

API Standard Paragraphs Rotordynamic Tutorial: Lateral Critical Speeds, Unbalance Response, Stability, Train Torsionals, and Rotor Balancing

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API Standard Paragraphs Rotordynamic Tutorial: Lateral Critical Speeds, Unbalance Response, Stability, Train Torsionals, and Rotor Balancing

1 Scope

1.1 Introduction

This document is intended to describe, discuss, and clarify the API Standard Paragraphs (SP) Section 6.8 which outlines the complete lateral and torsional rotordynamics and rotor balancing acceptance program designed by API to ensure equipment mechanical reliability. Background material on the fundamentals of these subjects (including terminology) along with rotor modeling utilized in this analysis is presented for those unfamiliar with the subject.

The standard paragraphs are introduced with references to the appropriate background material to enhance the understanding. This information is intended to be a primary source of information for this complex subject and is offered as an introduction to the major aspects of rotating equipment vibrations that are addressed during a typical lateral dynamics analysis. It is not intended to be a comprehensive guideline on the execution of rotordynamics analyses but is intended to:

- a) provide guidance on the requirements for analysis;
- b) aid in the interpretation of rotordynamics reports;
- c) provide guidance in judging the acceptability of results presented.

1.2 Organization

The document is divided into six sections:

1. Overview
2. Lateral Dynamic Analysis
3. Stability Analysis
4. Torsional Analysis
5. Balancing of Machinery
6. Standard Paragraphs

The individual sections have been prepared in a stand alone manner. As a result, necessary material may be repeated in a succeeding section to provide sufficient clarity to the discussion.

Sections two through four have a parallel organization:

- Modeling criteria
- Analysis techniques and results
- Machine specific considerations
- Testing
- Applications and examples

1.3 Standard Paragraphs

In order to aid turbomachinery purchasers, the American Petroleum Institute's Subcommittee on Mechanical Equipment has produced a series of specifications that define mechanical acceptance criteria for new rotating equipment. Experience accumulated by turbomachinery purchasers over the past ten years indicates that if the API standards are properly applied, the user can be reasonably assured that the installed unit is fundamentally reliable and will, barring problems with the installation and operator misuse, provide acceptable service over its design life.

An integral component of these individual equipment specifications is contained in the API Standard Paragraphs, those specifications that are generally applicable to all types of rotating equipment. The criteria associated with lateral and torsional rotordynamics and balancing have been categorized as standard paragraphs. In rotating equipment specifications published by API (for example, API Standard 617—*Axial and Centrifugal Compressors and Expander-compressors for Petroleum, Chemical and Gas Industry Services*) there is a section on rotordynamics and balancing. The backbone of those sections is the standard paragraphs augmented by additional information that is applicable only to the type of unit considered in the standard.

The Dynamics Standard Paragraphs originated in the Centrifugal Compressor Standard, API 617. A timeline for the first through eighth editions with major highlights is [1]:

- 1st Edition June 1958—dynamics section added. A critical speed separation margin defined and a balance/vibration criterion established. Vibration limit in operating speed range for mechanical test is a step function.
- 2nd Edition April 1963—torsional analysis added as an option.
- 3rd Edition October 1973—balance criteria and separation margin for laterals below operating speed range adjusted, rotor response required when specified, torsional analysis required on motor driven and geared units, vibration limit in operating speed range changes from step function to $(12,000/\text{MCOS})^{0.5}$ or 2.0 mils whichever is less and vibration limit added at trip speed.
- 4th Edition November 1979—AF (amplification factor) limited when passing through lateral critical speeds, lateral analysis required including a rotor response and transient torsional analysis for all synchronous motor-driven units.
- 5th Edition April 1988—removed limits on AF while passing through lateral critical speeds, acceptance defined basis rotor response analysis, lateral analysis report requirements spelled out in detail, operating seal clearance criteria added, separation margin for laterals tied to amplification factor, amplification factor less than 2.5 acceptable in any operating case, a dynamics shop test included, and balance criteria modified.
- 6th Edition February 1995—lateral paragraphs reorganized and criteria set for comparison with shop test evaluation and transient torsional required with variable speed motors, and vibration limit in operating speed range changes to $(12,000/\text{maximum continuous speed})^{0.5}$ or 1.0 mils whichever is less (had been 2.0 mils).
- 7th Edition July 2002—formula for allowable separation margins for lateral criticals changes, unbalance response test criteria clarified, stability analysis requirement included, and entire section updated.
- Proposed 8th Edition—active magnetic bearing specifications expanded including stability criteria, inclusion of damper seals into lateral analysis, high-speed balance specifications revamped, requirements for lateral and torsional reports added and influence of VFD, generators and motor faults and excitations on torsional analysis.

The complete text of the Dynamic Section of the Standard Paragraphs is included at the end of the document.

1.4 Definitions and References

Definitions are incorporated into each section of the document. Due to very large number of references employed, they are identified at the end of each relevant section.

1.5 Fundamental Concepts of Rotating Equipment Vibrations

In order to understand the results of a rotordynamics design analysis, it is necessary to first gain an appreciation for the physical behavior of vibratory systems. Begin by noting that all real physical systems/structures (such as buildings, bridges, and trusses) possess natural frequencies. Just as a tuning fork has a specific frequency at which it will vibrate most violently when struck, a rotor has specific frequencies at which it will tend to vibrate during operation. Each resonance is essentially comprised of two associated quantities: the frequency of the resonance and the associated deflections of the structure during vibration at the resonance frequency. Resonances are often called “modes of vibration” or “modes of motion,” and the structural deformation associated with a resonance is termed “mode shape.”

The modes of vibration are important only if there is a source of energy to excite them, like a blow to a tuning fork. The natural frequencies of rotating systems are particularly important because all rotating elements possess finite amounts of unbalance that excite the rotor at the shaft rotation frequency (synchronous frequency) and its multiples. When the synchronous rotor frequency equals the frequency of a rotor natural frequency, the system operates in a state of resonance, and the rotor's response is amplified if the resonance is not critically damped. The unbalance forces in a rotating system can also excite the natural frequencies of nonrotating elements, including bearing housings, supports, foundations, piping, and the like.

Although unbalance is the excitation mechanism of greatest concern in a rotordynamics analysis, unbalance is only one of many possible lateral loading mechanisms. Lateral forces can be applied to rotors by the following sources: impeller aerodynamic loadings, misaligned couplings and bearings, rubs between rotating and stationary components, and so on. A more detailed list of rotor excitation mechanisms of particular interest is found in the API Standard Paragraphs, 6.8.1.1. This subject is discussed in detail in 3.5, as well as scattered in the appropriate sections of this document.

The vibration behavior of a rotor can be described with the aid of a simple physical model. Assume that a rotor-bearing system is analogous to the simple mass-spring-damper system presented in Figure 1-1.

From physics, the governing equation of motion for this system can be written as Equation 1-1 and Equation 1-2:

$$ma + cv + kx = F(t) \quad (1-1)$$

$$m\ddot{\chi} + c\dot{\chi} + kx = F(t) \quad (1-2)$$

where

m is the mass of the block, kg (lbm);

$a = \ddot{\chi}$ is the acceleration, m/s² (in./s²);

c is the viscous damping coefficient, N-s/m (lbf-s/in.);

$v = \dot{\chi}$ is the velocity, m/s (in./s);

k is the stiffness of the elastic element, N/m (lbf/in.);

x is the displacement of the block, m (in.);

$F(t)$ is the force applied to the block (time-dependent function), N (lbf).

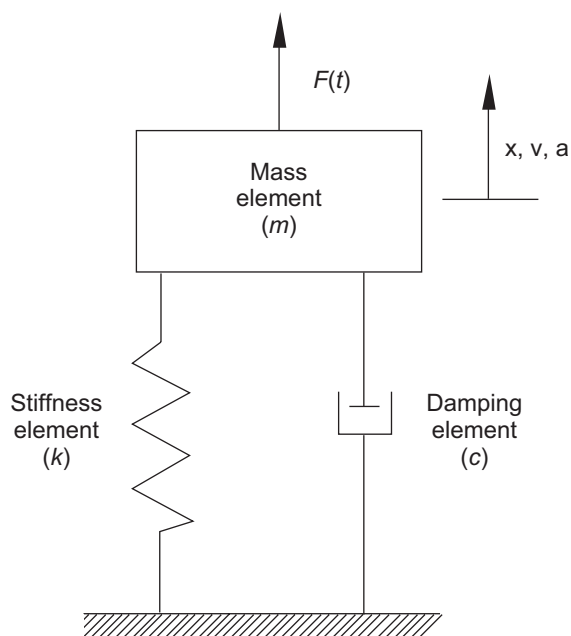


Table of Typical Units and Conversions

Quantity	SI Units (MLT System of Units)	US Customary Units (FLT System of Units)	US-to-SI Conversion ¹
g (gravitational acceleration)	9.88 m/s ²	386.4 in./s ²	- NA -
M (mass) [=Weight/ g]	kg	lbm	2.2046
c (damping)	N-s/mm	lbf-s/in.	0.17513
k (stiffness)	N/mm	lbf/in.	0.17513
F (force)	N	lbf	4.4482
x (displacement)	μm	mils	25.4
v (velocity)	mm/s	in./s	25.4
a (acceleration)	g	g	1.0

NOTE 1 Multiply the quantities listed above (in US Customary Units shown) by the US to SI conversion factors to obtain the quantity in the SI units listed in the table.

NOTE 2 NA = not applicable.

NOTE 3	Common Units	US Customary Units	SI Units
	s = seconds	in. = inches	mm = millimeters
	g = acceleration	mil = 1.0×10^{-3} in.	μm = micrometers
		lbf = pound (force)	kg = kilograms
		lbm = pound (mass)	N = Newtons = kg-m/s ²

Figure 1-1—Simple Mass-spring-damper System

In this example, the displacement response of the block to the applied force is counteracted by the block's mass and the support's stiffness and damping characteristics. The undamped natural frequency of this system is calculated by determining the eigenvalue of the second order homogeneous ordinary differential equation ($F = 0$) for the case where the damping term is neglected ($c = 0$) as seen in Equation 1-3.

$$\omega = \sqrt{\frac{k}{m}} \quad (1-3)$$

where

ω is the undamped natural frequency, rad/s.

Since real, physical systems include damping, this needs to be included in the analysis. The damped natural frequency of the homogeneous system ($F = 0$) is defined in Equation 1-4.

$$\omega_d = \sqrt{\frac{k}{m} - \left(\frac{c}{2m}\right)^2} \quad (1-4)$$

where

ω_d is the damped natural frequency, rad/s.

If the system was excited (hit by a hammer), this damped natural frequency is the frequency of vibration that would be seen as the system responds. Note that the damped natural frequency of the system, ω_d , is equal to the undamped natural frequency of the system only when system damping is negligible. This observation underscores the fact that an undamped critical speed analysis should, in general, not be used to define the critical speeds of a rotating machine.

If the exciting force is sinusoidal, i.e.,

$$F(t) = A \sin(\omega t) \quad (1-5)$$

The response will be:

$$x(t) = B \sin(\omega t + \theta) \quad (1-6)$$

where

$\left(\frac{B}{A}\right)$ is the amplitude ratio;

θ is the phase angle, rad.

In the case of a rotor with unbalance, the unbalance force is defined in Equation 1-7:

$$F_{UB} = me_u \omega^2 \quad (1-7)$$

where

m is the mass of rotor, kg (lbm);

e_u is the mass eccentricity, m (in.);

ω is the rotational speed, rad/s;

F_{UB} is the unbalance force, N (lbf).

This result is called “forced response analysis” and is analogous to the unbalance response analysis performed in rotordynamics studies. The amplitude ratio depends upon frequency of the excitation and the damping in the system. Figure 1-2 shows the amplitude ratio versus the excitation frequency. Maximum amplitude ratio is seen where the excitation frequency equals the natural frequency of the system. Amplitude ratio also increases as damping decreases with the amplitude becoming infinite at zero damping (a situation that is not physically practical).

There is a phase difference between the excitation and response. This phase difference is a function of damping and reaches 90 degrees at the natural frequency. Figure 1-3 shows the phase angle versus excitation frequency.

If a transient rather than a sinusoidal excitation excites the system, the actual response normally looks like that shown in Figure 1-4.

In this case, the response is at the natural frequency. It decays with time based on the amount of damping. This response is called “stable.” In Figure 1-5, the response grows with time. This response is called “unstable.”

While the simple, single degree of freedom system described above is useful for examining the general concepts of vibration theory, this system is clearly not representative of a turbomachine. Recognize that the early development of rotor dynamics analysis took place in the late 19th to early 20th Centuries and calculations could only be performed with pencil and paper. Thus, it was required to utilize sound, simplifying assumptions to permit solutions to be performed. In 1869, Lord Rankine published an analysis that concluded operation of a rotating shaft above its first resonant speed was impossible. This reflected the common feeling of many of the learned mechanics experts of the day. However, in 1889, Gustav Laval built a steam turbine which he operation above the first resonance.

It was apparent that a more accurate representation of a rotating assembly was required. Henry Jeffcott (under charter of the British Royal Society) performed an analysis with the results published in 1919. In that analytical work, he proposed the model as shown in Figure 1-6. It is a two degree of freedom model with all mass concentrated in a central disk. The disk is mounted on a massless flexible shaft and It is connected to ground by rigid supports. This model may be analyzed and solved to indicate the presence of the first undamped critical speed. This form of model is known as the Laval (Europe) or Jeffcott (outside Europe) model.

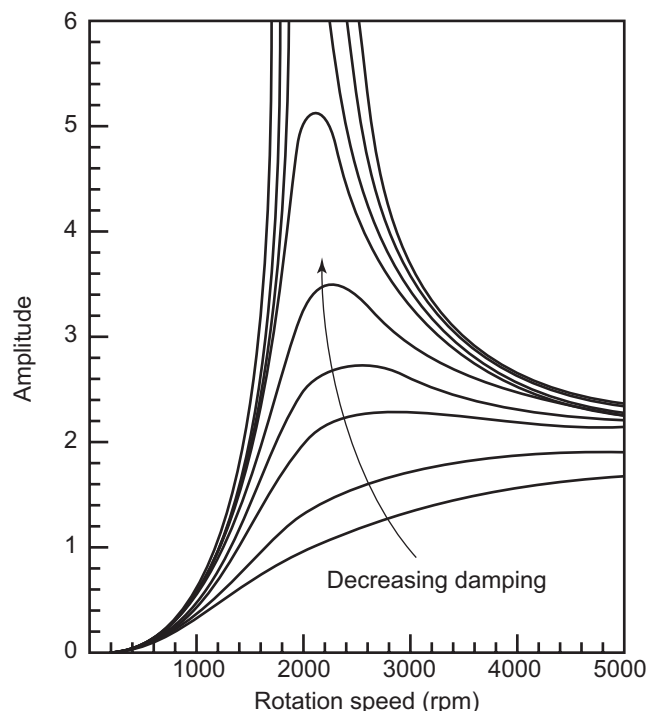


Figure 1-2—Amplitude Ratio Versus Excitation Frequency



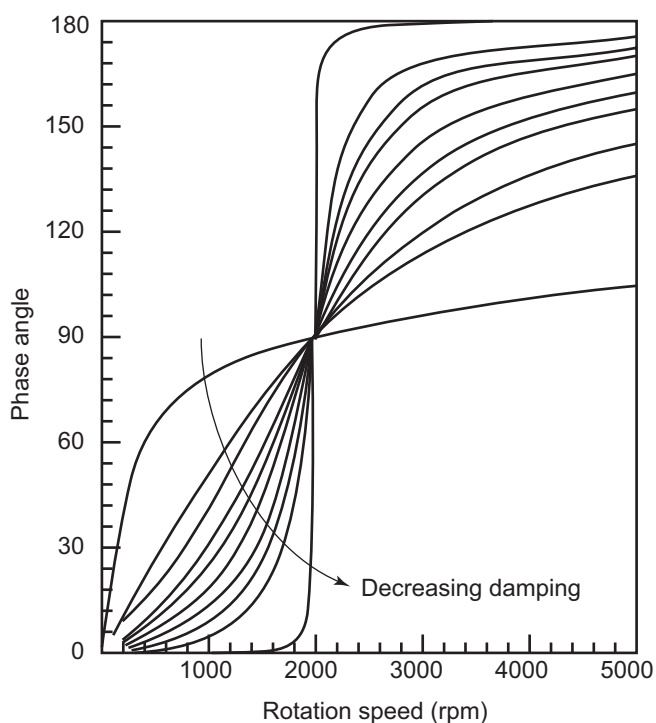


Figure 1-3—Phase Angle Versus Excitation Frequency

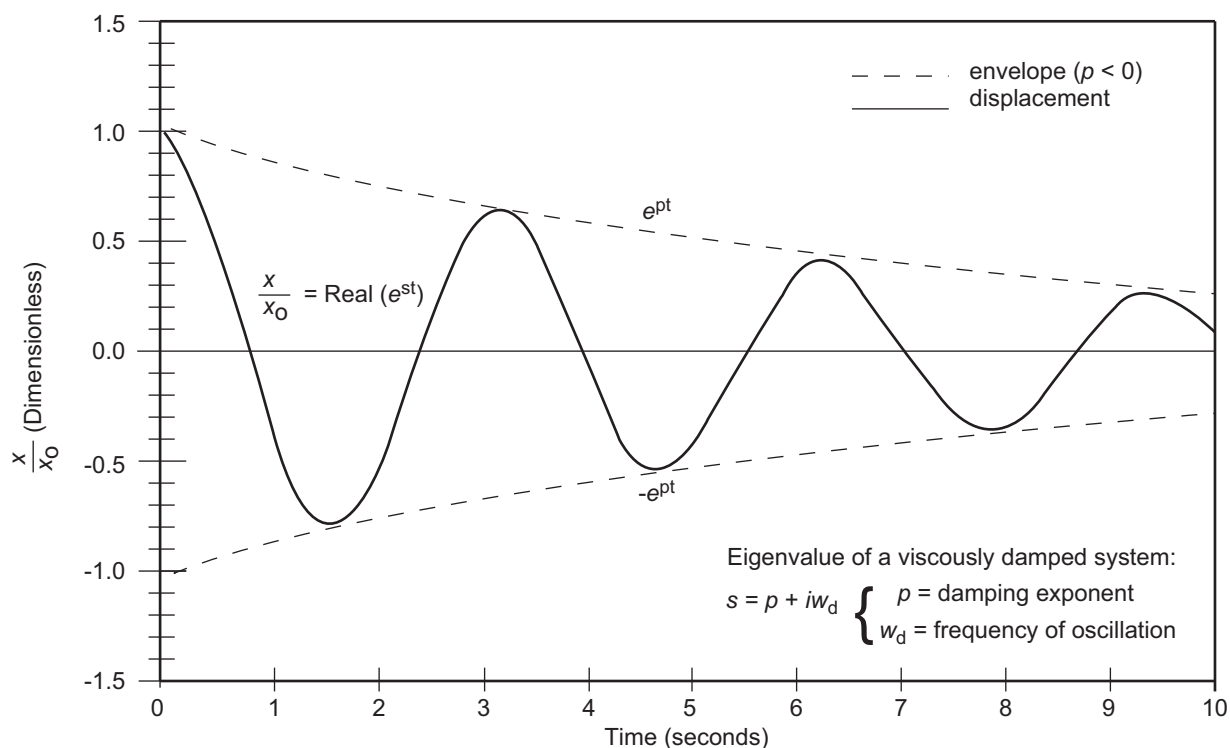


Figure 1-4—Response of a Spring-mass System to Transient (Stable)

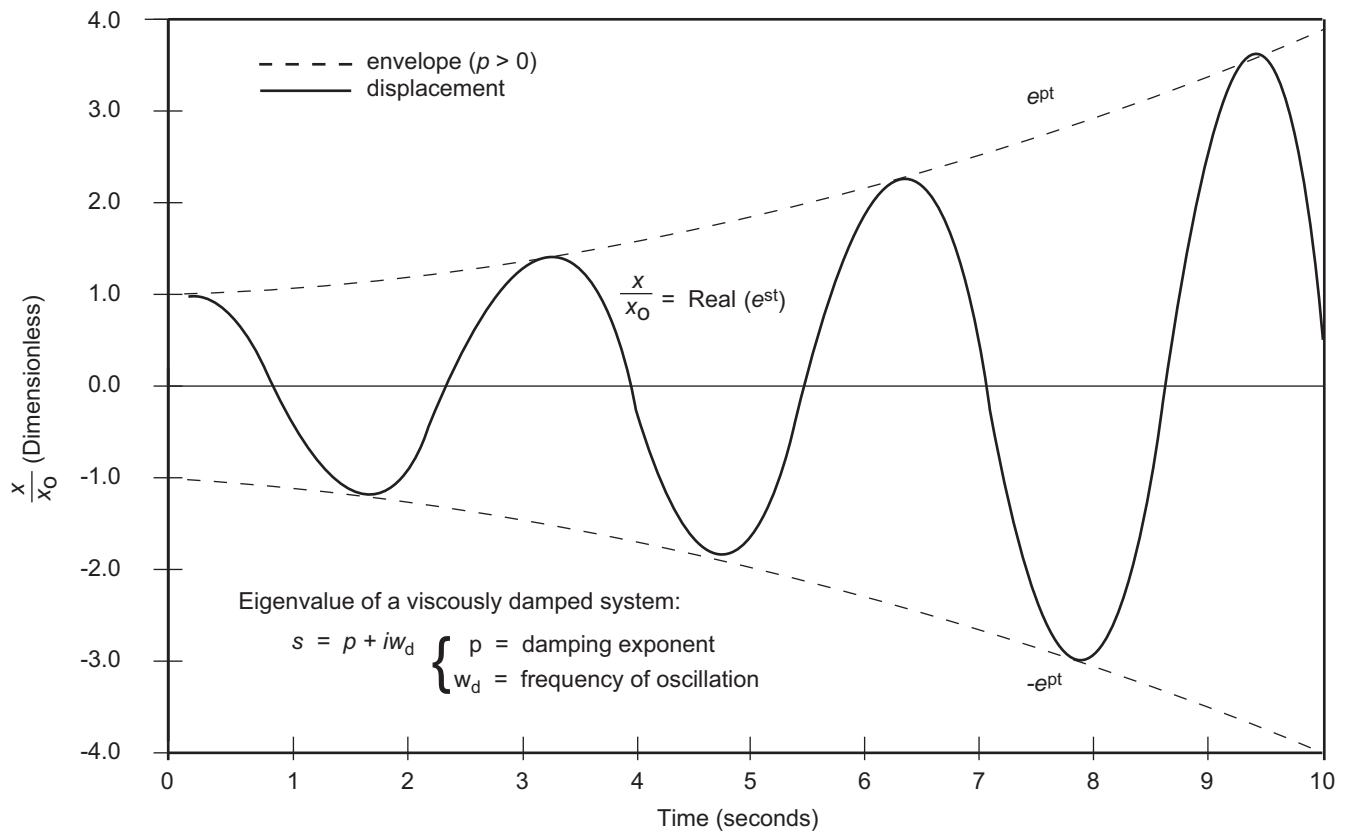


Figure 1-5—Response of a Spring-mass System to Transient (Unstable)

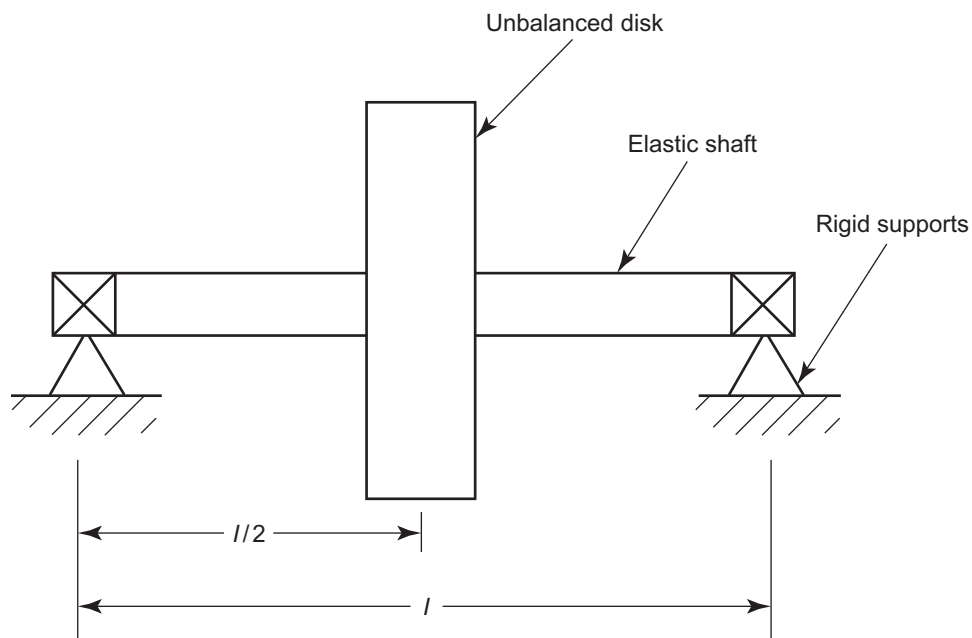


Figure 1-6—Jeffcott Form for Rotor Model

The Laval-Jeffcott Model is used extensively to gain insight into the potential rotor dynamic performance of rotors. While it is able to provide simple, qualitative results, more complete and comprehensive models were required. The next level of complexity added was to replace the rigid shaft supports with springs as shown in Figure 1-7. Then, damping was added Figure 1-8 and Figure 1-9. As computing capabilities began to be available, more comprehensive, accurate models were developed. These are described in considerably detail in the rest of this document.

In order to consider the effect of the combined shaft and bearing stiffness, we need to review the effective combined stiffness as shown in Figure 1-7. As shown, the effective stiffness, K_{eq} , is the result of the individual stiffnesses added. The equation that describes this is:

$$K_{eq} = \frac{2}{\left(\frac{2}{K_{shaft}}\right) + \left(\frac{1}{K_{brg}}\right)} \quad (1-8)$$

$$K_{shaft} = \frac{48EI}{L^3} \quad (1-9)$$

$$I = \frac{\pi D^4}{64} \quad (1-10)$$

where

- K_{shaft} is the shaft stiffness, N/m (lbf/in.);
- E is the Young's modulus for shaft material, N/m² (lbf/in.²);
- L is the shaft section length, m (in.);
- I is the area moment of inertia, m⁴ (in.⁴);
- D is the shaft diameter, m (in.).

This equation indicates that the stiffness of the combined shaft-bearing system will be less than the stiffness of the single most flexible element. In this example, the shaft is the single most flexible element ($K_{shaft} = 8756.5$ N/mm or 50,000 lbf/in.). According to Equation 1-8, the effective stiffness of the combined shaft-bearing spring system is only 7297.7 N/mm (41,670 lbf/in.).

To carry this analysis one step higher in complexity, consider the sketch of a rotordynamic system displayed in Figure 1-8. Figure 1-9 shows the model with masses, springs, and dampers.

Note that this system is identical to the system just discussed except that viscous damping elements have been added to the bearing model. All oil film bearings generate significant viscous damping forces. Figure 1-10 displays the calculated response of the disk to a harmonic load acting at the disk for various values of shaft stiffness.

Note that as the shaft stiffness decreases, the peak response frequency decreases while the amplitude of the peak response and the sharpness of the peak both increase. These observations are understood by noting that the decrease in shaft stiffness decreases the relative deflection of the shaft in the bearings and diminishes the magnitude of the damping forces provided by the bearings. Thus, one may conclude that the effect of damping provided by the bearings is maximized when the shaft stiffness is large relative to the bearing stiffness.

These general concepts of vibrations will be demonstrated in considerably more detail in the sections that follow.

1.6 References

- [1] Pettinato, B.C., Kocur, J.A. and Swanson, E.E. (2011), "Evolution and Trend of API 617 Compressor Rotordynamic Criteria", Turbomachinery Society of Japan: *Turbomachinery*, 39, pp 292-303.

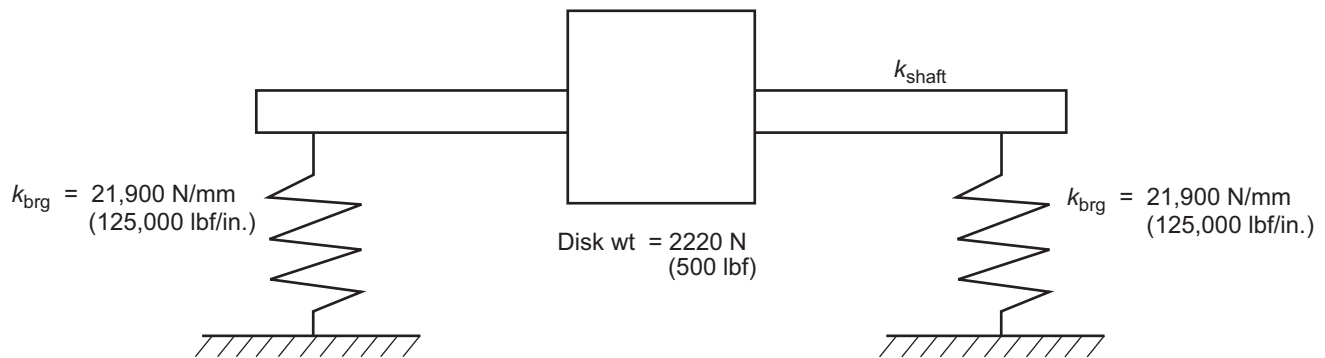


Figure 1-7—Simplified Model of a Beam-type Rotating Machine

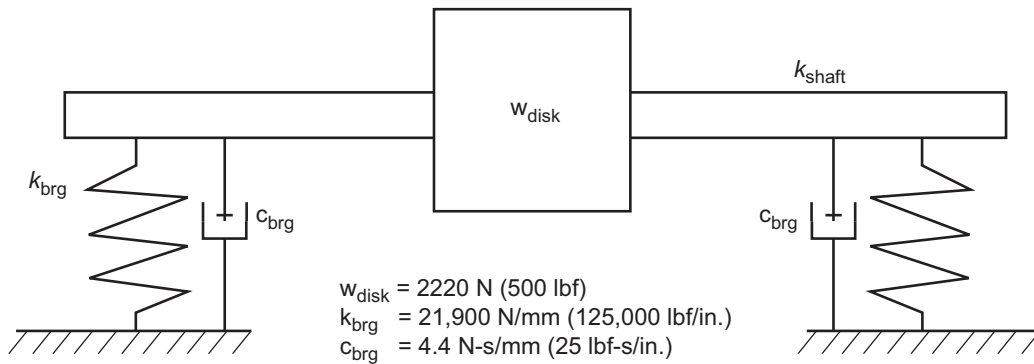
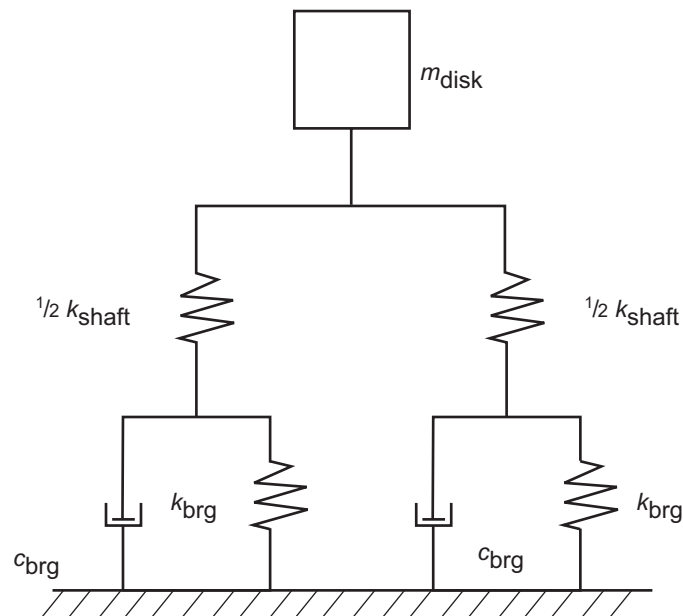
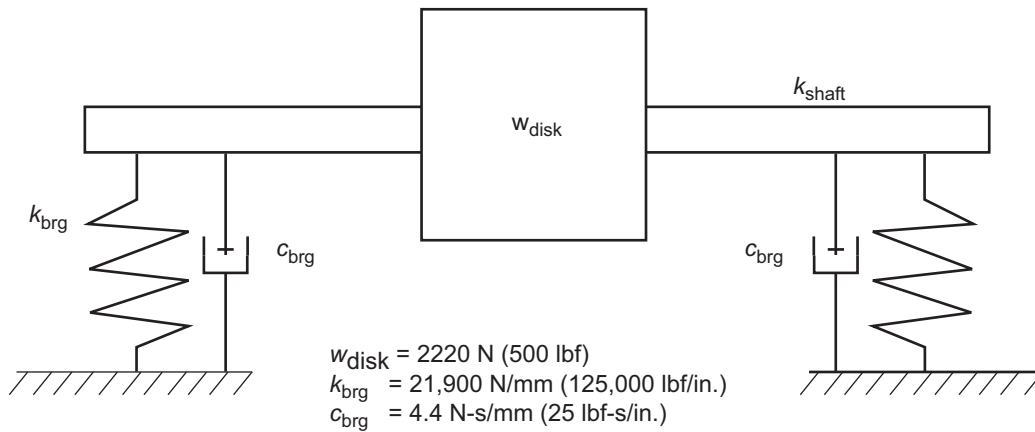


Figure 1-8—Simplified Model of a Beam-type Rotating Machine with Damping

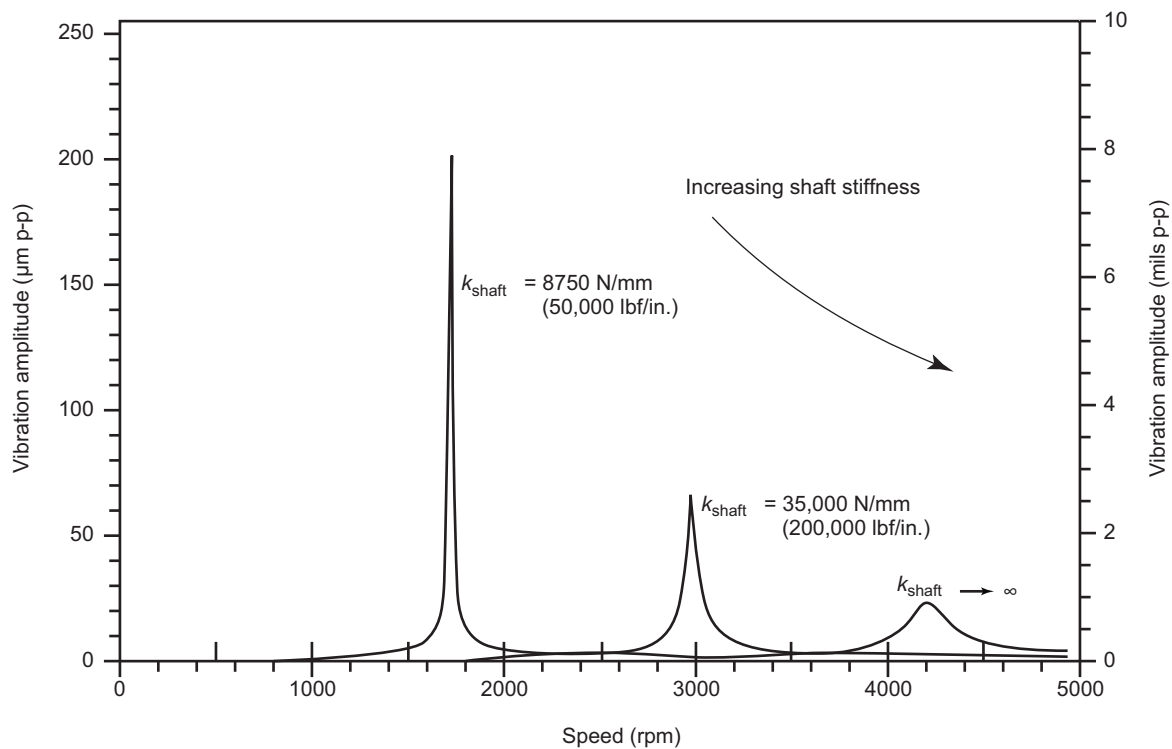


Dynamic System Schematic

Figure 1-9—Spring-mass-damper Model of Beam Type Rotating Machine



System Schematic



Calculated Responses for Several Values of Shaft Stiffness

Figure 1-10—Synchronous Response of Beam Type Machine for Various Shaft Stiffness Values

SECTION 2—LATERAL ROTORDYNAMICS

2.1 Introduction

The mechanical reliability of rotating equipment depends heavily upon decisions made by both the purchaser and vendor prior to equipment manufacture. Units that are designed using the appropriate application of sophisticated computer-aided engineering methods will be less problematic than units designed without the benefit of such analysis. Even with performance and mechanical acceptance tests prior to delivery and installation, the discovery of design-related problems during these tests may compromise the planned cost of the unit and/or its delivery schedule.

For this reason, rotordynamics analysis tools have been developed and are continuing to evolve. In the late 19th century, there was a widespread belief that machines could not be operated above the first critical speed and designs focused on machines that operated below that critical. Once it was proven that operation above the first critical was possible, it was still difficult to execute the designs due to the complexity of the analysis and the limits of computational power. In the mid-20th century, critical breakthroughs were made in the analytical tools. Once this was coupled with the explosion of computational power, analysis methodology began to experience significant growth. Today, very comprehensive analyses may be performed on computers and analysts are continuing to develop more accurate and complex modeling techniques. In addition, enhanced measurement capabilities are providing the opportunity to improve this modeling work.

Today, these analysis tools and procedures have become a standard fundamental design tool for a class of turbomachinery that includes centrifugal and axial compressors, centrifugal pumps, steam turbines, gas turbines, electric motors, expander turbines, and gears. Application of this technology requires communication between the purchaser and manufacturer to be very effective. This document has been prepared to facilitate this communication by providing the means to achieve a common understanding or platform upon which to hold meaningful discussion. This document identifies the analysis requirements as well as providing an aid to interpret and understand results of that analysis.

The general class of machines to which this document applies are primarily custom designed, i.e. while they belong to a machine class, the specific design features like bearing span, rotor weight, operating speed, etc. fall within design ranges selected for the specific application. This is different from “standard design” turbomachinery, e.g. aircraft gas turbines which are complex but which have many identical machines built. Most of the principles identified in this document apply to those machines, but the details have not been directed at that class of machine.

2.2 Rotor-bearing System Modeling

Modeling is the single most important process in performing any engineering analysis of a physical system. If the model does not accurately simulate the proposed design, the sophisticated analysis and evaluation of the design will do little good. The steps taken to model rotating equipment are listed in sequence below.

- a) Generate a mass-elastic lateral model of the unit's rotating assembly.
- b) Calculate the static bearing reactions (including miscellaneous static load mechanisms such as gear loading, loads resulting from partial arc steam admission, and so forth).
- c) Calculate the linearized fluid film bearing coefficients and squeeze film damper behavior. Determine the dynamic support stiffness.
- d) Calculate the linearized floating ring oil seal coefficients (if present).
- e) Calculate all other excitation mechanisms (such as aerodynamic effects and labyrinth seal effects).

The flow chart in Figure 2-1 outlines the generalized procedure for the lateral analysis. These steps will be discussed in greater detail in this section.

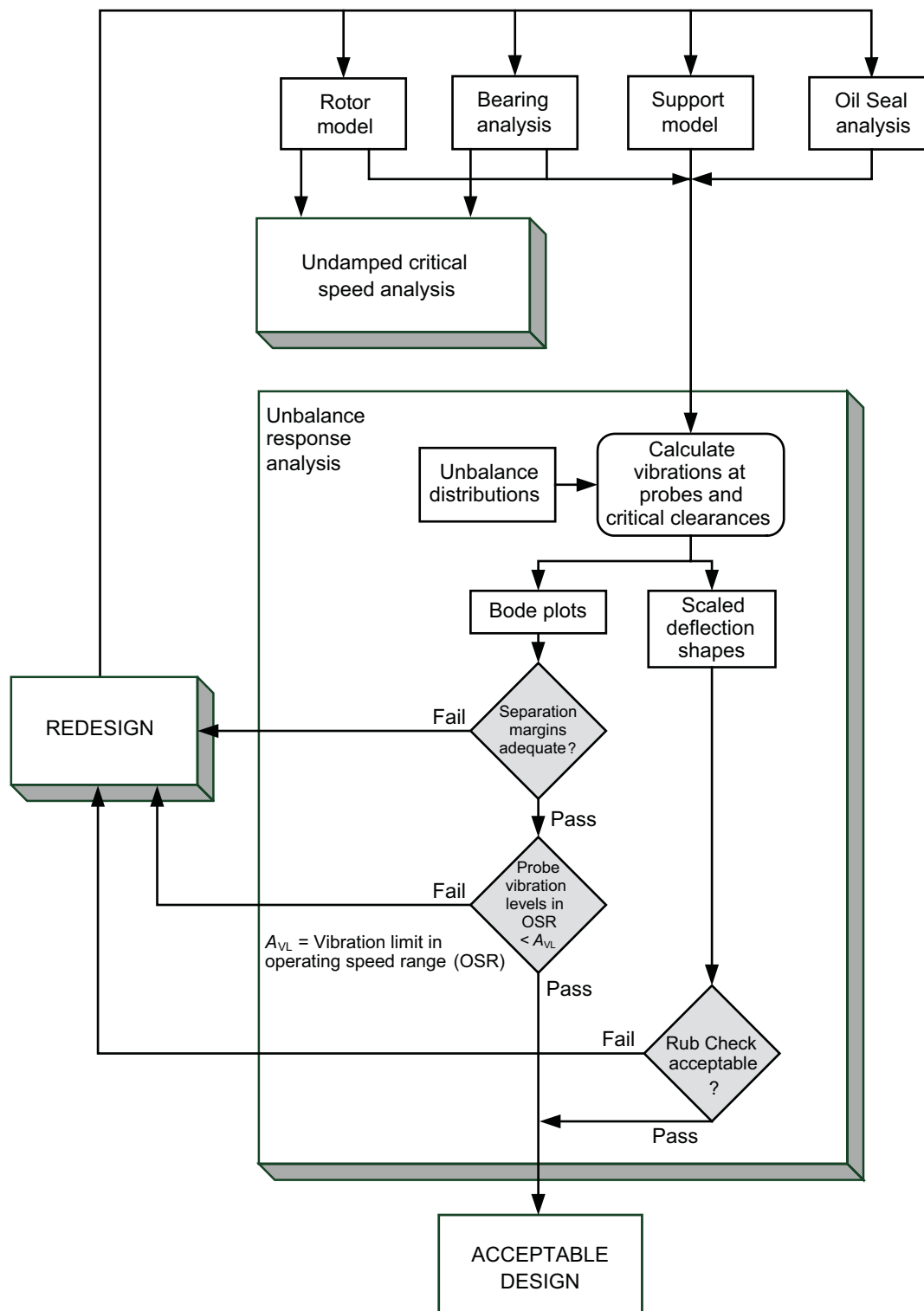


Figure 2-1—Overview of Lateral Analysis Procedure

2.3 Rotor Modeling Methods and Considerations

2.3.1 General

This section will describe some of the methods that have been successfully employed over a period of many years to model the important elements of a rotor assembly. The reader is cautioned that tie-bolt rotors are potentially subject to greater modeling complications than solid shaft rotor designs. For example, the rotor bending stiffness characteristics may be related to the tie-bolt stretch. The nonlinear axial face friction forces between rotor segments may become significant if the segments move relative to each other during rotor operation. Finally, thin shell rotors may simply not be adequately represented by direct application of cylindrical beam elements. In such cases, either a sophisticated finite element analysis of the rotating element or a modal test benchmark may be necessary to build or verify accurate prediction of results.

An accurate model of a rotor system has adequate representation of the rotor's mass and elastic (mass-inertia-stiffness) properties. For the purpose of performing a basic rotordynamics design audit, two simple building blocks are joined together to form a complete model of the rotating assembly. These elements are the shaft lumped mass-inertia elements and the disk lumped mass-inertia elements. Shaft elements contribute mass, inertia and stiffness to the global model, whereas disk elements contribute mass and inertia only. More complicated element types can be used at the cost of introducing complexity to the model. In general, however, most turbomachinery can be adequately modeled using the lumped mass-inertia shaft and disk elements presented in this tutorial.

Once the type of elements to be used in the analysis has been established, it simply remains for the engineer to partition or discretize the subject rotor's geometry using a sufficient number of the selected elements. Schematics of a rotor and its associated lumped parameter model are displayed in Figure 2-2. Some general constraints must be placed on the use of the lumped mass-inertia shaft elements, however, to ensure that accurate rotor models emerge from the process. Clearly, if too few elements are used, the resulting model may not possess sufficient resolution to accurately capture some of the detailed mass-elastic properties of the rotating assembly. If a large number of elements are used to model the rotor, then numerical problems may result. A secondary benefit of minimizing the number of elements used to produce a rotor model is a reduction of the amount of time needed by the engineer to generate data files and for the computer to perform calculations.

2.3.2 Division of Rotor into Discrete Sections

The modeling process starts with the analyst's dividing the rotor into a series of stations connected by shaft elements that begin and end at step changes in the outside diameter (OD) or inside diameter (ID). The stations are also chosen to coincide with the axial locations of the concentrated mass or shrunk on members, such as impellers, turbine disks, thrust collars, balance pistons, spacer sleeves, couplings, dry gas seals, etc. Additionally, stations are located at the journal bearing and seal centerlines, radial probe locations, and at the ends of the rotor.

The number of stations needed to represent the physical rotor depends on: the number and shape of modes to be examined [1], and the number to adequately approximate a continuous body with a discretized model. The following simple guidelines are proposed for modeling a long uniform shaft segment.

- a) The length to diameter ratio of any section should not exceed 1.0 (less than or equal to 0.6 is preferred [2]).
- b) The length to diameter ratio of any section should not be much less than 0.10.

The first guideline is proposed to ensure that the model possesses sufficient resolution to permit accurate calculation of the undamped critical speeds. The number of critical speeds calculated should be sufficient that the next undamped critical speed above maximum continuous speed or trip is identified. The second guideline is proposed to ensure that large differences in L/D values are avoided as this practice may generate numerical calculation problems. When a large L/D difference exists, precision errors accumulate during the numerical manipulation of the shaft element stiffnesses. It should be noted that some transfer matrix based programs are capable of modeling well

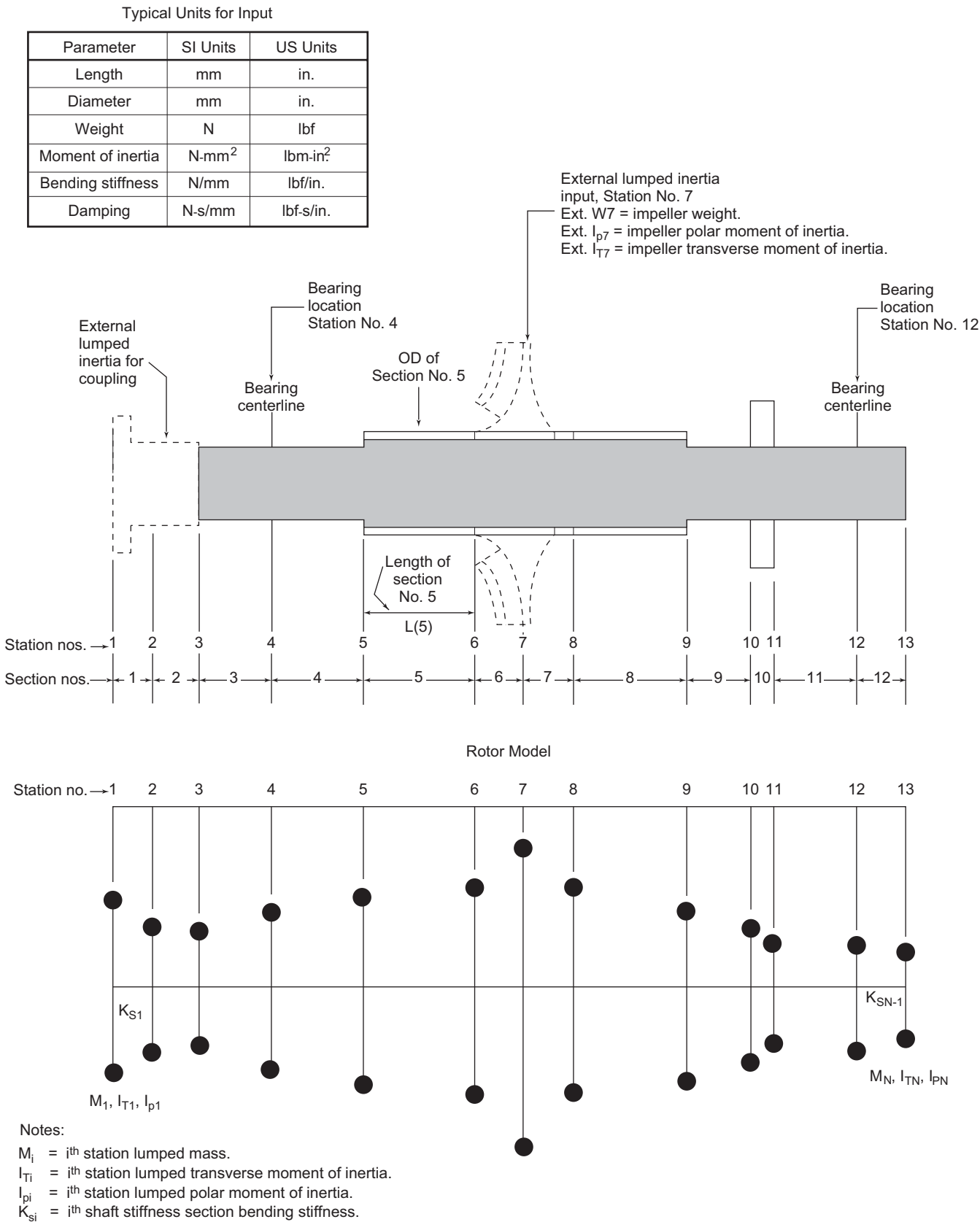


Figure 2-2—Schematic of a Lumped Parameter Rotor Model

outside these guidelines without any significant degradation of model accuracy. In general, the guidelines provide a safe modeling procedure for those cases where sufficient program benchmarking has not been performed.

Whenever the analyst is unsure of how to model a given feature in the rotating element, he or she may always proceed by determining the sensitivity of calculated results to various ways of modeling the feature in question. For example, if one strictly adheres to the two modeling guidelines proposed above, a short circumferential groove machined into the shaft cannot be modeled. Such grooves are often found on compressor shafts to locate split rings at the ends of the aerodynamic assembly and to lock thrust collars onto the shaft. Such design features can be ignored when analysis indicates that decreasing the diameter of the entire element encompassing the groove does not affect the critical speeds or the associated mode shapes. When a given geometric feature possesses a strong influence on calculated results, the designer must examine the possibility that the rotor's design may be fundamentally flawed.

On those occasions when the analyst has difficulty modeling a rotating assembly because the rotor geometry cannot be readily described using rudimentary shaft elements, then an equivalent model can be formulated from more sophisticated analysis. For example, the bending characteristics of a stub shaft bolted to the second stage impeller on an overhung gas pipeline compressor have been determined using a finite element analysis of the shaft and impeller sections. The finite element mesh is displayed in Figure 2-3. Note that the large counter-bored bolt holes dramatically decrease that stub shaft's lateral bending stiffness. Once the static bending analysis of the component is accomplished, an equivalent lumped parameter beam-type model of the type used in rotordynamics analysis can be formulated that possesses identical bending stiffnesses at the lumped mass and inertia locations.

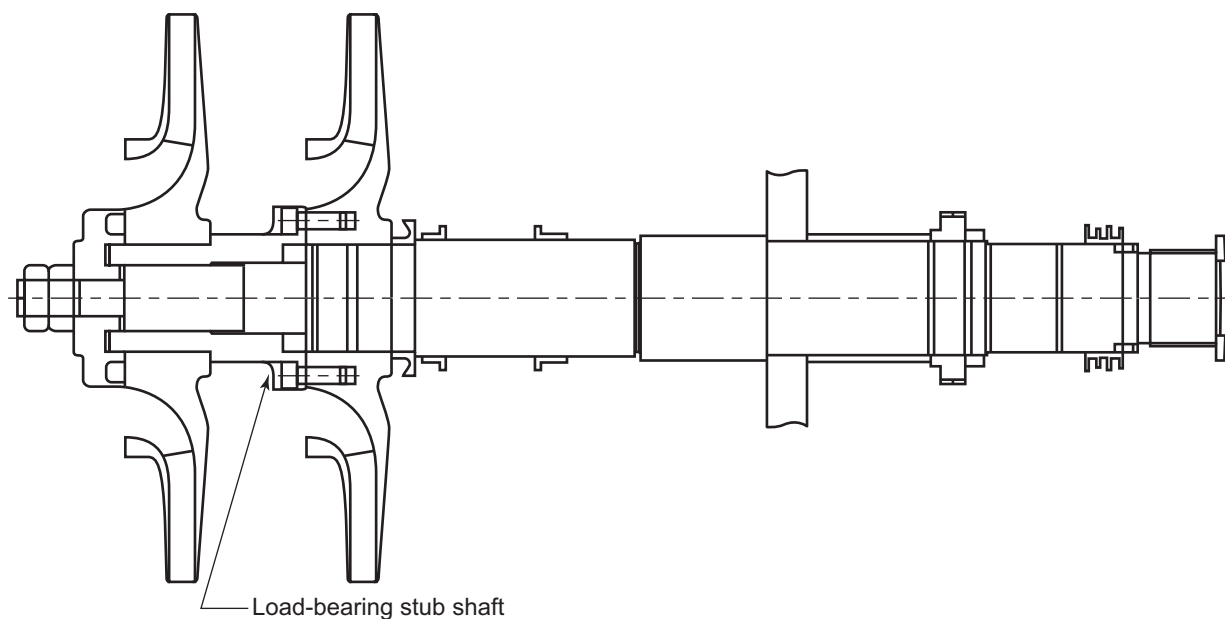
2.3.3 Addition of External Masses and Inertial Loadings

Components that are shrunk on turbomachinery shafts (impellers, sleeves, thrust collars, and so on) affect the bending stiffness of the rotating element to varying degrees. The amount of shrink fit and contact length determine the amount of contribution to the bending stiffness of the rotor. The model used to predict the unit's critical speeds may have to be refined according to data collected during mechanical testing of the actual machine if the critical speeds differ by more than 5 %.

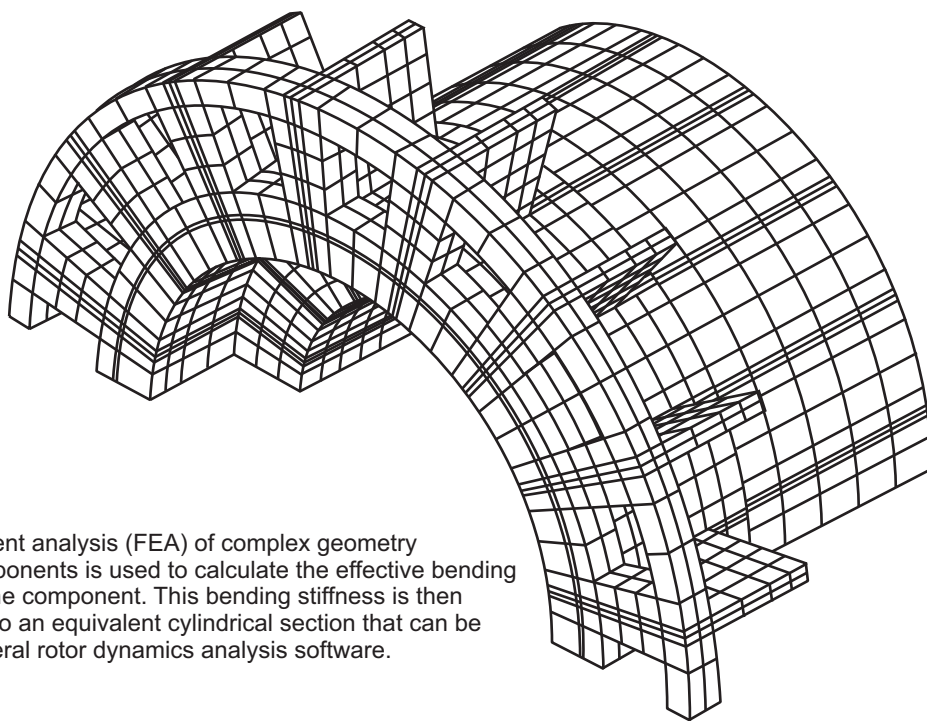
Components shrunk or fitted onto the shaft affect the mass and inertia characteristics of the rotating assembly, and must be added to the model. This is most often accomplished by adding lumped masses and inertias at the centers of gravity of the shrunk-on components. It is occasionally necessary, as in the case of motor cores, to generate detailed inertia distributions of the shrunk-on component. Most rotors will include at least several of the following additional masses:

- a) impellers/disks/blades*,
- b) couplings*,
- c) balance rings*,
- d) check nuts, nose cones,
- e) fans*,
- f) sleeves,
- g) balance pistons*,
- h) thrust collars*,
- i) dry gas seals*.

NOTE Items marked with (*) are typically represented by mass and inertia.



Cross-section of rotating element
with complex geometry component



A finite element analysis (FEA) of complex geometry rotating components is used to calculate the effective bending stiffness of the component. This bending stiffness is then converted into an equivalent cylindrical section that can be input into lateral rotor dynamics analysis software.

3D Finite element model of stub shaft

Figure 2-3—3D Finite Element Model of a Complex Geometry Rotating Component

In the rotor lateral model, a station located at the center of gravity of coupling hub should be specified for the coupling associated with the distributed coupling weight and moment of inertia. Coupling vendors provide the location of the center of gravity of each half of the assembled coupling and the weight distribution on the drawings. When the center of gravity of coupling hub is beyond the shaft end, model manipulation is needed to provide the same overhung moment. One method to accomplish this is to include a “virtual” element with very low mass and very high modulus of elasticity to locate the center of gravity.

Particular machines will have specific masses that must be added, including the following:

- a) armature windings in electric motors*,
- b) shrunk-on gear meshes*,
- c) wet impeller mass and inertia in pumps*,

NOTE Items marked with (*) are typically represented by mass and inertia.

It is imperative that the rotor model properly account for these masses and any additional rotating masses that may be peculiar to a particular system. Some of these rotor modeling aspects that are specific to different machine types are discussed in Section 2.8.

2.3.4 Addition of Stiffening Due to Shrink Fits and Irregular Sections

Most rotating assemblies have nonintegral collars, sleeves, impellers, and so forth that are shrunk onto the shaft during rotor assembly. If the amount and length of the shrink fit and the size of the shrunk-on component are sufficiently large, then the shrunk-on component must be modeled as contributing to the shaft stiffness. The vendor must determine the importance of shrink fits for particular cases. Often, this can be accomplished only by experience with units of similar type. Smalley et al. [3] provide an empirical method based on static deflections. A modal test of a vertically or horizontally hung rotor will give some indication of the stiffening effect of shrunk-on components, but such measurements will likely exaggerate such effects because the fits will tend to be relieved as a result of centrifugal growth at normal operating speeds.

In some cases, a shaft segment consists of a series of short grooves and steps. Such segments are often found on turbine shafts at main labyrinth packing locations. Since the change in the diameter of the shaft segment encompassing the grooves does not affect the rotor critical speeds and the associated mode shapes, the stepping segment is usually simplified as one element using the average shaft diameter to evaluate the mass and bending stiffness of the stepping shaft segment.

Noncircular rotor cross-sections are common in the midspan areas of electric motors and generators. These electrical machines frequently possess integral or welded-on arms in the midspan area to support the rotor core. These structures add significant stiffening to the rotor midspan. This contribution to the lateral bending stiffness of the rotating assembly must be accounted for, as it is incorrect to model the stiffness of motor rotors using the base shaft only. Older steam turbines of built-up construction may also possess noncircular midspan rotor cross-sections. Additional details on these machinery-specific modeling aspects are given in Section 2.8.

2.3.5 Location of Bearings, Seals, and Radial Probes

It is well understood that bearings and seals can dramatically alter the vibration behavior of a rotating machine. Each fluid film support bearing and floating ring oil seal is typically represented using a set of eight linearized dynamic coefficients. The linearized models of the bearings and oil seals are assumed to act at the centerlines of the associated bearing and sealing lands.

A dry gas seal only adds lumped external mass and inertia to the shaft station located at the center of gravity of the dry gas seal's rotating assembly. The principle stiffness and damping of gas seals are relatively small, and are not included in the rotordynamics model for lateral critical speed and unbalance response analyses.

Other important, tight clearance seal locations within a machine should also have their locations included as additional rotor model stations. The unbalance response analysis must ensure these locations are not easily susceptible to rubbing. In the case of abrasable, floating ring and compliant seals, the vibration amplitude at these locations should be calculated, but it may be allowable for the level of the amplitude to exceed the minimum running clearances in these locations. The linearized models of seals are assumed to act at the centerlines of the associated sealing lands.

The axial locations of the radial bearing probes' centerlines should be included as additional stations in the rotor model, as this is where the vibration of the rotating machine will be measured.

2.3.6 Determination of Material Properties

The material properties required to generate the rotor model are density, Young's modulus of elasticity, shear modulus and Poisson's Ratio.

Manufacturers must determine the importance of temperature on shaft material properties for particular applications. For high-temperature cases, vendors should be aware of the influence of temperature on the elastic modulus of shaft materials in the rotor lateral models. Sometimes temperature variation exists along the shaft, resulting in the elastic modulus changing along the axial length. The shafts of most rotating machinery are made of carbon and low-alloy steels, such as AISI 4340 and ASTM A470. The properties of carbon and low-alloy steels change considerably over the temperature range from 0 °F to 650 °F. Figure 2-4 shows the trends of the changes in elastic modulus with temperature for several low-alloy steels, including AISI 4340, which are determined during "static" tensile loading and dynamic loading [4].

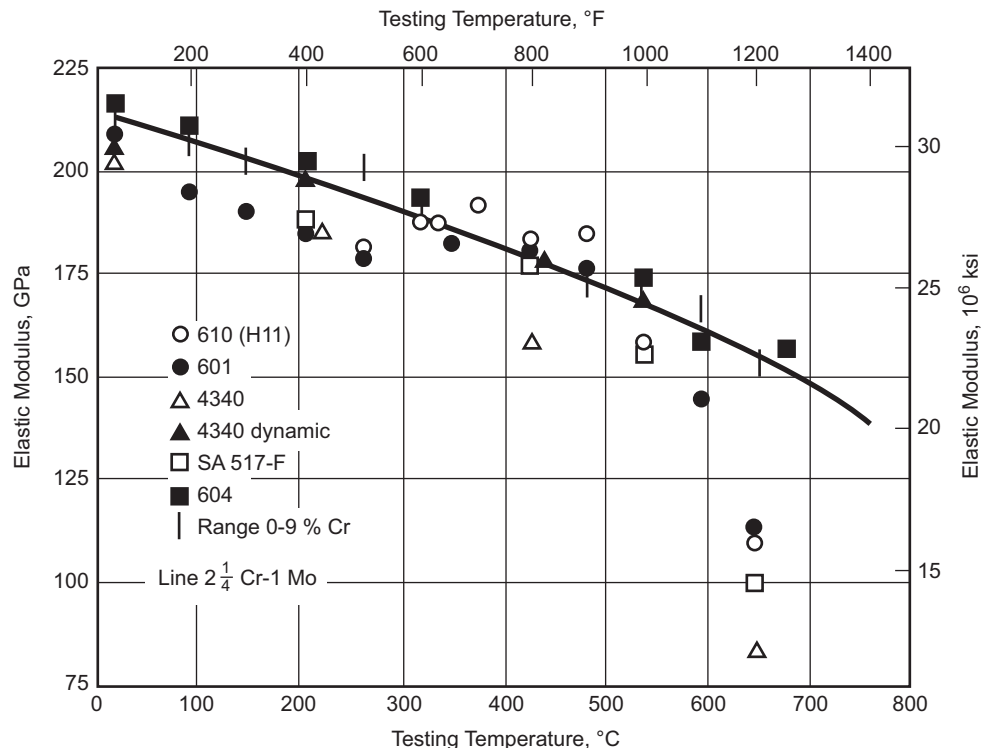


Figure 2-4—Elastic Modulus vs. Temperature [4]



2.3.7 Rotor Model Example

Results of the complete modeling process are displayed for an eight-stage 12-megawatt (16,000-horsepower) steam turbine rotor. A larger version of this drawing was used to describe shaft geometry. The measured rotor weight was used to check the results of the modeling process. The resulting tabular description of the model is presented in Table 2-1. Note that the translational and rotational inertias shown in this table are formed by the sum of externally applied inertias (from turbine blades and disks) and shaft inertias calculated for each of the shaft sections. A cross-section of the rotor model is displayed in Figure 2-5.

2.3.8 Built-up Rotors

Some rotors are constructed of axially segmented sections, which are stacked and bolted together. The turboexpander in Figure 2-6 and the turbocompressor in Figure 2-7 are examples of built-up rotors. The segments are radially located by either rabbet or spline fits and are held together axially by through bolts. When properly designed, the joints are very stiff, and can be approximated as being an integral piece of metal.

Rotor modeling of a curvic coupling joint is illustrated in Figure 2-8. The hatched areas represent the elements used to model the rotor stiffness, while the unhatched areas are modeled as masses and inertias only. These methods have been shown to be sufficiently accurate when the rotor is properly designed, and assembled with appropriate preloading of the stacked components. While a more precise joint model can be obtained by using finite element methods or by verifying through modal analysis testing, such methods are typically unnecessary.

2.3.9 Train Lateral

2.3.9.1 General

Most machinery trains consist of a series of two-bearing rotors connected by spacer-type, flexible couplings as shown in Figure 2-9. Individual units are typically isolated from each other dynamically by the flexible coupling such that they can be treated as separate units. A lateral rotordynamic analysis of the entire coupled system (train lateral analysis) is therefore rarely required. There are, however, unique circumstances in which the train lateral analysis should be considered.

- 1) The coupling spacer natural frequency is within the region shown in Figure 2-10 as 'train lateral recommended'.
- 2) The coupling is not of the flexible spacer type. These situations would include hard coupled turbomachinery, typically consisting of a single fixed joint, as well as piloted and unpiloted spline couplings.

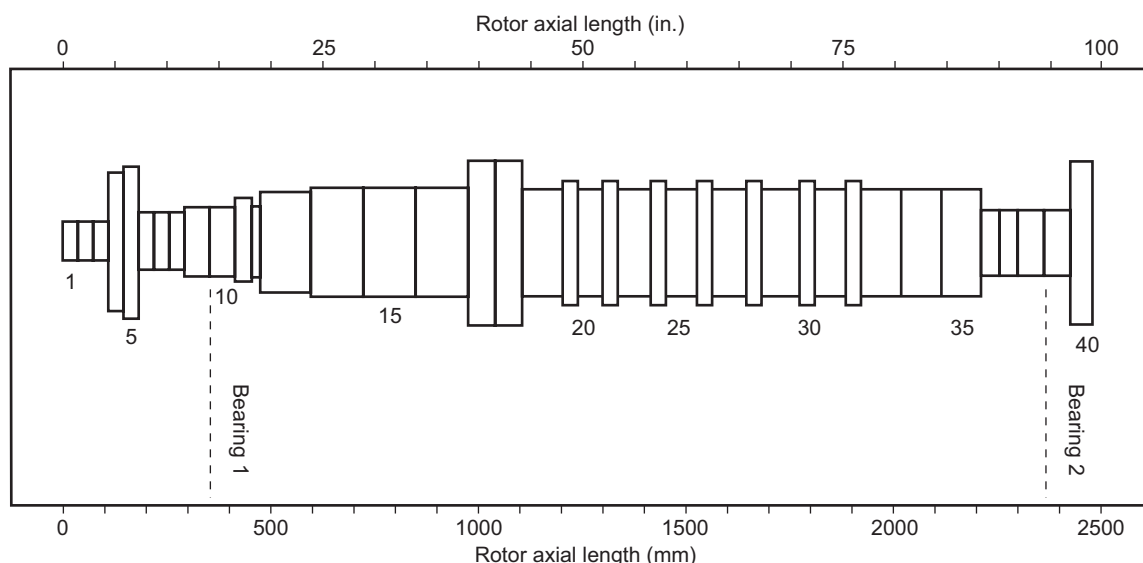


Figure 2-5—Rotor Model Cross-section of an Eight Stage 12 MW (16,000 HP) Steam Turbine

Table 2-1—Computer Model Generated for the Eight-Stage Steam Turbine Rotor

Station No.	Axial Location (in.)	Wt. (lbm)	Length (in.)	Shaft OD (in.)	Shaft ID (in.)	I (in. ⁴)	Polar Inertia, IP (lbm-in. ²)	Transverse Inertia, IT (lbm-in. ²)	Ex10 ⁻⁶ (lb/in. ²)
1	0.00	1.762	1.500	3.25	0	5.48	2.321	1.491	28.7
2	1.50	3.523	1.500	3.25	0	5.48	4.652	2.991	28.7
3	3.00	3.523	1.500	3.25	0	5.48	4.652	2.991	28.7
4	4.50	23.786	1.300	12.35	0	1140.08	421.909	214.386	28.7
5	5.80	47.029	1.200	13.69	0	1724.18	1005.320	508.760	28.7
6	7.00	29.793	1.723	5.00	0	30.68	600.712	304.538	28.7
7	8.72	9.579	1.723	5.00	0	30.68	29.941	17.336	28.7
8	10.45	9.579	1.723	5.00	0	30.68	29.941	17.336	28.7
9	12.17	13.195	2.100	6.00	0	63.62	52.789	30.671	28.7
10	14.27	17.970	2.390	6.00	0	63.62	80.869	48.077	28.7
11	16.66	17.007	1.190	7.50	0	155.32	95.364	53.119	28.7
12	17.85	12.185	1.010	6.50	0	87.62	77.377	39.974	28.7
13	18.86	43.105	4.260	9.00	0	322.06	413.466	265.154	28.7
14	23.12	117.719	5.680	10.00	0	490.87	906.536	563.578	28.7
15	28.80	109.175	4.910	10.00	0	490.87	1364.690	901.689	28.7
16	33.71	109.172	4.910	10.00	0	490.87	1364.650	901.648	28.7
17	38.62	515.401	2.500	15.00	0	2485.05	1402.150	811.401	28.7
18	41.12	515.404	2.500	15.00	0	2485.05	2629.720	1316.590	28.7
19	43.62	59.702	3.570	10.00	0	490.87	1215.930	650.785	28.7
20	47.19	138.463	1.250	11.25	0	786.28	9424.440	4816.280	28.7
21	48.44	127.568	2.590	10.00	0	490.87	9288.250	4722.120	28.7
22	51.03	157.523	1.570	11.57	0	879.64	12322.200	6252.180	28.7
23	52.60	158.524	2.680	10.00	0	490.87	12334.700	6260.170	28.7
24	55.28	162.368	1.560	11.56	0	876.60	13086.000	6636.280	28.7
25	56.84	165.258	2.940	10.00	0	490.87	13122.200	6660.060	28.7
26	59.78	170.604	1.570	11.57	0	879.64	14030.000	7115.410	28.7
27	61.35	171.937	3.060	10.00	0	490.87	14046.700	7126.750	28.7
28	64.41	174.432	1.570	11.57	0	879.64	14517.900	7362.870	28.7
29	65.98	177.100	3.300	10.00	0	490.87	14551.300	7386.300	28.7
30	69.28	177.100	1.570	11.57	0	879.64	14551.300	7386.290	28.7
31	70.85	170.763	2.730	10.00	0	490.87	14472.100	7332.250	28.7
32	73.58	197.509	1.880	11.88	0	977.77	17066.500	8649.220	28.7
33	75.46	211.795	4.015	10.00	0	490.87	17245.100	8779.610	28.7
34	79.47	146.058	4.785	10.00	0	490.87	1115.940	677.903	28.7
35	84.26	32.353	2.240	9.00	0	322.06	423.397	220.752	28.7
36	86.50	27.875	1.640	6.50	0	87.62	244.917	132.617	28.7
37	88.14	15.406	1.640	6.50	0	87.62	81.369	44.136	28.7
38	89.78	19.540	2.520	6.50	0	87.62	103.196	59.591	28.7
39	92.30	24.801	2.760	6.50	0	87.62	130.976	79.978	28.7
40	95.06	111.748	2.000	21.62	0	10724.00	314.579	165.750	28.7
41	97.06	77.957	0.000	21.62	0	0.00	4554.850	2292.050	28.7
4475.292 97.059									
Bearing Reactions: 2190.92 lbm at Station 10 2284.38 lbm at Station 39									

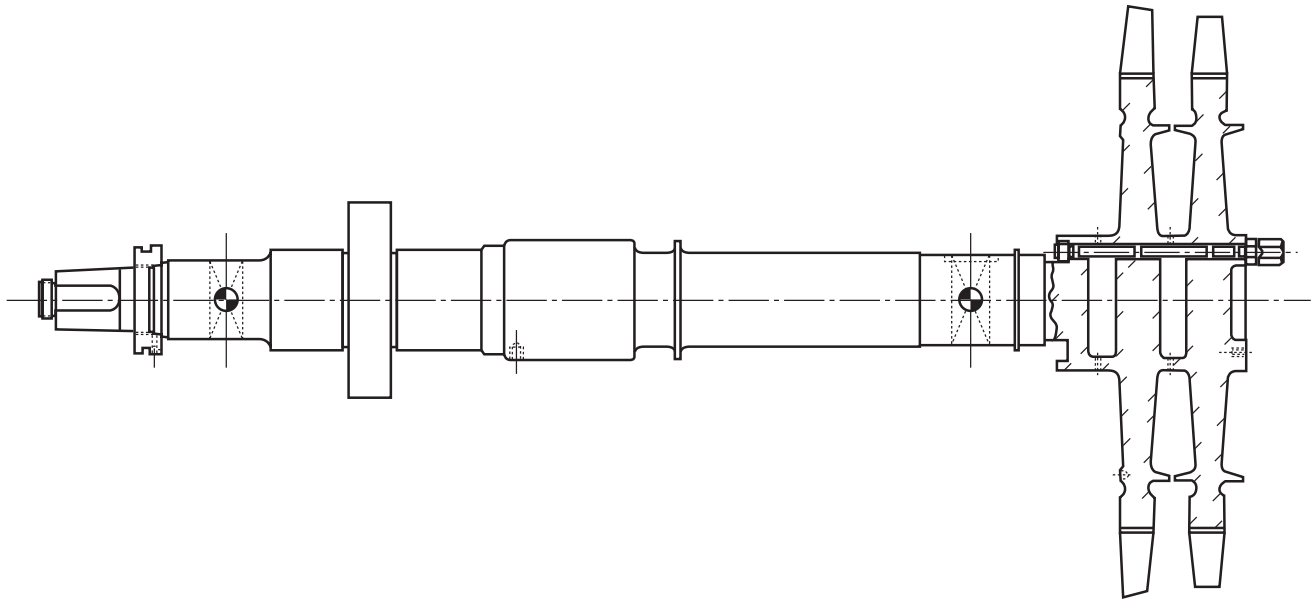


Figure 2-6—Turboexpander with Curvic Coupling Fits

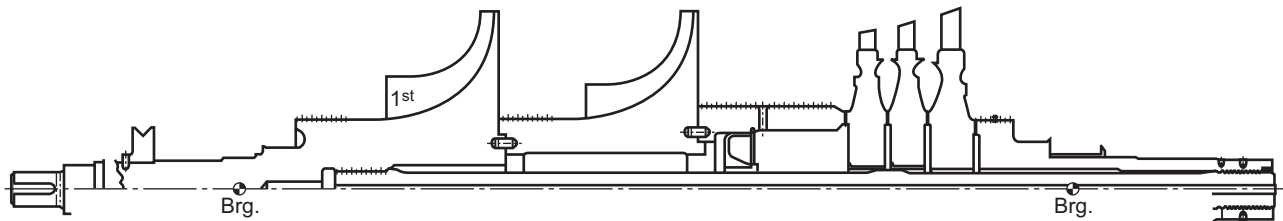


Figure 2-7—Turbocompressor with Rabbet and Curvic Coupling Fits

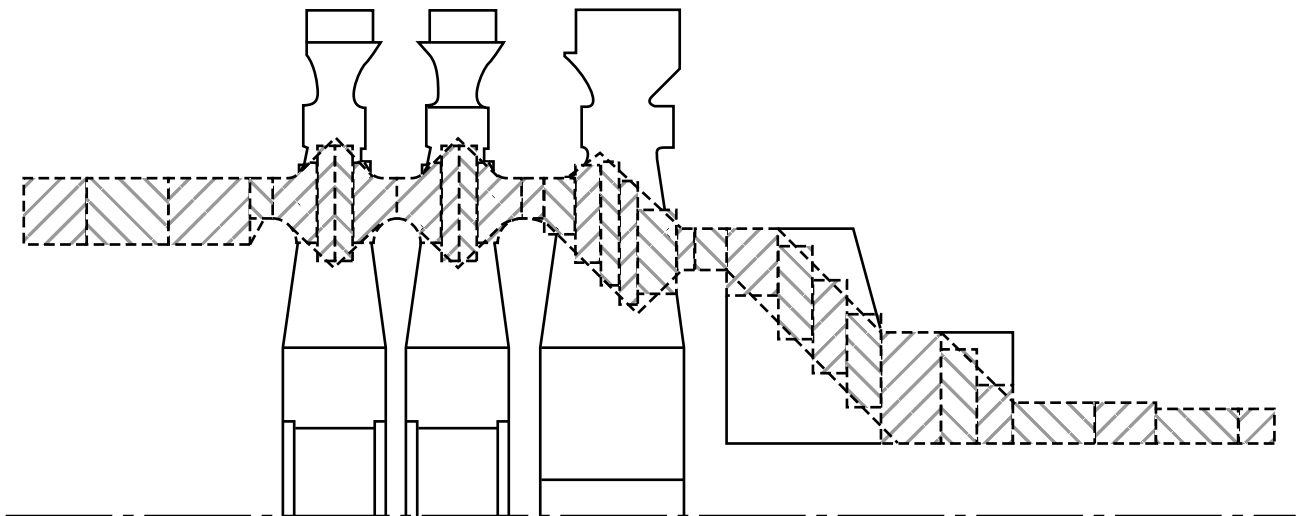


Figure 2-8—Modeling of Curvic Coupling Joints

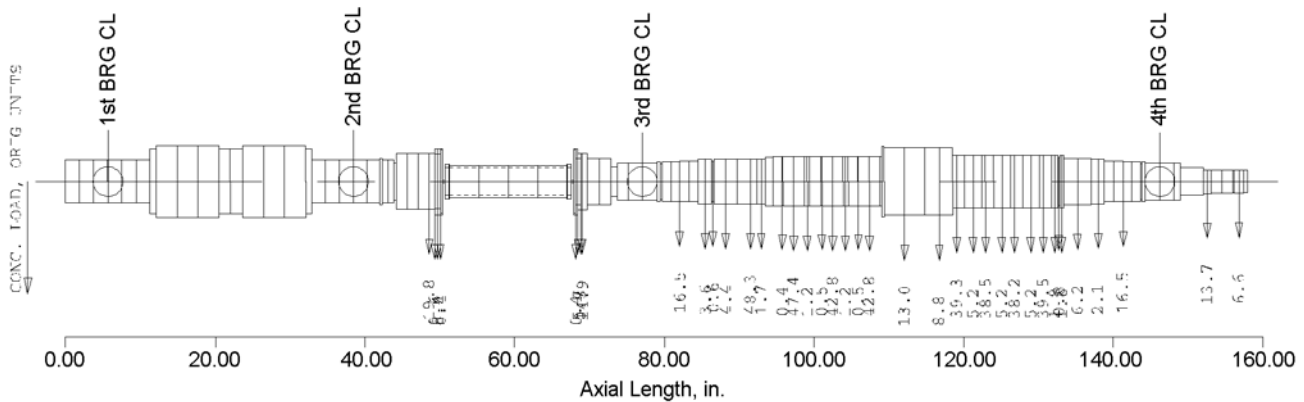


Figure 2-9—Train Lateral Model

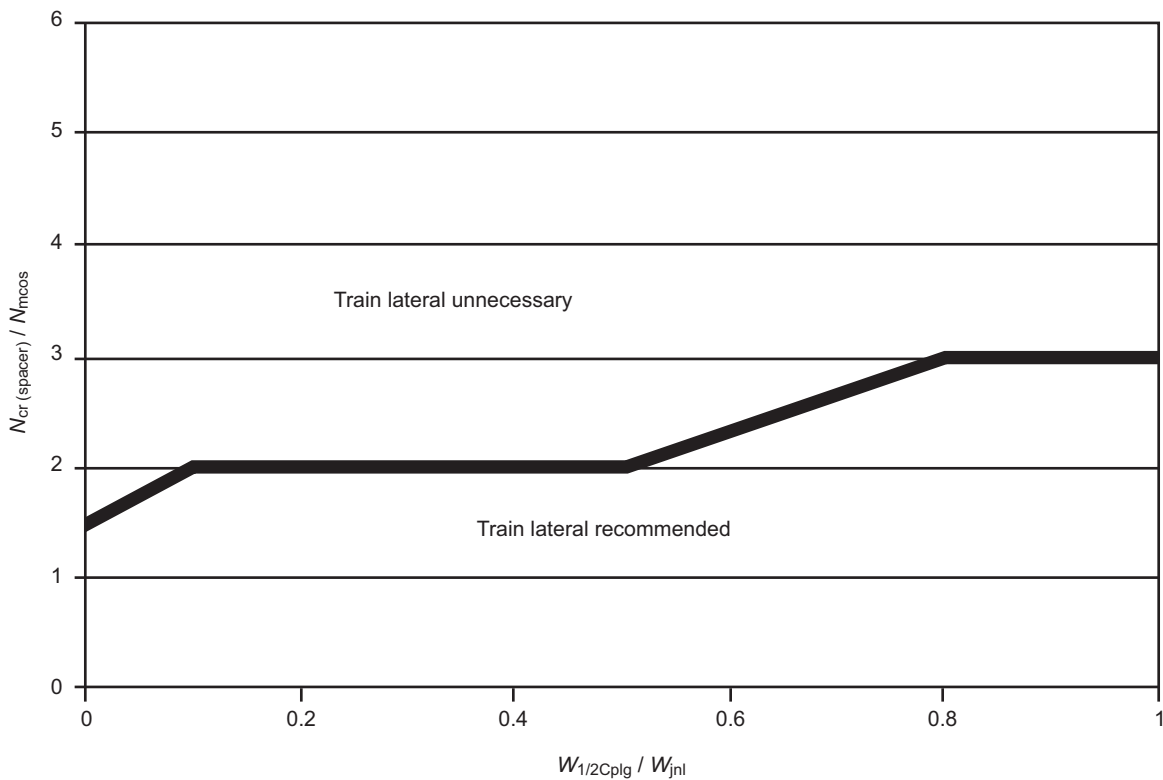


Figure 2-10—Train Lateral Guideline Diagram (W_{int} = Static Bearing Reaction) [5]

Note that Figure 2-10 shows an increase in the recommended coupling critical speed ratio (coupling spacer critical divided by the train maximum continuous speed) from 2.0 to 3.0 for coupling half weights that approach the journal load. This reflects the increased influence of the coupling on train dynamics as the coupling weight approaches the journal static reaction. For most petrochemical turbomachinery applications, however, the coupling weight is a small percentage of the rotor weight and a ratio ≥ 2.0 is recommended. For ratios less than 2.5, a simply supported tube calculation for the coupling critical speed is not sufficient.

For trains with more than two bodies, a partial train lateral analysis consisting only of the bodies meeting the specified criteria is acceptable, since other units in the train, connected by couplings with adequate spacer natural frequencies, would be sufficiently isolated and have little effect on the rotordynamic characteristics of the remaining train.

Figure 2-11 displays the first four mode shapes of a three body string representing the first critical speed of each body and the second critical of the gas turbine. The first unit is a gas turbine supported by bearings 1, 2, and 3. The second unit is a compressor supported by bearings 4 and 5. The third unit is a motor supported on bearings 6, 7, and 8. The gas turbine and compressor are joined by a coupling with spacer natural frequency 1.67 times running speed. The compressor and motor are joined by a coupling with spacer natural frequency 10.28 times running speed.

The mode shapes indicate that the coupling between the gas turbine and compressor has significant modal displacement. The seventh mode (2680 cpm) is the bending mode of this coupling in combination with the shaft ends. In addition, there is some modal participation between the compressor and gas turbine at the 6th mode (2155 cpm), which must be traversed. By contrast, the motor is well isolated from the compressor, and by the criteria listed, a partial train lateral analysis of the turbine and compressor is sufficient.

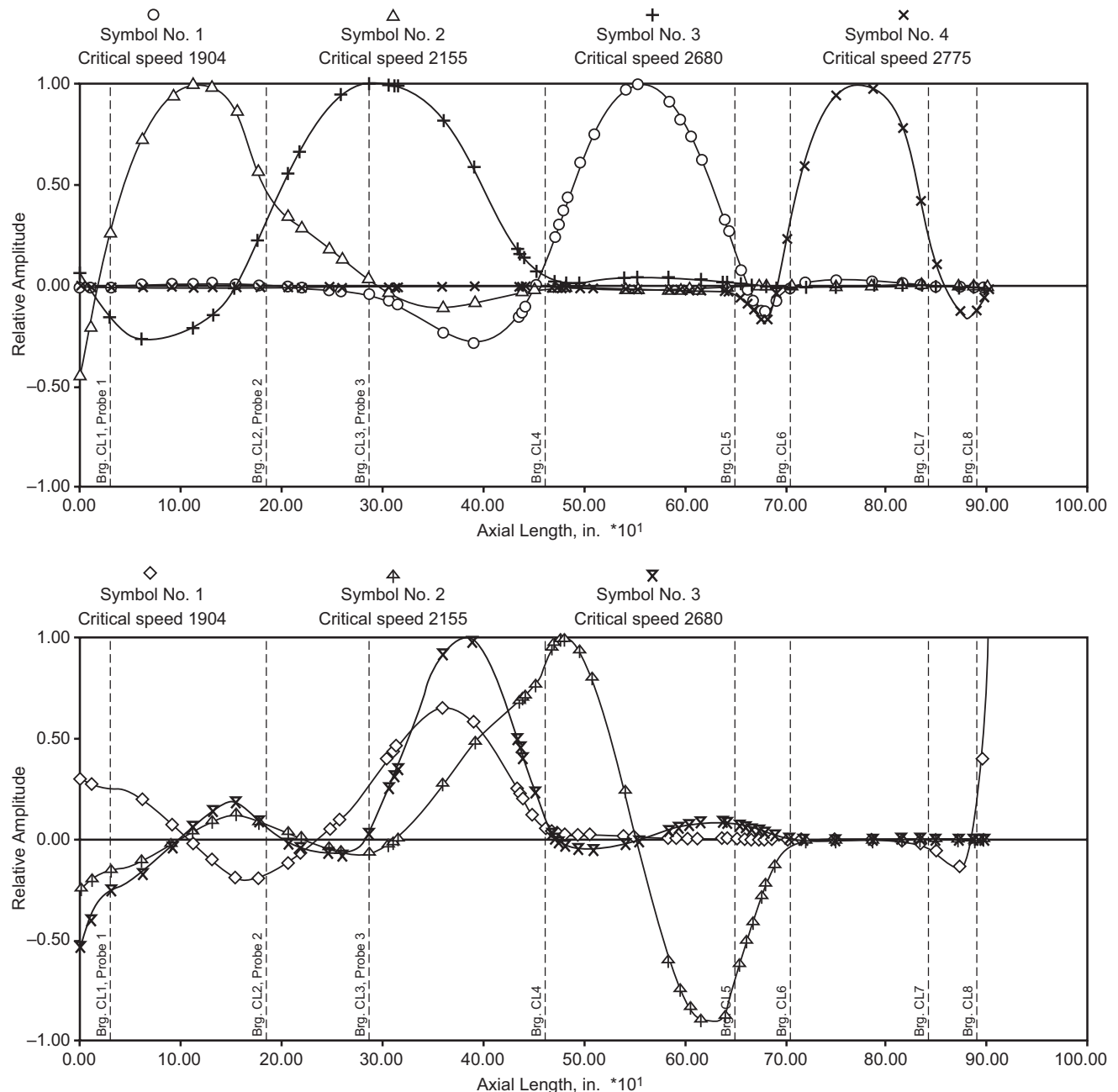


Figure 2-11—Train Lateral Modes

The lateral train analysis of a geared train, if performed can be divided into two subsystems:

- 1) Driver – Coupling – Gear
- 2) Pinion – Coupling – Driven Machine(s)

The bearing analyses of the gear and pinion are normally at the rated conditions.

Prior to performing a train lateral rotordynamic analysis, each of the individual unit rotordynamic analyses should be completed. The modeling of the train consists of coupling the unit models together through the coupling(s). The coupling half weight properties, as used in the unit model, are no longer relevant and the entire coupling including the hubs, flexible joints, and spacer are modeled instead.

2.3.9.2 Coupling Models

For flexible couplings, the hubs are assumed to be integral with the shaft ends while the flexible joints are modeled as either angular and radial springs or equivalent beam elements. The bending stiffness is to be supplied by the coupling vendor or calculated [6,7]. The spacer is modeled as a hollow shaft (tube) using the techniques described in Section 2.3.2. A model using equivalent beam elements is illustrated in Figure 2-12.

Gear type couplings are modeled the same as flexible couplings, except that the joints are assumed to have no bending moment transfer and can be modeled with either zero or very low bending stiffness.

Solid flange couplings do not correspond to API 671 standards. For these couplings, the coupling hubs are assumed integral to the shaft end, and the flange portions are modeled as an equivalent beam element or bending spring. These data are either supplied by the coupling vendor, derived by finite element analysis or calculated based on either analytical or empirical methods [7,8,9,10].

Spline type couplings are typically modeled with separate radial and torsional stiffnesses at the joints [11].

2.3.9.3 Static Bearing Loads

The static bearing loads of the string are handled by superposition. That is, the load at each bearing would be the same as the uncoupled case for a multi-span system where each span is supported by two bearings. For multi-bearing and rigid coupled systems, the bearing loads should be determined by including static sag and misalignment effects.

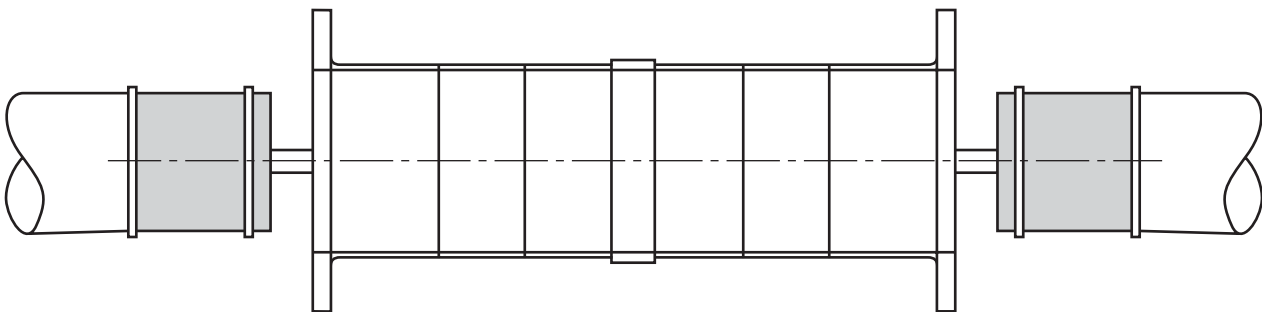


Figure 2-12—Equivalent Coupling Model

2.3.10 References

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2.4 Support Stiffness Effects

2.4.1 Introduction

The accurate modeling of the effective bearing stiffness and damping coefficients acting upon the rotor is required to accurately predict the lateral rotordynamics. An important consideration in predicting these *effective* coefficients is the influence of the pedestal support stiffness and damping characteristics. These support characteristics act in series with the bearing characteristics and the combined effective characteristics, i.e. bearing stiffness and damping properties acting on the rotating shaft, may be quite different from either of these [1-6,16]. The analysis of machine vibration response based on rigid bearing supports usually predicts critical speeds that are substantially higher than actual values [1-4]. The dynamic support stiffness, combined with bearing stiffness and damping, produces effective stiffness and damping coefficients that are generally lower than those for the bearing alone, resulting in lower critical speeds than would occur if the supports were rigid.

For oil film bearing supported machines, Nicholas and Barrett [3] found that for the four rotors analyzed, neglecting support flexibility resulted in predicted first critical errors that range from 14 % to 21 % high and second critical errors that range from 40 % to 88 % high. In addition, the dynamic support flexibility produces effective damping coefficients that are lower than the bearing damping coefficients alone, resulting in larger amplification factors than would occur if the supports were rigid.

Some high-speed machines operate on ball bearings. These machines include micro-turbines (in applications for oil and gas) and gas generators and power turbines, among others. Ball bearings are usually very stiff compared to fluid film bearings and can be regarded as rigid connections in most cases. This makes substructure representation important in these cases. In addition, a roller bearing supported rotor may rely on structural damping to control rotor vibration. In this case, accounting for support dynamic characteristics is required to accurately predict rotordynamic performances.

Also, for active magnetic bearing (AMB) supported rotors, the effects of support flexibility might need to be considered and included in the rotordynamic model. The issues are discussed in more detail in the AMB section of this tutorial.

In addition to the bearing type, a machine's sensitivity to bearing supports dynamic characteristics is dependent on the rotor properties, machine construction and excitation mechanisms. For this reason, different requirements are set by API standards in terms of support dynamic effects. For instance, in the API 617 standard for axial and centrifugal compressors, it is required to account for supports' dynamic stiffness and damping in the rotordynamic analysis, when the ratio between the bearings supports and the bearing stiffness is equal or lower than 3.5. Furthermore, it requires avoidance of support resonances within machine operating speed range. In the API 546 standard for brushless, synchronous machines, some stricter requirements are imposed for separation margins between frame and foundation system natural frequencies relative to electrical line frequency, operating speed and its multiples. These are given to avoid structural modes' excitation within electrical machinery.

Since rotating machinery is designed, marketed and sold, for the most part, based on analytical predictions, an accurate method of easily incorporating the support flexibility effect into rotordynamic analyses is of paramount importance. Therefore, industrial practices have advanced to include the effects of support flexibility into rotordynamic analyses. A basic and typical method is to model the supports with stiffness and damping coefficients which are constant over the entire speed range [3,7]. In most cases, the support stiffness is based on static deflections of the bearing pedestal (experimentally and/or analytically calculated). While this approach can be successfully utilized to predict both the location and amplification of rotor critical speeds [3], it will not show more than a single support or foundation resonance.

Detailed models of support structures have been incorporated into rotordynamic analyses in an effort to predict the support-rotor resonance interactions. The usual approach has been to use a modal model from a finite element analysis of the structure. Li and Gunter [8] use a component mode synthesis technique whereas Queitzsch [9] uses analytical frequency response functions (FRF) to represent the supporting structure. These methods have been proven successful, but they may be time-consuming.

The method proposed by Nicholas, Whalen and Franklin [1] utilizes experimental FRF data to represent the bearing support structure. The experimental data are determined from modal analysis techniques where the response of the structure to a known force is measured. The resulting FRF data are plotted as a function of frequency. If the units of the measured FRF function are displacement divided by the force, the resulting data are called "dynamic compliance" [10].

The application of experimental FRF data to rotordynamic analyses has been discussed previously [1,2,4,5,11,16,17]. One of the biggest advantages of this method is that the support mass is included implicitly in the FRF data along with the support stiffness [1]. The FRF data can easily be incorporated into the rotordynamic support model used in References [1,2,3,5,7,15,16,17], either as a constant dynamic stiffness over a narrow speed range or as a frequency dependent dynamic stiffness over the entire speed range [1].

The modal analysis technique used in determining the dynamic compliance data is detailed herein and in Reference [1,15]. The data can then be employed in a forced response rotordynamic analysis, using various levels of flexible support model sophistication. Example results are compared to actual test stand speed amplitude plots from a steam turbine running on the test stand with a known midspan unbalance.

2.4.2 Representation of Support Dynamic Characteristics

A typical outline drawing of a steam turbine case is shown in Figure 2-13. The steam end bearing is housed in a bearing case that is supported by a flex plate to allow for axial thermal expansion. The exhaust end bearing case is supported within the exhaust casing which sits on two sets of thick horizontal plates with gussets for added stiffness. These plates along with the flex plate are attached to the baseplate.

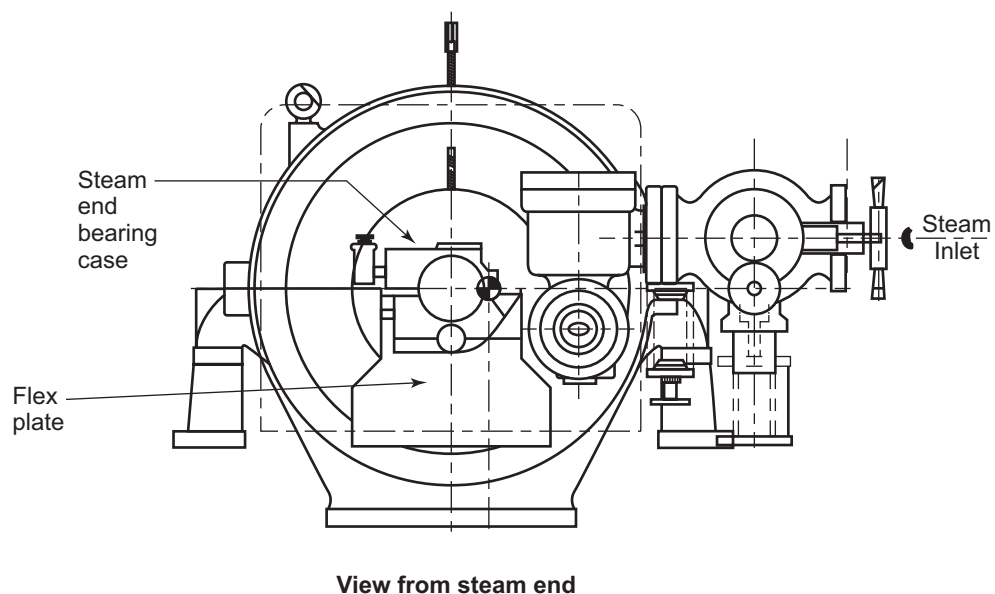
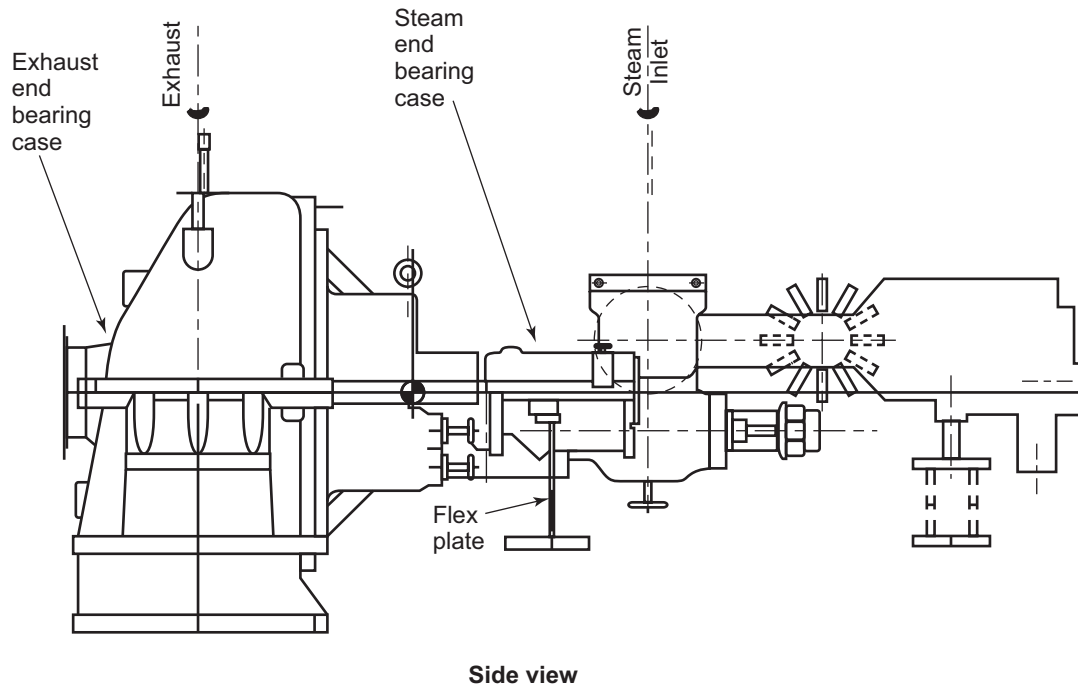


Figure 2-13—Steam Turbine Support Schematic

A model for this complex support is illustrated in Figure 2-14. The first level of flexibility is the bearing fluid film which is represented by eight principal (xx,yy) and cross-coupled (xy,yx) stiffness and damping coefficients. For tilting pad bearings, the second level of flexibility is the pad [6] and the pad pivot [12]. This effect may be accounted for in the tilting pad bearing analysis [3,6,11,12]. The next level of flexibility is everything past the pad pivot. This may include the bearing case, the supporting plates, the baseplate, the columns and the foundation. In some instances, in addition to the direct stiffness and damping components, the support can be characterized by a cross-coupling effect among orthogonal axes on the same support, or even crosstalk between different supports through machine or support structure.

Each component of the dynamic system (rotor, bearings, supports and foundation) is characterized by its own dynamics, thereby influencing, to some extent, the dynamics of the overall system. Thus, to determine the dynamic performances of the entire system, the dynamics characteristics of each single component needs to be considered. Usually, this results in combining the dynamic properties of each component into a single equivalent dynamic system for rotordynamic analyses.

Several methods exist to combine the effects of supports dynamics into rotordynamic analyses, each of them resulting in a different level of complexity and accuracy on vibration performance prediction. The modeling approaches can be distinguished based on the level of dynamic coupling within and between pedestals (direct terms only, with cross-coupling, with crosstalk between pedestals) as well as by the approach used to analytically combine the rotor/bearing and support system dynamics. Examples of system integration approaches include the use of full support matrices (K, C, M), state space representation, modal methods, transfer functions, and single degree of freedom (SDOF) approximations.

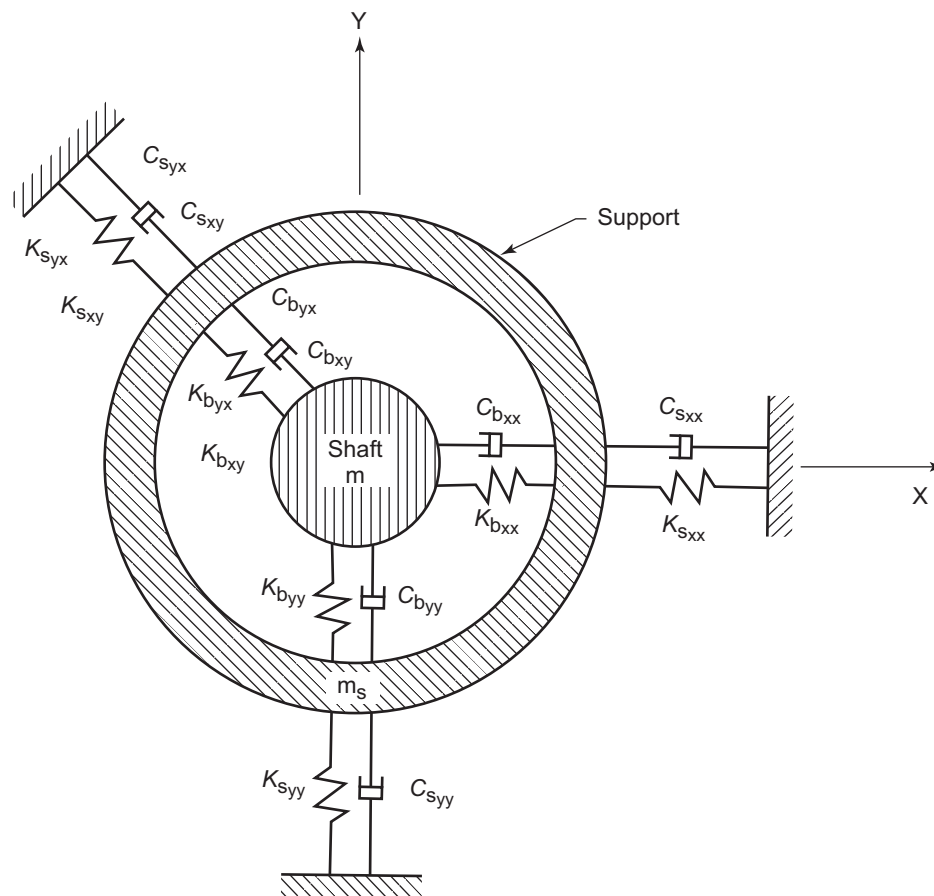


Figure 2-14—Journal Bearing Fluid Film and Flexible Support Model

Application of each method is dependent on the origin of the support stiffness dynamic properties (i.e. numerical or experimental) and on the level of accuracy required by the analysis and the machine's sensitivity to the supports' dynamics.

2.4.3 Dynamic Support Stiffness Characterization

In general, the support flexibility is dynamic. That is, the support stiffness and damping properties are a function of the applied excitation frequency, ω .

Predicting the support stiffness and damping characteristics is not always straightforward. Although it is possible to make reasonable assumptions about the dynamic characteristics for some support configurations, most situations require the characteristics to be predicted from finite element analyses of the support structure or to be determined from test data. Further complicating the situation is the fact that the support characteristics may vary with the vibration frequency of the rotor.

When experimentally determining the support FRF, several classical and advanced techniques are available and implemented in commercial instrumentation and identification software. The identification can be conducted in difficult environmental conditions or even in operation [10].

The block diagram of the test system used by Nicholas et al. [1] to determine the stiffness and damping properties of an actual bearing support is illustrated in Figure 2-15. An impact hammer is used to excite the bearing case at the bearing centerline. An internal load cell registers the force imparted on the bearing case by the hammer. Mounted on the case at the bearing centerline is an accelerometer that senses the case motion that results from the impact force. The force and acceleration signals are used as input to a frequency or spectral analyzer. The acceleration is double integrated and the resulting displacement is divided by the force from the impact hammer. Using these signals over a specified frequency range, the dynamic compliance FRF, $\alpha(\omega)$, is calculated and is a complex quantity, containing both amplitude and phase information. The inverse of the dynamic compliance is the dynamic stiffness, $K_{\text{dyn}}(\omega)$:

$$K_{\text{dyn}}(\omega) = \frac{1}{\alpha(\omega)} \quad (2-1)$$

Equivalent support stiffness and damping can be found from:

$$k_s(\omega) = \text{Re}(K_{\text{dyn}}(\omega)) \quad (2-2)$$

$$c_s(\omega) = \frac{\text{Im}(K_{\text{dyn}}(\omega))}{\omega} \quad (2-3)$$

These calculations can be used to determine the support characteristics at each vibration frequency being analyzed and combined with the bearing coefficients at each frequency in order to perform rotordynamic calculations.

By measuring the pedestal response orthogonal to the applied excitation force, but on the same pedestal, the cross-coupled compliance is obtained experimentally. Conversely, when the excitation force and the measured vibration response are located at different pedestals, crosstalk FRFs can be obtained.

An example of a compliance FRF plot is shown in Figure 2-16 for a steam turbine case. The exhaust end vertical compliance results from a vertically mounted accelerometer sensing vertical acceleration from a vertical excitation (principal compliance). Two different excitation sources are shown: an impact hammer and an electromagnetic exciter or shaker. Note that for frequencies below 200 Hz (12,000 cpm), both excitation sources give very nearly identical results. The impact hammer offers the advantage of being significantly quicker to set up and conduct the actual dynamic compliance testing.

The compliance plot in Figure 2-16 plots the *magnitude* of the complex compliance FRF.

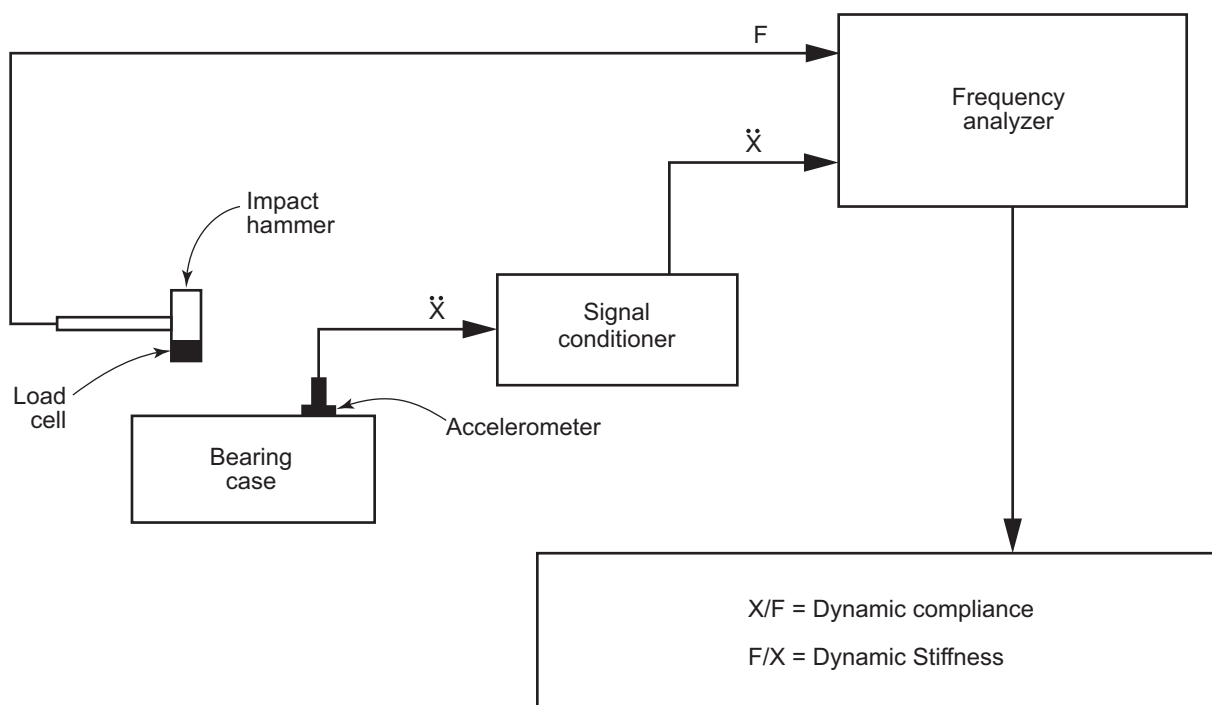


Figure 2-15—Dynamic Stiffness Analysis Diagram

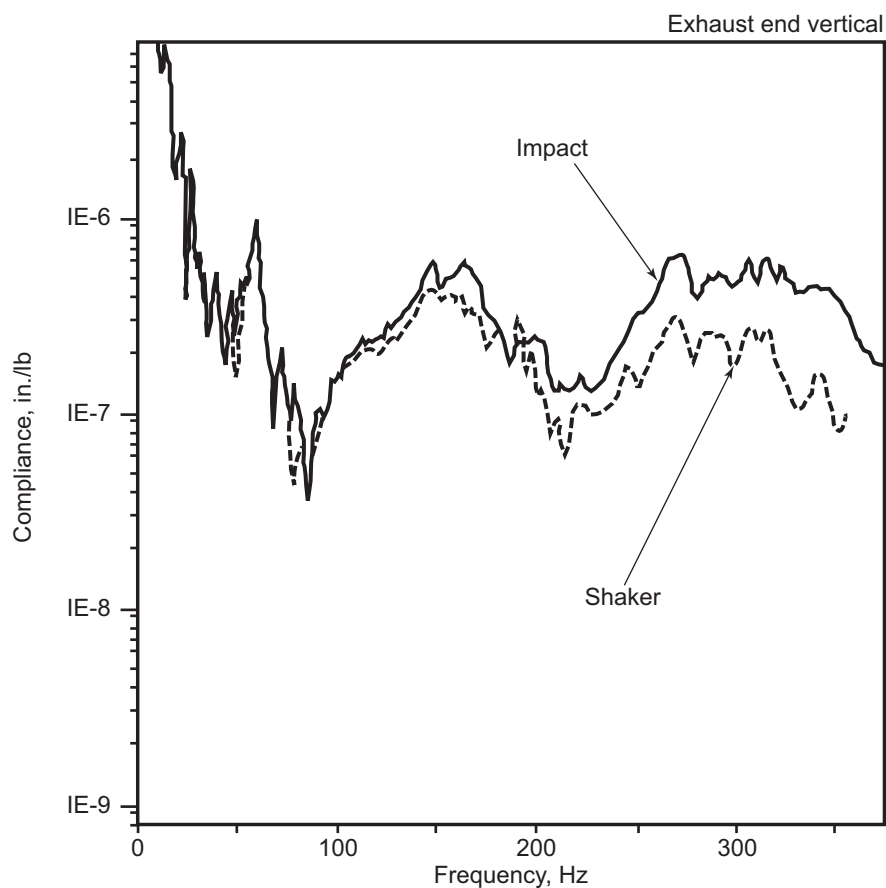


Figure 2-16—Exhaust End Dynamic Compliance Plots

When the pedestal dynamics is represented by a SDOF spring-mass-damper system, as discussed in [1,10], the magnitude of the dynamic compliance is:

$$|\alpha(\omega)| = \frac{|X|}{|F|} = \frac{1}{\sqrt{(K_s - m_s \omega^2)^2 + (C_s \omega)^2}} \quad (2-4)$$

where

F is the applied force in N (lbf);

X is the resulting displacement in meters (in.).

Inverting, the magnitude of the dynamic stiffness is obtained:

$$|K_D(\omega)| = \frac{|F|}{|X|} = \sqrt{(K_s - m_s \omega^2)^2 + (C_s \omega)^2} \quad (2-5)$$

Thus, for small support damping, the dynamic stiffness' magnitude becomes:

$$|K_D(\omega)| = |(K_s - m_s \omega^2)| \quad (2-6)$$

Note that since $|K_D(\omega)|$ is an absolute value, the magnitude can never be negative.

2.4.4 SDOF Representation of Support Dynamic Effects

A simplified support model for a rotor supported on tilting pad journal bearings is shown in Figure 2-17. The single support mass with two degrees of freedom model illustrated in Figure 2-14 is reduced to two single degrees of freedom (SDOF) support spring-mass-damper systems in the horizontal X and vertical Y directions. Unlike the model in Figure 2-14, the SDOF models discussed from here on assume that the support has no cross-coupling. The X and Y direction equations are uncoupled since the cross-coupled terms are zero, or near zero, for tilting pad bearings. For illustrative purposes, only the Y direction is considered in Figure 2-17, but an identical system also exists in the X direction.

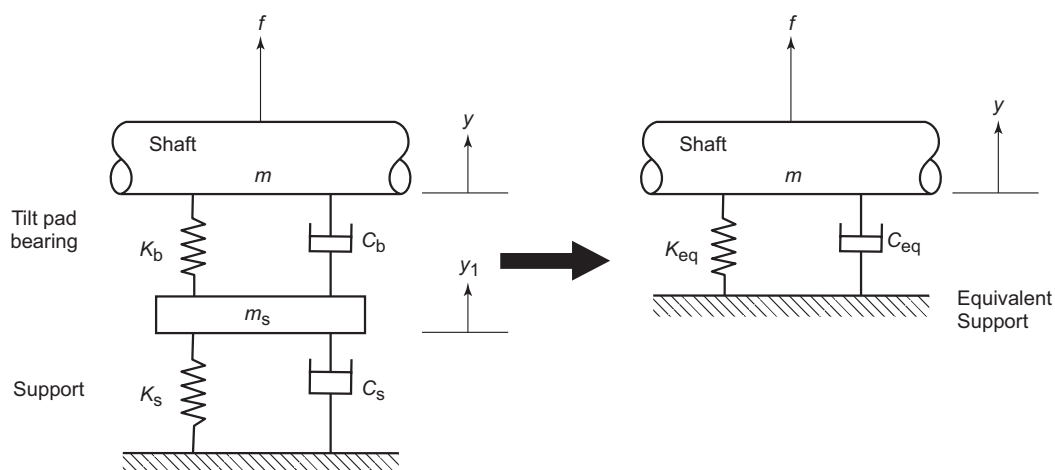


Figure 2-17—Single Degree of Freedom Flexible Support Model

The Y displacement shown in Figure 2-17 is the absolute rotor response; Y_1 is the support or pedestal response and $Y-Y_1$ is the relative rotor response. Since most vibration probes are mounted on the bearing case to monitor shaft motion, it is the relative response that is of primary importance for correlation purposes.

From Figure 2-17, the bearing stiffness and damping are combined with the support mass, stiffness and damping to yield an equivalent support model. In this model, the bearing stiffness and damping, K_b and C_b , are functions of the shaft rotational speed, Ω . The equivalent support properties are also speed dependent while the support stiffness and damping, K_s and C_s , may be constant or speed dependent. The combination of pedestal and bearing properties for tilt pad bearing with no cross-coupling terms, both on bearing and support, yield the equivalent stiffness and damping properties in horizontal (x) direction [1,3]:

$$K_{eq_{xx}} = \frac{\hat{K}_{s_{xx}} K_{b_{xx}} (\hat{K}_{s_{xx}} + K_{b_{xx}}) + \omega^2 (K_{b_{xx}} C_{s_{xx}}^2 + \hat{K}_{s_{xx}} C_{b_{xx}}^2)}{(\hat{K}_{s_{xx}} + K_{b_{xx}})^2 + \omega^2 (C_{s_{xx}} + C_{b_{xx}})^2} \quad (2-7)$$

$$C_{eq_{xx}} = \frac{K_{b_{xx}}^2 C_{s_{xx}} + \hat{K}_{s_{xx}}^2 C_{b_{xx}} + \omega^2 C_{s_{xx}} C_{b_{xx}} (C_{s_{xx}} + C_{b_{xx}})}{(\hat{K}_{s_{xx}} + K_{b_{xx}})^2 + \omega^2 (C_{s_{xx}} + C_{b_{xx}})^2} \quad (2-8)$$

and similarly for the vertical (y) direction:

$$K_{eq_{yy}} = \frac{\hat{K}_{s_{yy}} K_{b_{yy}} (\hat{K}_{s_{yy}} + K_{b_{yy}}) + \omega^2 (K_{b_{yy}} C_{s_{yy}}^2 + \hat{K}_{s_{yy}} C_{b_{yy}}^2)}{(\hat{K}_{s_{yy}} + K_{b_{yy}})^2 + \omega^2 (C_{s_{yy}} + C_{b_{yy}})^2} \quad (2-9)$$

$$C_{eq_{yy}} = \frac{K_{b_{yy}}^2 C_{s_{yy}} + \hat{K}_{s_{yy}}^2 C_{b_{yy}} + \omega^2 C_{s_{yy}} C_{b_{yy}} (C_{s_{yy}} + C_{b_{yy}})}{(\hat{K}_{s_{yy}} + K_{b_{yy}})^2 + \omega^2 (C_{s_{yy}} + C_{b_{yy}})^2} \quad (2-10)$$

In equations 2-7 through 2-10, $\hat{K}_{s_{xx}} = K_{s_{xx}} - m_{sx}\omega^2$ and $\hat{K}_{s_{yy}} = K_{s_{yy}} - m_{sy}\omega^2$. Note that when using experimental FRF data, it is not necessary to calculate the support mass (m_{sx}, m_{sy}) explicitly, since $\hat{K}_{s_{xx,yy}}$ is the real part of this function.

Equations and modeling techniques outlined by Barrett et al. [4] or by Vázquez et al. [5] can easily consider support cross-coupling in the horizontal and vertical directions, as well as crosstalk from one support to another.

Examples of using the SDOF model and equations 2-7 through 2-10 are summarized in the tables below for two steam turbines operating on tilting pad journal bearings [3]. Table 2-2 is for a 13.1 kN steam turbine running at 4291 rpm. Table 2-3 is for a 22.5 kN steam turbine running at 5833 rpm. Since the bearing case flexibility is modeled, the support mass is assumed to be the approximate bearing case mass of 227 kg for both turbines. A small amount of support damping is assumed.

From Table 2-2, the equivalent stiffness values are 8 % to 36 % lower while the equivalent damping values are 62 % to 74 % lower compared to the original bearing characteristics. From Table 2-3, the equivalent stiffness values are 2 % to 33 % higher while the equivalent damping values are 71 % to 77 % lower compared to the original bearing characteristics. Typically, support flexibility has a much more predominate effect on damping compared to stiffness. Table 2-3 actually shows an increase in equivalent stiffness while the equivalent damping decreases by more than 70 %.

Table 2-2—Bearing, Support and Equivalent Characteristics, 13.1 kN Steam Turbine

Parameter	Bearing	Support	Support/Bearing	Equivalent
K_{xx} (kN/m)	124,861	350,240	2.81	114,879
K_{yy} (kN/m)	225,905	350,240	1.55	145,700
C_{xx} (kN-s/m)	327	56.4	0.17	125
C_{yy} (kN-s/m)	434	56.4	0.13	112

Table 2-3—Bearing, Support and Equivalent Characteristics, 22.5 kN Steam Turbine

Parameter	Bearing	Support	Support/Bearing	Equivalent
K_{xx} (kN/m)	158,308	525,360	3.32	210,144
K_{yy} (kN/m)	224,154	525,360	2.34	227,656
C_{xx} (kN-s/m)	599	69.0	0.12	172
C_{yy} (kN-s/m)	687	69.0	0.10	161

2.4.5 SDOF Representation of Support Dynamic Effects Including Bearing Cross-Coupling

When stiffness and damping cross-coupling exists in the bearing oil film, e.g. when a fixed geometry bearing is present, the equivalent stiffness and damping equations become much more difficult since the X and Y direction equations of motion are coupled by the cross terms. Details of equivalent stiffness and damping equations including bearings cross-coupling terms are discussed in [15].

Defining the following additional terms:

K_{bxy}, K_{byx} is the bearing stiffness cross-coupling;

C_{bxy}, C_{byx} is the bearing damping cross-coupling;

$$A = K_{bxx} + i\omega C_{bxx}$$

$$B = K_{bxy} + i\omega C_{bxy}$$

$$D = K_{byx} + i\omega C_{byx}$$

$$E = K_{byy} + i\omega C_{byy}$$

$$G = K_{sxx} - m_{sx}\omega^2 + i\omega C_{sxx}$$

$$H = K_{syy} - m_{sy}\omega^2 + i\omega C_{syy}$$

$$R = (A + G) \times (E + H) - B \times D$$

Assuming the support has no cross-coupling, the equivalent stiffness and damping characteristics of the combined bearing support system become (based on work by Lloyd Barrett):

$$i\omega C_{eq_{xx}} + K_{eq_{xx}} = G \left(1 - \frac{G(E+H)}{R} \right)$$

$$i\omega C_{eq_{xy}} + K_{eq_{xy}} = \frac{BGH}{R}$$

(2-11)

$$i\omega C_{eq_{yx}} + K_{eq_{yx}} = \frac{DGH}{R}$$

$$i\omega C_{eq_{yy}} + K_{eq_{yy}} = H \left(1 - \frac{H(A+G)}{R} \right)$$

(2-12)

Unfortunately, it would be very cumbersome to separate the real and imaginary parts from equations 2-11 and 2-12 and solve for the equivalent properties by hand. However, they can easily be separated and solved for using a computer code or a commercial math program.

For zero bearing cross-coupling, $B = D = 0$, and equations 2-11 and 2-12 reduce to:

$$i\omega C_{eq_{xx}} + K_{eq_{xx}} = \frac{(A)G}{A+G}$$

(2-13)

$$i\omega C_{eq_{yy}} + K_{eq_{yy}} = \frac{(E)H}{E+H}$$

(2-14)

Now, the real and imaginary parts of equations 2-13 and 2-14 can be easily separated, resulting in equations 2-7 through 2-10.

2.4.6 Forced Response Correlation

A practical case where SDOF support model representation was successfully applied to predict the forced response of a real machine is reported by Nicholas et al. [1]. The machine investigated consisted of a nine stage 1097 kg (2418 lbm) steam turbine rotor operating on 5-pad tilting pad bearings with 127 and 102 mm (5.0 and 4.0 in.) diameter journals on the exhaust and steam ends, respectively. The rotor tested was run up to a trip speed of 6150 rpm with 16,560 g-mm (23 oz-in.) of unbalance placed at the center wheel rim.

The resulting speed-amplitude plot is shown in Figure 2-18 for the steam end probes (see Reference [1] for the exhaust end probe plots). The probes, which are mounted on the bearing case, are clocked 45° from top-dead-center and are referred to as “right probe” and “left probe.” From Figure 2-18, both probes exhibit split or dual first critical speed peaks. Split critical speeds often indicate strong support interaction with the rotor-bearing system. Figure 2-19 shows a phase-amplitude plot for the exhaust end right probe. The inner loop at 2872 rpm also indicates strong support interaction.

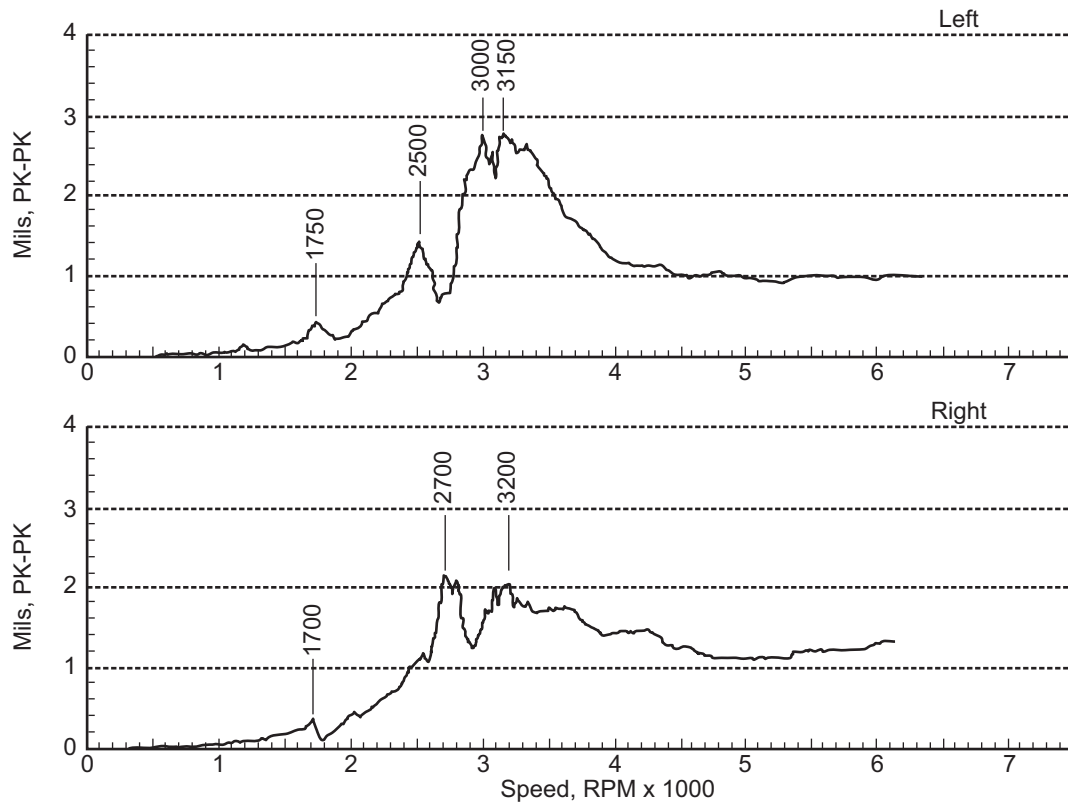
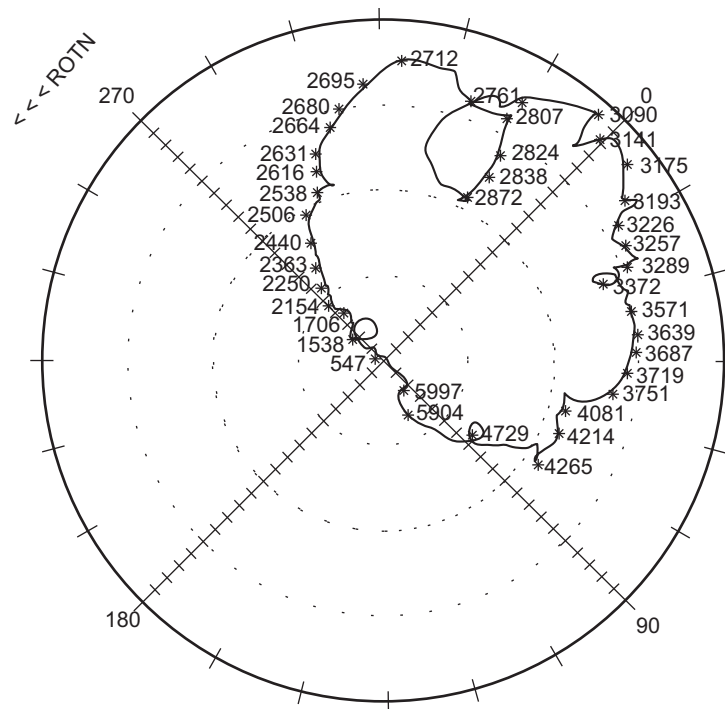


Figure 2-18—Steam End Test Stand Response



Polar Plot—Exhaust End Right Probe

Full Scale Amp = 2 Mils, PK-PK Amp per Div = 0.1 Mils, PK-PK

Figure 2-19—Exhaust End Test Stand Response