

Australian Standard[®]

Measurement of water flow in open channels

Part 2.4: General—Estimation of uncertainty of a flow rate measurement



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 - Irrigation Association of Australia
 - National Measurement Institute
 - Plumbing Products Industry Group
 - University of New South Wales
 - University of South Australia
 - Water Services Association of Australia
-

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Measurement of water flow in open channels

Part 2.4: General—Estimation of uncertainty of a flow rate measurement

Originated as AS 3778.2.4—1990.
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PREFACE

This Standard was prepared by the Standards Australia Committee CE-024, Measurement of Water Flow in Open Channels and Closed Conduits, to supersede AS 3778.2.4—2001.

The objective of this Standard is to specify methods for measuring discharge in large rivers and estuaries by the moving-boat technique.

This Standard is identical to and reproduced from ISO 5168:2005, *Measurement of fluid flow—Procedures for the evaluation of uncertainties*.

As this Standard is reproduced from an international standard, the following applies:

- (a) Its number appears on the cover and title page while the international standard number appears only on the cover.
- (b) In the source text ‘ISO 5168’ should read ‘AS 3778.2.4’.
- (c) A full point substitutes for a comma when referring to a decimal marker.

The terms ‘normative’ and ‘informative’ have been used in this Standard to define the application of the appendix to which they apply. A ‘normative’ appendix is an integral part of a Standard, whereas an ‘informative’ appendix is only for information and guidance.

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INTRODUCTION

Whenever a measurement of fluid flow (discharge) is made, the value obtained is simply the best estimate that can be obtained of the flow-rate or quantity. In practice, the flow-rate or quantity could be slightly greater or less than this value, the uncertainty characterizing the range of values within which the flow-rate or quantity is expected to lie, with a specified confidence level.

GUM is the authoritative document on all aspects of terminology and evaluation of uncertainty and should be referred to in any situation where this International Standard does not provide enough depth or detail. In particular, GUM (1995), Annex F, gives guidance on evaluating uncertainty components.

Australian Standard
Measurement of water flow in open channels
Part 2.4: General—Estimation of uncertainty of a flow rate measurement

1 Scope

This International Standard establishes general principles and describes procedures for evaluating the uncertainty of a fluid flow-rate or quantity.

A step-by-step procedure for calculating uncertainty is given in Annex A.

2 Normative references

The following referenced documents are indispensable for the application of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO 9300, *Measurement of gas flow by means of critical flow Venturi nozzles*

ISO Guide to the expression of uncertainty in measurement (GUM), 1995

International vocabulary of basic and general terms in metrology (VIM), 1993

3 Terms and definitions

For the purposes of this document, the terms and definitions given in VIM (1993), GUM (1995) and the following apply.

3.1 uncertainty

parameter, associated with the results of a measurement, that characterizes the dispersion of the values that could reasonably be attributed to the measurand

NOTE Uncertainties are expressed as an absolute value and do not take a positive or negative sign.

3.2 standard uncertainty

$u(x)$

uncertainty of the result of a measurement expressed as a standard deviation

3.3 relative uncertainty

$u^*(x)$

standard uncertainty divided by the best estimate

NOTE 1 $u^*(x) = u(x)/x$.

NOTE 2 $u^*(x)$ can be expressed either as a percentage or in parts per million.

NOTE 3 Relative uncertainty is sometimes referred to as dimensionless uncertainty.

NOTE 4 The best estimate is in most cases the arithmetic mean of the related uncertainty interval.

3.4 combined standard uncertainty

$u_c(y)$

standard uncertainty of the result of a measurement when that result is obtained from the values of a number of other quantities, equal to the positive square root of a sum of terms, the terms being the variances or covariances of these other quantities weighted according to how the measurement result varies with changes in these quantities

3.5 relative combined uncertainty

$u_c^*(y)$

combined standard uncertainty divided by the best estimate

NOTE 1 $u_c^*(y)$ can be expressed as a percentage or parts per million.

NOTE 2 $u_c^*(y) = u_c(y)/y$.

NOTE 3 Relative combined uncertainty is sometimes referred to as dimensionless combined uncertainty.

NOTE 4 The best estimate is in most cases the arithmetic mean of the related uncertainty interval.

3.6 expanded uncertainty

U

quantity defining an interval about the result of a measurement that can be expected to encompass a large fraction of the distribution of values that could reasonably be attributed to the measurand

NOTE 1 The fraction can be viewed as the coverage probability or the confidence level of the interval.

NOTE 2 $U = k u_c(y)$

3.7 relative expanded uncertainty

U^*

expanded uncertainty divided by the best estimate

NOTE 1 U^* can be expressed as a percentage or in parts per million.

NOTE 2 $U^* = k u_c^*(y)$.

NOTE 3 Relative expanded uncertainty is sometimes referred to as dimensionless expanded uncertainty.

NOTE 4 The best estimate is in most cases the arithmetic mean of the related uncertainty interval.

3.8 coverage factor

k

numerical factor used as a multiplier of the combined standard uncertainty in order to obtain an expanded uncertainty

NOTE A coverage factor is typically in the range 2 to 3.

3.9 Type A evaluation

〈uncertainty〉 method of evaluation of uncertainty by the statistical analysis of a series of observations

3.10

Type B evaluation

(uncertainty) method of evaluation of uncertainty by means other than the statistical analysis of a series of observations

3.11

sensitivity coefficient

c_i

change in the output estimate, y , divided by the corresponding change in the input estimate, x_i

3.12

relative sensitivity coefficient

c_i^*

relative change in the output estimate, y , divided by the corresponding relative change in the input estimate, x_i

4 Symbols and abbreviated terms

4.1 Symbols

a_i	estimated semi-range of a component of uncertainty associated with input estimate, x_i , as defined in Annex B
A_t	area of the throat
b_i	breadth associated with a vertical i
b'_i	upper bound of an asymmetric uncertainty distribution as defined in Annex B
c_i	sensitivity coefficient used to multiply the uncertainty in the input estimate, x_i , to obtain the effect of a change in the input quantity on the uncertainty of the output estimate, y
c_i^*	relative sensitivity coefficient used to multiply the relative uncertainty in input estimate, x_i , to obtain the effect of a relative change in the input quantity on the relative uncertainty of the output estimate, y
C_c	calibration coefficient
C	discharge coefficient
C_V	coefficient of variation
d_i	depth associated with a vertical i
d_o	orifice diameter
$d_{o,0}$	orifice diameter measured at temperature $T_{0,x}$
d_p	pipe diameter
$d_{p,0}$	pipe diameter measured at temperature $T_{0,x}$
$\bar{E}$	mean meter error, expressed as a fraction

E_j	j th meter error, expressed as a fraction
f	functional relationship between estimates of the measurand, y , and the input estimates, x_i , on which y depends
$\frac{\partial f}{\partial x_i}$	partial derivative with respect to input quantity, x_i , of the functional relationship, f , between the measurand and the input quantities
F	flow factor, equal to $\frac{q}{\sqrt{\Delta p_r}}$
F_{exp}	flow factor for a new design
F_{Redp}	$(19\,000 \cdot \beta / \text{Re}_{\text{dp}})^{0.8}$
F_{ref}	reference flow factor
F_s	factor, assumed to be unity, that relates the discrete sum over the finite number of verticals to the integral of the continuous function over the cross-section
k	coverage factor used to calculate the expanded uncertainty, U
k_t	coverage factor derived from a table; see D.12
K	meter factor
$\overline{K}$	mean meter factor
K_j	j th K -factor;
l_b	length of crest
l_h	gauged head
l_1	distance from the upstream tapping to the upstream face
L_1	l_1 divided by the pipe diameter, d_p
l'_2	distance from the downstream tapping to the downstream face
L'_2	l'_2 divided by the pipe diameter, d_p
m	particular item in a set of data
m'	number of data sets to be pooled
m''	number of verticals
M'_2	$2L'_2 / (1 - \beta)$
n	number of repeat readings or observations
n'	exponent of l_h , usually 1,5 for a rectangular weir and 2,5 for a V-notch

n''	number of depths in a vertical at which velocity measurements are made
N	number of input estimates, x_i , on which the measurand depends
p_0	upstream pressure
Δp_{mt}	pressure difference across the orifice meter
Δp_r	pressure difference across the radiator
$P(a_i)$	probability that an input estimate, x_i , has a value of a_i
q	volume flow-rate
q_{ma}	mass flow;
Q	flow, expressed in cubic metres per second, at flowing conditions
R	specific gas constant
Re_{dp}	Reynolds number related to d_p by the expression $V d_p \rho / \mu$
$s_{\text{mt,po}}$	pooled experimental standard deviation of the orifice plate readings
s_{pe}	standard deviation of a larger set of data used with a smaller data set
s_{po}	standard deviation pooled from several sets of data
$s_{\text{r,po}}$	pooled experimental standard deviation for the radiator readings
$s(x)$	experimental standard deviation of a random variable, x , determined from n repeated observations
$s(\bar{x})$	experimental standard deviation of the arithmetic mean, $\bar{x}$
t	Student's statistic
T_0	upstream absolute temperature
$T_{0,x}$	temperature at which measurement x is made
T_{op}	operating temperature
$u_{\text{c,corr}}(v)$	combined uncertainty for those components for multiple meters that are correlated
$u_{\text{c,uncorr}}(v)$	combined uncertainty for those components for multiple meters that are uncorrelated
u_{cal}^*	instrument calibration uncertainty from all sources, formerly called systematic errors or biases
u_{cri}^*	relative uncertainty in point velocity at a particular depth in vertical i due to the variable responsiveness of the current meter
u_{d}^*	relative standard uncertainty in the coefficient of discharge

u_{ei}^*	relative uncertainty in point velocity at a particular depth in vertical i due to velocity fluctuations (pulsations) in the stream
u_{lb}^*	relative standard uncertainty in the measurement of the crest length
u_{lh}^*	relative standard uncertainty in the measurement of the gauged head
$u_{m''}^*$	relative uncertainty due to the limited number of verticals
u_{pi}^*	relative uncertainty in mean velocity, V_p , due to the limited number of depths at which velocity measurements are made at vertical, i
$u^*(Q)$	combined relative standard uncertainty in the discharge;
u_{sm}	standard uncertainty of a single value based on past experience
$u(x_{i,corr})$	correlated components of uncertainty in a single meter
$u(x_{i,uncorr})$	uncorrelated components of uncertainty in a single meter
$u^*(x_i)$	standard uncertainty associated with the input estimate, x_i
$u_c^*(y)$	combined standard uncertainty associated with the output estimate, y
$u^*(x_i)$	relative standard uncertainty associated with the input estimate x_i
$u_c^*(y)$	combined relative standard uncertainty associated with the output estimate, y
$U^*(y)$	relative expanded uncertainty associated with the output estimate
$U(y)$	expanded uncertainty associated with the output estimate, y
U_{CMC}	combined uncertainty of the calibration rig
$U_{AS-overall-E}$	type A uncertainty in meter error
$U_{AS-overall-K}^*$	type A uncertainty in the K -factor
V	mean velocity in the pipe
V_i	mean velocity associated with a vertical i
x_i	estimate of the input quantity, X_i
x_m	m th observation of random quantity, x
x_0	dimension at temperature $T_{0,x}$
$\bar{x}$	arithmetic mean or average of n repeated observations, x_m , of randomly varying quantity, x
y	estimate of the measurand, Y
Δx_i	increment in x_i used for numerical determination of sensitivity coefficient

Δy	increment in y found in numerical determination of sensitivity coefficient
Z_n	Grubbs test statistic for outliers
β	orifice plate diameter ratio, equal to d_o/d_p
φ_{cf}	critical flow function
Φ_F	ratio of the factor F for a new design compared to the old design
λ	expansion coefficient
μ	dynamic fluid viscosity
ρ	fluid density
ν	degrees of freedom
ν_{eff}	effective degrees of freedom
ν_{po}	degrees of freedom associated with a pooled standard deviation

4.2 Subscripts

c	combined
corr	correlated
do	orifice diameter
dp	pipe diameter, effective
ex	external
i	of the i th input
j	of the j th set
$k = 2$	obtained with a coverage factor of 2
m	of the m th observation
n	of the n th observation
N	of the N th input
nom	nominal value of
op	operating temperature
pe	from past experience
po	pooled
sm	based on a single measurement
t	tolerance interval

uncorr	uncorrelated
x	of x
$\bar{x}$	of the mean value of x
95	with a 95 % confidence level

5 Evaluation of the uncertainty in a measurement process

The first stage in an uncertainty evaluation is to define the measurement process. For the measurement of flow-rate, it will normally be necessary to combine the values of a number of input quantities to obtain a value for the output. The definition of the process should include the enumeration of all the relevant input quantities.

Annex E enumerates a number of categories of sources of uncertainty. This categorization can be of value when defining all of the sources of uncertainty in the process. It is assumed in the following sections that the sources of uncertainty are uncorrelated; correlated sources require different treatment (see Annex F).

Consideration should also be given to the time over which the measurement is to be made, taking into account that flow-rate will vary over any period of time and that the calibration can also change with time.

If the functional relationship between the input quantities $X_1, X_2, \dots, X_N$, and output quantity Y in a flow measurement process is specified in Equation (1):

$$Y = f(X_1, X_2, \dots, X_N) \quad (1)$$

then an estimate of Y , denoted by y , is obtained from Equation (1) using input estimates $x_1, x_2, \dots, x_N$, as shown in Equation (2):

$$y = f(x_1, x_2, \dots, x_N) \quad (2)$$

Provided the input quantities, X_i , are uncorrelated, the total uncertainty of the process can be found by calculating and combining the uncertainty of each of the contributing factors in accordance with Equation (3):

$$u_c(y) = \sqrt{\sum_{i=1}^N [c_i u(x_i)]^2} \quad (3)$$

Where the extent of interdependence is known to be small, Equation (3) may be applied even though some of the input quantities are correlated; ISO 5167-1:2003 ^[1] provides an example of this.

Each of the individual components of uncertainty, $u(x_i)$, is evaluated using one of the following methods:

- Type A evaluation: calculated from a series of readings using statistical methods, as described in Clause 6;
- Type B evaluation: calculated using other methods, such as engineering judgement, as described in Clause 7.

Uncertainty sources are sometimes classified as “random” or “systematic” and the relationship between these categorizations and Type A and Type B evaluations is given in Annex I.

The sensitivity coefficients, c_i , provide the links between uncertainty in each input and the resulting uncertainty in the output. The methods of calculating the individual sensitivity coefficients, c_i , are described in detail in Clause 8.

6 Type A evaluations of uncertainty

6.1 General considerations

Type A evaluations of uncertainty are those using statistical methods, specifically, those that use the spread of a number of measurements.

Whilst no correction can be made to remove random components of uncertainty, their associated uncertainty becomes progressively less as the number of measurements increases. In taking a series of measurements, it should be recognized that, as the purpose is to define the random fluctuations in the process, the timescale for the data collection should reflect the anticipated timescale for the fluctuations. Collecting readings at millisecond intervals for a process that fluctuates over several minutes will not characterize those fluctuations adequately.

In many measurement situations, it is not practical to make a large number of measurements. In this case, this component of uncertainty may have to be assigned on the basis of an earlier Type A evaluation, based on a larger number of readings carried out under similar conditions. Caution should be exercised in making these estimates (see Annex D), as there will always be some uncertainty associated with the assumption that the earlier measurements were taken under truly similar conditions.

The methods of calculating the uncertainty in a mean and in a single value reflect the reduction in uncertainty obtained by averaging several readings [Equations (4) to (8)] and are explained in more detail in D.4 to D.6.

6.2 Calculation procedure

Further explanation of the equations given below can be found in Annex D.

The standard uncertainty of a measured value, x_i , is calculated from a sample of measurements, $x_{i,m}$, in accordance with Equations (4) to (8):

- a) Calculate the average value of the measurements in accordance with Equation (4); see D.1:

$$\bar{x}_i = \frac{1}{n} \sum_{m=1}^n x_{i,m} \quad (4)$$

- b) Calculate the standard deviation of the sample in accordance with Equation (5); see D.2:

$$s(x_i) = \sqrt{\frac{1}{(n-1)} \sum_{m=1}^n (x_{i,m} - \bar{x}_i)^2} \quad (5)$$

The standard uncertainty of a single sample is the same as its standard deviation and is given by Equation (6):

$$u(x_i) = s(x_i) \quad (6)$$

- c) Calculate the standard deviation of the mean value in accordance with Equation (7); see D.4:

$$s(\bar{x}_i) = \frac{s(x_i)}{\sqrt{n}} \quad (7)$$

The standard uncertainty of the mean value is then given by Equations (8):

$$u(\bar{x}_i) = s(\bar{x}_i) \quad (8)$$

The use of the mean of several readings is a key technique for reducing uncertainty in readings subject to random variations. For the derivation of Equation (7) see Dietrich [2].

NOTE The approach outlined here represents a simplification and, when the functional relationship defined by Equation (1) is highly non-linear and uncertainties are large, the more rigorous approach described in the GUM (1995), 4.1.4, could yield a more robust answer.

7 Type B evaluation of uncertainties

7.1 General considerations

Type B evaluations of uncertainty are those carried out by means other than the statistical analysis of series of observations.

As explained in D.9, Type A uncertainties result in a bandwidth of 1 standard deviation that would encompass 68 % of the possible values of the measured quantity. In making Type B assessments, it is necessary to ensure that a similar confidence level is obtained such that the uncertainties obtained by different methods can be compared and combined.

Type B assessments are not necessarily governed by the normal distribution and the limits assigned can represent varying confidence levels. Thus, a calibration certificate could give the meter factor for a turbine meter with 95 % confidence while an instrument resolution uncertainty defines with 100 % confidence the range of values that will be represented by that number rather than the next higher or lower. The equations for obtaining the standard uncertainty for various common distributions are given in 7.3 to 7.8.

7.2 Calculation procedure

Type B evaluations of uncertainty require a knowledge of the probability distribution associated with the uncertainty. The most common probability distributions are presented in 7.3 to 7.8; the shapes of the distributions are shown in Annex B.

7.3 Rectangular probability distribution

Typical examples of rectangular probability distributions include

- maximum instrument drift between calibrations,
- error due to limited resolution of an instrument's display,
- manufacturers' tolerance limits.

The standard uncertainty of a measured value, x_i , is calculated from Equation (9):

$$u(x_i) = \frac{a_i}{\sqrt{3}} \quad (9)$$

where the range of measured values lies between $x_i - a_i$ and $x_i + a_i$. The derivation of Equation (9) is given by Dietrich [2].

7.4 Normal probability distribution

Typical examples include calibration certificates quoting a confidence level or coverage factor with the expanded uncertainty. Here, the standard uncertainty is calculated from Equation (10):

$$u(x_i) = \frac{U}{k} \quad (10)$$

where

U is the expanded uncertainty;

k is the quoted coverage factor; see Annex C.

Where a coverage factor has been applied to a quoted expanded uncertainty, care should be exercised to ensure that the appropriate value of k is used to recover the underlying standard uncertainty. However, if the coverage factor is not given and the 95 % confidence level is quoted, then k should be assumed to be 2.

7.5 Triangular probability distribution

Some uncertainties are given simply as maximum bounds within which all values of the quantity are assumed to lie. There is often reason to believe that values close to the bounds are less likely than those near the centre of the bounds, in which case the assumption of rectangular distribution could be too pessimistic. In this case, the triangular distribution, as given by Equation (11), may be assumed as a prudent compromise between the assumptions of a normal and a rectangular distribution.

$$u(x_i) = \frac{a_i}{\sqrt{6}} \quad (11)$$

7.6 Bimodal probability distribution

When the error is always at the extreme value, then a bimodal probability distribution is applicable and the standard uncertainty is given by Equation (12):

$$u(x_i) = a_i \quad (12)$$

Examples of this type of distribution are rare in flow measurement.

7.7 Assigning a probability distribution

When the source of the uncertainty information is well defined, such as a calibration certificate or a manufacturer's tolerance, the choice of probability distribution will be clear-cut. However, when the information is less well defined, for example when assessing the impact of a difference between the conditions of calibration and use, the choice of a distribution becomes a matter of the professional judgement of the instrument engineer.

7.8 Asymmetric probability distributions

The above cases are for symmetrical distributions, however, it is sometimes the case that the upper and lower bounds for an input quantity, X_i , are not symmetrical with respect to the best estimate, x_i . In the absence of information on the distribution, GUM recommends the assumption of a rectangular distribution with a full range equal to the range from the upper to the lower bound. The standard uncertainty is then given by Equation (13):

$$u(x_i) = \frac{a_i + b'_i}{\sqrt{12}} \quad (13)$$

where $(x_i - a_i) < X_i < (x_i + b'_i)$.

A more conservative approach would be to take a rectangular distribution based on the larger of two asymmetric bounds.

$$u(x_i) = \text{the greater of } \frac{a_i}{\sqrt{3}} \text{ or } \frac{b'_i}{\sqrt{3}} \quad (14)$$

If the asymmetric element of uncertainty represents a very significant proportion of the overall uncertainty, it would be more appropriate to consider an alternative approach to the analysis such as a Monte Carlo analysis; see Annex K.

A common example of an asymmetric distribution is seen in the drift of instruments due to mechanical changes, for example, increasing friction in the bearings of a turbine meter or erosion of the edge of an orifice plate.

8 Sensitivity coefficients

8.1 General

Before considering methods of combining uncertainties, it is essential to appreciate that it is insufficient to consider only the magnitudes of component uncertainties in input quantities, it is also necessary to consider the effect each input quantity has on the final result. For example, an uncertainty of 50 µm in a diameter or 5 % in a thermal expansion coefficient is meaningless in terms of the flow through an orifice plate without knowledge of how the diameter or thermal expansion impact the measurement of flow-rate. It is, therefore, convenient to introduce the concept of the sensitivity of an output quantity to an input quantity, i.e. the sensitivity coefficient, sometimes referred to as the influence coefficient.

The sensitivity coefficient of each input quantity is obtained in one of two ways:

- analytically; or
- numerically.

8.2 Analytical solution

When the functional relationship is specified as in Equation (1), the sensitivity coefficient is defined as the rate of change of the output quantity, y , with respect to the input quantity, x_i , and the value is obtained by partial differentiation in accordance with Equation (15):

$$c_i = \frac{\partial y}{\partial x_i} \quad (15)$$

However, when non-dimensional uncertainties (for example percentage uncertainty) are used, non-dimensional sensitivity coefficients shall also be used in accordance with Equation (16):

$$c_i^* = \frac{\partial y}{\partial x_i} \frac{x_i}{y} \quad (16)$$

In certain special cases where, for example, a calibration experiment has made the functional relationship between the input and output simple, the value of c_i or c_i^* can be unity. Example 1 in Annex G gives an example for a calibrated nozzle.

8.3 Numerical solution

Where no mathematical relationship is available, or the functional relationship is complex, it is easier to obtain the sensitivity coefficients numerically, by calculating the effect of a small change in the input variable, x_i , on the output value, y .

First calculate y using x_i , and then recalculate using $(x_i + \Delta x_i)$, where Δx_i is a small increment in x_i . The result of the recalculation can be expressed as $y + \Delta y$, where Δy is the increment in y caused by Δx_i .

The sensitivity coefficients are then calculated in accordance with Equation (17):

$$c_i \approx \frac{\Delta y}{\Delta x_i} \quad (17)$$

They are calculated in non-dimensional, or relative, form in accordance with Equation (18):

$$c_i^* \approx \frac{\Delta y}{\Delta x_i} \frac{x_i}{y} \quad (18)$$

Table 1 shows how a typical spreadsheet could be set up to calculate a specific sensitivity coefficient for any function where $y = f(x_1, x_2, \dots, x_N)$.

Table 1 — Spreadsheet set-up for calculating sensitivity coefficients

Sensitivity coefficient	Increment	x_1	x_2	...	x_i	x_N	y	c	c^*
—	—	x_1	x_2	...	x_i	x_N	$y = f(x_1, x_2, \dots, x_N) = y_{\text{nom}}$	—	—
c_1	$\Delta x_i \approx 10^{-6} \cdot x_1$	$x_1 + \Delta x_i$	x_2	...	x_i	x_N	$y_i = f(x_1 + \Delta x_i, x_2, \dots, x_N)$	$\frac{(y_1 - y_{\text{nom}})}{\Delta x_1}$	$c_1 \cdot \frac{x_1}{y_{\text{nom}}}$

The analytical solution calculates the gradient of y with respect to x_i at the nominal value, x_i , whereas the numerical solution obtains the average gradient over the interval x_i to $(x_i + \Delta x_i)$. The increment used (Δx_i) should therefore be as small as practical and certainly no larger than the uncertainty in the parameter x_i . However, a complication can arise if the increment is so small as to result in changes in the calculated result, y , that are comparable with the resolution of the calculator or computer spreadsheet. In these circumstances the calculation of c_i can become unstable. The problem can be avoided by starting with a value of Δx_i equal to the uncertainty in x_i and progressively reducing Δx_i until the value of c_i agrees with the previous result within a suitable tolerance. This iteration process can, of course, be automated with a computer spreadsheet.

9 Combination of uncertainties

Once the standard uncertainties of the input quantities and their associated sensitivity coefficients have been determined from either Type A or Type B evaluations, the overall uncertainty of the output quantity can be determined in accordance with Equation (19):

$$u_c(y) = \sqrt{\sum_{i=1}^N [c_i u(x_i)]^2} \quad (19)$$

Where relative uncertainties have been used, relative sensitivity coefficients shall also be used, in accordance with Equation (20):

$$u_c^*(y) = \sqrt{\sum_{i=1}^N [c_i^* u^*(x_i)]^2} \quad (20)$$

Equations (19) and (20) assume that the individual input quantities are uncorrelated; the treatment of correlated uncertainties is discussed in C.6. Correlation arises where the same instrument is used to make several measurements or where instruments are calibrated against the same reference.

In general, the choice of absolute or relative uncertainties is of little consequence. However, once the decision has been made, care is needed to ensure that all uncertainties are expressed in the same terms. Measurements with arbitrary zero points give rise to problems if uncertainties are expressed in relative terms. For example, an uncertainty of 1 mm in a diameter of 500 mm gives a relative uncertainty of 0,2 % and, if expressed in inches, the uncertainty becomes 0,039 4 in out of 19,69 in, leaving the relative uncertainty unchanged. However, if the uncertainty in a temperature of 20 °C is 0,5 °C, the relative uncertainty is 2,5 %, but by expressing the values in degrees Fahrenheit, the temperature becomes 68 °F and the uncertainty becomes 0,9 °F, giving a relative uncertainty of 1,3 %. Relative uncertainties cannot be used in these circumstances and absolute uncertainties should be used. A relative uncertainty can only be used when it is based on a measurement that is used to calculate the end result.

10 Expression of results

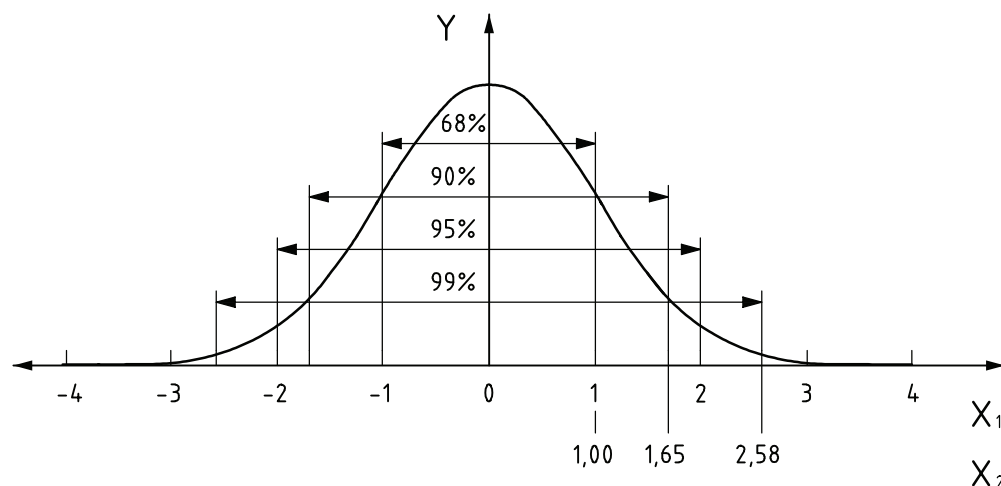
10.1 Expanded uncertainty

In Equations (19) and (20), the overall result is obtained from a summation of the contributions of the standard uncertainty of each input source to the uncertainty of the result. The resulting combined uncertainty is, therefore, a standard uncertainty; by referring to Figure 1, it can be seen that, with an effective k factor of 1, the bandwidth defined by a standard uncertainty will only have a confidence level of about 68 % associated with it. There is, therefore, a 2:1 chance that the true value will lie within the band, or a 1 in 3 chance that it will lie outside the band. Such odds are of little value in engineering terms and the normal requirement is to provide an uncertainty statement with 90 % or 95 % confidence level; in some extreme cases, 99 % or higher might be required. To obtain the desired confidence level, an expanded uncertainty, U , is used in accordance with Equation (21):

$$U = k u_c(y) \quad (21)$$

or, where relative uncertainties are being used, in accordance with Equation (22):

$$U^* = k u_c^*(y) \quad (22)$$



Key

- X_1 standard deviation
- X_2 coverage factor
- Y percent of readings in bandwidth

Figure 1 — Coverage factors for different levels of confidence with the normal, or Gaussian, distribution

It is recommended that for most applications a coverage factor, $k = 2$, be utilized to provide a confidence level of approximately 95 %; the choice of coverage factor will depend on the requirement of the application. Values of k for various levels of confidence are given in Table 2.

Table 2 — Coverage factors for different levels of confidence with the normal, or Gaussian, distribution

Confidence level, %	68,27	90,00	95,00	95,45	99,00	99,73
Coverage factor, k	1,000	1,645	1,960	2,000	2,576	3,000

If the random contribution to uncertainty is large compared with the other contributions and the number of readings is small, the above method provides an optimistic coverage factor. In this case, the procedure outlined in Annex C should be used to estimate the actual coverage factor. A criterion that can be used to determine whether the procedure described in Annex C should be applied is as follows.

Generally, if an uncertainty evaluation involves only one Type A evaluation and that Type A standard uncertainty is less than half the combined standard uncertainty, there is no need to use the method described in Annex C to determine a value for the coverage factor, provided that the number of observations used in the Type A evaluation is greater than 2.

The uncertainty associated with an expanded uncertainty can be denoted using subscripts.

EXAMPLE U_{95} or $U_k = 2$.

10.2 Uncertainty budget

In reports providing an uncertainty estimate, an uncertainty budget table should be presented, (or referenced) providing at least the information set out in Table 3.

Table 3 — Uncertainty budget

Symbol	Source of uncertainty	Input uncertainty	Probability distribution	Divisor [see Equations (9) to (14)]	Standard uncertainty $u(x_i)$	Sensitivity coefficient c_i	Contribution to overall uncertainty $[c_i u(x_i)]^2$
$u(x_1)$	e.g. calibration	5	Normal	2	2,5	0,5	1,56
$u(x_2)$	e.g. output resolution	1	Rectangular	$\sqrt{3}$	0,58	2,0	1,35
...							
$u(x_i)$							
$u(x_N)$							
u_c	Combined uncertainty	—	—	—	$u_c(y) = \sqrt{\Sigma}$	$\leftarrow^a$	$= \Sigma [c_i u(x_i)]^2$
U	Expanded uncertainty	$= k u_c(y)$	$\leftarrow^a$	k	$\leftarrow^a$	—	—

^a The arrows in the last two rows of the table indicate that, whereas in the upper rows the calculation proceeds from left to right, in these rows the calculation of the final expanded uncertainty proceeds from right to left.

Table 3 is presented here in absolute terms and each input and corresponding standard uncertainty will have the units of the appropriate input parameter. The table may, equally validly, be presented in relative terms, in which case all input and resulting standard uncertainties will be in percentages or parts per million. Where the inputs are all standard uncertainties, the columns headed "Input uncertainty," "Probability distribution" and "Divisor" may be omitted.

If the computation of a combined uncertainty is in response to a requirement for a test result to have a specified level of uncertainty and the analysis shows that level to be exceeded, the budget table can be of particular value in identifying the largest sources of uncertainty as an indicator of the problem areas which should be addressed.

After the expanded uncertainty has been calculated for a minimum confidence level of 95 %, the measurement result should be stated as follows.

- “The result of the measurement is [value].”
- “The uncertainty of the result is [value] (expressed in absolute or relative terms as appropriate).”
- “The reported uncertainty is based on a standard uncertainty multiplied by a coverage factor $k = 2$, providing a confidence level of approximately 95 %.”

In cases where the procedure of Annex C has been followed, the actual value of the coverage factor should be substituted for $k = 2$. In cases where a confidence level greater than 95 % has been used, the appropriate k factor and confidence level should be substituted.

In reporting the result of any uncertainty analysis, it is important to make a clear statement of whether the reported uncertainty is that of a single value, of a mean of a specified number of values, or of a curve fit based on a specified number of values.

Annex A (normative)

Step-by-step procedure for calculating uncertainty

A.1 Dimensional and non-dimensional uncertainty

Decide whether dimensional or non-dimensional uncertainty estimates (for example parts per million or per cent) will be used to prevent any confusion. In making this decision, the guidance of Clause 9 concerning parameters with arbitrary zeroes should be borne in mind.

A.2 Mathematical relationship

Determine the mathematical relationship between the input quantities and the output quantity in accordance with Equation (1):

$$Y = f(X_1, X_2, \dots, X_N)$$

NOTE The equation numbers referred to in this annex correspond with the equation numbers in the body of the text.

A.3 Standard uncertainty

A.3.1 General

Identify the sources of uncertainty in each of the input quantities; see Annex E. Estimate the standard uncertainty for each source. The calculation method for each component is dependent upon the uncertainty estimates provided and associated probability distributions. The data available usually allow the standard uncertainty to be calculated using one of the following methods.

A.3.2 Type A evaluations — Standard deviation of the mean of repeated measurements

$$u(\bar{x}_i) = s(\bar{x}_i)$$

See Equation (8).

A.3.3 Type B evaluations — Based on subjective assessment and experience

A.3.3.1 Rectangular probability distribution

$$u(x_i) = \frac{a_i}{\sqrt{3}}$$

See Figure B.1 and Equation (9).

A.3.3.2 Normal probability distribution

$$u(x_i) = \frac{U}{k}$$

See Figure B.2 and Equation (10).

A.4 Sensitivity coefficients

A.4.1 General

The sensitivity coefficients can be calculated either dimensionally or non-dimensionally using either analytical or numerical methods. The choice of dimensional or non-dimensional sensitivity coefficients will be determined by the choice made in A.1.

A.4.2 Dimensional

$$c_i = \frac{\partial y}{\partial x_i} \approx \frac{\Delta y}{\Delta x_i}$$

See Equations (15) and (17).

A.4.3 Non-dimensional

$$c_i^* = \frac{\partial y}{\partial x_i} \frac{x_i}{y} \approx \frac{\Delta y}{\Delta x_i} \frac{x_i}{y}$$

See Equations (16) and (18).

A.5 Combined uncertainty

A.5.1 General

Decide whether any inputs are correlated. If there are no correlations, calculate the combined standard uncertainty of the measurement from A.5.2 or A.5.3. If there are correlations, follow the guidance given in Annex F.

A.5.2 Dimensional

$$u_c(y) = \sqrt{\sum_{i=1}^N [c_i u(x_i)]^2}$$

See Equation (19).

A.5.3 Non-dimensional

$$u_c^*(y) = \sqrt{\sum_{i=1}^N [c_i^* u^*(x_i)]^2}$$

See Equation (20).

A.6 Unreliable input quantities

Where unreliable input quantities are used, for example small sample sizes, the procedure of Annex C should be used to obtain the coverage factor for the calculation of expanded uncertainty in A.7.