nature of the apparent flame speed and the burning velocity. The apparent flame speed is the flame speed actually measured by an observer during an experiment. The apparent flame speed includes expansion of the combustion products. As result the apparent flame speed is larger than the burning velocity which describes the amount of reactants that experience combustion during a time increment. The following equations fit experimental observations, and thus are apparent flame speeds. During the calculation, burning velocity was calculated using the apparent flame speed and the expansion ratio.

The laminar burning velocity, S_o , for a hydrocarbon

fuel vapor mixed uniformly in air is readily available (Gordon and McBride, 1971; Barnett and Hibbard, 1957; Kuo, 1986). Significant variations of S_o occur with alkynes ($\text{C}_n\text{H}_{2n-2}$) and alkenes (C_nH_{2n}) as a function of the number of carbon atoms in the fuel molecule. However, alkanes exhibit very little change in S_o with the carbon atom number. Information regarding S_o for a particular fuel is readily available, and consequently, the numerical value of S_o is considered to incorporate the effect of fuel type.

Physical conditions influencing the burning velocity include: equivalence ratio, pressure, temperature, and turbulence level. These are incorporated in a mathematical function yielding the burning velocity in a hydrocarbon vapor-air mixture. The mixture is assumed premixed and of uniform concentration throughout the vapor space above the liquid surface inside the storage tank. Each of the above four physical characteristics are now considered regarding the effect on burning velocity.

Equivalence ratio, Φ , defined as the actual mass air-fuel ratio divided by the stoichiometric air-fuel ratio, represents the relative concentration of the hydrocarbon fuel in the air. Both Kanury (1975) and Kuo (1986) show the variation of laminar burning velocity with Φ . For all the hydrocarbon fuels shown, stoichiometric conditions where $\Phi = 1$ yield a maximum S_o . Otherwise, the laminar burning velocity is decreased in proportion to the deviation from the stoichiometric condition. Lee (1984) shows the influence of Φ on S_o for methane. Other hydrocarbons show the same characteristic variation of S_o with Φ . For the calculation of S_o , Lee's (1984) data is used to represent the change in laminar burning velocity with Φ for hydrocarbon fuels,

$$\frac{S_o}{S_o|_{\Phi=1}} = -2.614 + 7.228\Phi - 3.614\Phi^2. \quad (3.2.11)$$

The effect of pressure on the laminar burning velocity when $S_o < 0.5$ m/s is such that S_o decreases with increasing pressure. When $0.5 < S_o < 1.0$ m/s, S_o is independent of pressure, and when $S_o > 1.0$ m/s, then S_o increases with increasing pressure. For the hydrocarbons: 100 Octane gasoline, and JP4, the respective laminar burning velocities in air at standard conditions are: 0.37, 0.36, and 0.38 m/s (Kanury, 1975; Kuo, 1986). Thus, a reasonable assumption is that increasing pressure reduces the burning velocity for hydrocarbon fuels. Both Kanury (1975) and Kuo (1986) report experimental results identifying the influence of pressure on burning velocity as:

$$S_o \propto \left(\frac{P_u}{P_o} \right)^n \quad (3.2.12)$$

where a burning velocity of 0.4 m/s suggests that $n = -0.04$ (Kanury, 1975; Kuo, 1986). The influence of temperature actually involves two temperatures: reactant temperature and the flame temperature. Regarding the reactants temperature, the burning velocity is

given by the following relationship:

$$S = S_0 \left(\frac{P_u}{P_o} \right)^{-0.04} \left(\frac{T_u}{T_o} \right)^2 \quad (3.2.13)$$

where

S = burning velocity (m/s),
 P_u = reactant pressure (Pa),
 P_o = initial Pressure (Pa),
 T_u = reactant temperature (K),
 T_o = initial temperature (K).

The influence of reactant temperature is observed in the factor $(T_u/T_o)^2$. The flame temperature has a strong influence on the laminar burning velocity, S_o , but is already accommodated by Φ , P_u , T_u and the fuel type. Thus, no modification to S_o is required for the flame temperature.

Using the approach taken by Lee (1984), equation (3.2.13) is modified to accommodate the level of turbulence that may exist in the mixture,

$$S = aS_o \left(\frac{P_u}{P_o} \right)^{-0.04} \left(\frac{T_u}{T_o} \right)^2 \quad (3.2.14)$$

where a is the turbulence factor that ranges from 1 to 15. The laminar case is obtained when a equals 1. A large-scale balloon test at the China Lake Naval Weapons Center (Lind, and Whitson, 1977) yielded an apparent horizontal flame velocity of 3.9 to 23.7 m/s for different fuels, corresponding to a basic flame speed of 0.46 to 2.65 m/s. To achieve an extreme condition, Lee (1984) used fans to create a high level of turbulence and obtained a maximum burning velocity equal to 20 m/s. For the vapor space above the liquid surface inside a storage tank, the hydrocarbon-air mixture is assumed to be quiescent, suggesting low values for a . However, structural components in the storage tank vapor space, possibly generating turbulence during the combustion process, would increase the burning velocity. The obstructions inside a storage tank are considered as minimal and therefore assumed insignificant relative to the burning velocity variation.

The burning velocity starts at the laminar speed and increases with the generation of turbulence in the reactants. The turbulent factor varies throughout combustion and can be calculated as a function of time by the following equation (Abdel-Bayed et al., 1987):

$$a = \frac{U_t}{S_o} [1 - e^{-0.2(T_k/\tau_a)^{0.75}}]^{0.5} \quad (3.2.15)$$

where,

a = turbulent burning factor (ranges from 1 to 15),
 U_t = fully developed turbulent burning velocity (m/s),
 T_k = elapsed time from ignition (s),
 τ_a = integral time scale (s).

Equation (3.2.15) is a best fit curve through data taken by Abdel-Bayed (1987) for propagation of burning velocity due to turbulence from a point source ignition. The combustion calculation procedure used here determines the flame speed (as observed relative to the structure) for each increment of reactant mass burned. Consequently, the burning velocity, flame temperature, and volume increase associated with the combustion products are included in the flame speed calculation. The result is the pressure-time variation in the vapor space above the liquid surface inside the storage tank. Combining equations (3.2.11), (3.2.13), and (3.2.15) yields,

$$S = U_t [1 - e^{-0.2(T_k/\tau_a)^{0.75}}]^{0.5} \left(\frac{P_u}{P_o} \right)^{-0.04} \left(\frac{T_u}{T_o} \right)^2 \times (-2.614 + 7.288\Phi - 3.614\Phi^2) \quad (3.2.16)$$

The above expression for the burning velocity allows for variation of the fuel type, equivalence ratio, temperature, pressure, and changes in turbulence levels of the reactants.

3.2.3 Calculation of Pressure Rise. During combustion the pressure inside the tank is approximately uniform because the speed of the flame front is much smaller than the speed of sound in fuel vapor (speed of sound in air ≈ 330 m/s, speed of flame front less than 20 m/s).

The pressure rise is calculated at small increments in time using the flame temperature and burning velocity based on the properties that exist inside the tank at that instant of time. An incremental volume is determined by multiplying the burning velocity by the selected time increment and calculating a volume based on the procedure described in Section 3.1. The incremental volume is burned at constant volume combustion, ending at the constant volume flame temperature. The volumes of the reactants and products are then expanded to pressure equilibrium inside the tank.

At each time step there are three volumes: one containing the products, which are behind the combustion wave; one containing the combustion flame, which is the volume that is currently being burned; and the volume of the unburned reactants. The calculation starts by evaluating an ideal gas constant for the gases in each of the three volumes:

$$R_{mix} = \sum_{i=1}^n R_i M_i \quad (3.2.17)$$

where,

R_{mix} = ideal gas constant for the mixture (kJ/kg*K),
 R_i = ideal gas constant for the individual component (kJ/kg*K),
 M_i = mass fraction of the "i" component (kg/kg).

While balancing the pressures, the total mass remains constant, as does the number of moles of the products and reactants. However, the number of

moles in the volume of the flame front changes as the reactants are converted into products. This change in mole numbers is found by looking at the reaction equation and calculating the total number of moles on each side after the equation is balanced. The number of moles of products divided by the number of moles of reactants in the chemical equation gives the factor by which the number of moles changes during combustion.

The number of moles in each volume is given by:

$$\begin{aligned} N_1 &= \frac{P_1 V_1}{RT_1} \\ N_2 &= \frac{P_2 V_2}{RT_2} F \\ N_3 &= \frac{P_3 (V_T - V_1 - V_2)}{RT_3} \end{aligned} \quad (3.2.18)$$

where

N_1, N_2, N_3 = number of moles of products, flame, and reactants, respectively,

F = number of moles of products divided by the number of moles of reactants in the balanced chemical equation,

P_1, P_2, P_3 = pressure after combustion of the products, flame, and reactants, respectively (Pa),

V_1 = volume of products (m^3),

V_2 = volume of flame (m^3),

V_T = total volume of the vapor space (m^3),

$\bar{R}$ = ideal gas constant ($kJ/(Kmol \cdot K)$),

T_1, T_2, T_3 = temperature of the products or reactants, respectively (K).

To calculate the final pressure inside the tank after combustion, the specific heat capacity of the reactants and products, as functions of temperature, are needed. For the numerical calculation the specific heats are calculated by fitting a fourth order polynomial curve through data reported by Ferguson (1986), valid for $4000 > T > 1000$.

$$C_{pi} = R_u(a + bT + cT^2 + dT^3 + eT^4) \quad (3.2.19)$$

The specific heat capacity for the product mixture is then found by summing the ten species as given by

$$C_{pmix} = \sum_{i=1}^{10} C_{pi} N_i \quad (3.2.20)$$

where

C_{pmix} = specific heat capacity of the mixture ($kJ/(Kmol \cdot K)$),

C_{pi} = specific heat capacity of the species ($kJ/(Kmol \cdot K)$),

N_i = mole fraction of each species,

R_u = universal gas constant ($kJ/(Kmol \cdot K)$).

The specific heat capacity for the reactant mixture is also calculated using equations (3.2.19) and (3.2.20)

but using another set of coefficients valid for temperature between 400 to 1000 K.

The volume with highest pressure is allowed to expand, and the volume of lower pressure are compressed by the same amount until the pressure of all the volumes are the same. To do this, the internal energy for each volume is calculated after combustion as shown in equation (3.2.21):

$$U = C_{pmix} T N \quad (3.2.21)$$

where,

U = internal energy (kJ),

T = Temperature of volume (K),

N = Number of moles of volume.

The mixture temperature is found by adding the enthalpy of each volume, at their individual temperatures, and obtaining an enthalpy for the mixture, then solving for the temperature of the enthalpy of the mixture. Because the temperature difference between the flame and products is small, the enthalpy is found by multiplying the specific heat capacity at constant pressure by the temperatures of the respective volumes.

The internal energy is then calculated for the reactants and the products as shown in equation (3.2.21), noting that during the balancing of the pressures, the total internal energy of the products plus the reactants must remain constant. The pressure inside the tank is balanced by moving the boundary between the volumes in the direction of less pressure. As the boundary moves, the work done is calculated numerically by equation (3.2.22):

$$W = P \Delta V \quad (3.2.22)$$

where,

W = work done (kJ),

ΔV = change in volume of the products or reactants as the boundary moves (m^3).

The work done changes the internal energy of the volumes on both sides of the boundary.

The incremental change in volume is initially given a constant value of 0.005 of the flame volume before expansion. Using smaller incremental steps does not provide any noticeable change in the final balanced pressure. The boundary with the larger pressure difference is moved first, until the pressure difference on the other boundary is larger. The size of the increment is reduced to half when one step of movement reverses the pressure gradient. The calculation is completed when the pressure difference between the two sides of the boundaries is smaller than 0.1 Pa.

The calculation procedure evaluates the pressure at small increments in time. Figure 3.3 shows the convergence of pressure curves for different time increments. These curves show the solution converging at a time increment of 0.01 seconds, which is the maximum value used in the calculation procedure.

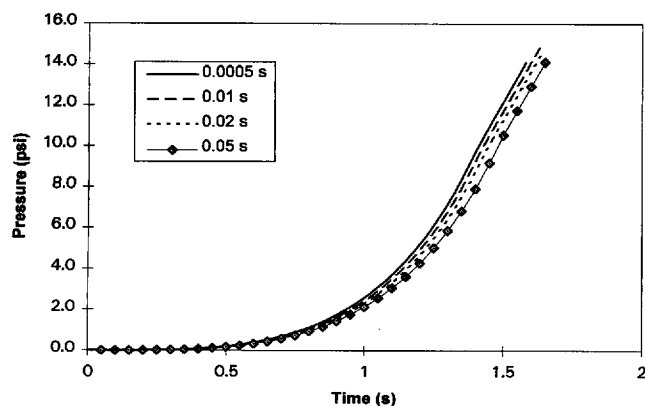


Fig. 3.3—Illustration of convergence with different time steps 30 ft. by 30 ft. empty tank, with methane

3.3 Venting

Venting is important during failure of the frangible joint, since a large opening will be formed to vent the over-pressure in the tank. In addition, there could be smaller vents that will activate during the over-pressure transient. The ability to simulate both types of venting was included in the combustion model. However, in the final calculations, the only venting included was that due to detachment of the roof.

A venting mass flow rate is calculated based on the pressure, temperature, and density of the vapor inside the tank, as well as the atmospheric pressure and temperature. The total mass that has left the tank during an individual time step is used to determine the pressure drop due to venting for that given time step. The mass flow rate is calculated assuming constant pressure and temperature inside the tank for the time increment. The pressure and temperature are then recalculated based on the amount of mass that is left inside the tank.

Two types of venting are considered in this analysis. For venting during frangible joint failure, a simplified analysis assuming adiabatic incompressible open-channel flow through a sluice gate is made (Fox and McDonald, 1985). For a given value of the joint opening area, the volume flow rate through the opening can be written as:

$$Q = C_D A \sqrt{\frac{2\Delta P}{\rho}} \quad (3.3.1)$$

where,

- ρ = density of the mixture (kg/m^3),
- ΔP = pressure difference (Pa),
- A = opening area on the roof to shell joint (m^2),
- C_D = coefficient of contraction, commonly between 0.6 to 0.9.

A different venting calculation is used for the venting pipe (Fox and McDonald, 1985). With flow through an orifice or a pipe, a determination of

laminar or turbulent flow is based on Reynold's number as described in equation (3.3.2).

$$R_e = \frac{2m}{\pi D \mu} \quad (3.3.2)$$

where,

- R_e = Reynold's number,
- m = mass flow rate (kg/m^3),
- D = diameter of the hole or pipe (m),
- μ = viscosity of the vapor ($\text{kg}/\text{m}^*\text{s}$).

When the Reynold's number is lower than 2300, the flow is laminar. Otherwise it is turbulent. The friction factor of the pipe, as well as the diameter and number of bends, influences the flow characteristics while venting the overpressure. For laminar flow in a pipe, the volume flow rate is:

$$Q = \frac{\Delta P \pi D^4}{128 \mu L (0.25 \pi D^2)} \quad (3.3.3)$$

For turbulent flow, the friction factor can be calculated as:

$$\frac{1}{f} = \left[-2.0 \log_{10} \left(\frac{\epsilon}{3.7D} + \frac{2.51}{R_e f^{0.9}} \right) \right]^{2.0} \quad (3.3.4)$$

where, f = friction factor, ϵ = roughness factor.

The mass flow rate for a small division of time within the individual time step is then calculated by the following equation (Abdel-Gayed, et al., 1987):

$$m = \frac{\rho \pi D^2}{4} \sqrt{\frac{2D}{fL + kD + fD \left(\frac{L_e}{D} \right) \left(\frac{\Delta P}{\rho} + g\Delta Z \right)}} \quad (3.3.5)$$

where,

- L = length of the pipe (m),
- k = minor loss coefficient,
- L_e/D = effective length for bends in pipes,
- g = acceleration of gravity (m/s^2),
- ΔZ = change in elevation between the entrance and exit of the vent pipe, m.

Since the time increment is small, the change in pressure due to venting over the time increment is sufficiently low for the temperature to be assumed to remain constant for venting during each time increment. After the mass flow rate is calculated for each small time step, a new pressure is found based on the amount of mass that has left the tank during that small time increment by the use of the ideal gas law.

3.4 Pressure Sensitivity Study

Predictions of the pressure rise inside tanks of different sizes containing different fuels were made. The location of the ignition source and the air-to-fuel ratio (by mass) were also varied. These calculations show the sensitivity of pressure rise to the different parameters inside the tank.

3.4.1 Effect of Ignition Source Location. The ignition source location can be anywhere in the tank,