

Australian Standard[®]

Reliability growth—Statistical test and estimation methods



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Australian Standard[®]

Reliability growth—Statistical test and estimation methods

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PREFACE

This Standard was prepared by the Standards Australia Committee QR-005, Dependability.

The objective of this Standard is to describe the power law reliability growth model and related projection model, and give step-by step guidance on their use. It includes statistical methods for reliability growth planning and tracking in the product design phase that may be used in conjunction with IEC 61164, *Programmes for reliability growth*, which has been adopted as AS IEC 61164—2004, and provides procedures to estimate some or all of the quantities required in the design phase of IEC 61014.

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INTRODUCTION

This International Standard describes the power law reliability growth model and related projection model and gives step-by-step directions for their use. There are several reliability growth models available, the power law model being one of the most widely used. This standard provides procedures to estimate some or all of the quantities listed in Clauses 4, 6 and 7 of IEC 61014.

Two types of input are required. The first one is for reliability growth planning through analysis and design improvements in the design phase in terms of the design phase duration, initial reliability, reliability goal, and planned design improvements, along with their expected magnitude. The second input, for reliability growth in the project validation phase, is for a data set of accumulated test times at which relevant failures occurred, or were observed, for a single system, and the time of termination of the test, if different from the time of the final failure. It is assumed that the collection of data as input for the model begins after the completion of any preliminary tests, such as environmental stress screening, intended to stabilize the product's initial failure intensity.

Model parameters estimated from previous test results may be used to plan and predict the course of future reliability growth programmes, provided the conditions are similar.

Some of the procedures may require computer programs, but these are not unduly complex. This standard presents algorithms for which computer programs should be easy to construct.

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Reliability growth—Statistical test and estimation methods

1 Scope

This International Standard gives models and numerical methods for reliability growth assessments based on failure data, which were generated in a reliability improvement programme. These procedures deal with growth, estimation, confidence intervals for product reliability and goodness-of-fit tests.

2 Normative references

The following referenced documents are indispensable for the application of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

References to international standards that are struck through in this clause are replaced by references to Australian or Australian/New Zealand Standards that are listed immediately thereafter and identified by shading. Any Australian or Australian/New Zealand Standard that is identical to the International Standard it replaces is identified as such.

IEC 60050(191):1990, *International Electrotechnical Vocabulary (IEV) – Chapter 191: Dependability and quality of service*

IEC 60300-3-5:2001, *Dependability management – Part 3-5: Application guide – Reliability test conditions and statistical test principles*

IEC 60605-4, *Equipment reliability testing – Part 4: Statistical procedures for exponential distribution – Point estimates, confidence intervals, prediction intervals and tolerance intervals*

IEC 60605-6, *Equipment reliability testing – Part 6: Tests for the validity of the constant failure rate or constant failure intensity assumptions*

~~IEC 61014:2003, *Programmes for reliability growth*~~

AS IEC 61014—2003, *Programmes for reliability growth* (identical to IEC 61014:2003)

3 Terms and definitions

For the purposes of this document, the terms and definitions of IEC 60050(191) and IEC 61014, together with the following terms and definitions, apply.

3.1**reliability goal**

desired level of reliability that the product should have at the end of the reliability growth programme

3.2**initial reliability**

reliability that is estimated for the product in earlier design stages before any potential failure modes or their causes have been mitigated by the design improvement

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3.3**reliability growth model for the design phase**

mathematical model that takes into consideration potential design improvements, and their magnitude to express mathematically reliability growth from start to finish during the design period

3.4**average product failure rate**

average product failure rate calculated from its reliability as estimated for a predetermined time period

NOTE The change in this failure rate as a function of time is a result of the modifications of the product design.

3.5**delayed modification**

corrective modification, which is incorporated into the product at the end of a test

NOTE A delayed modification is not incorporated during the test.

3.6**improvement effectiveness factor**

fraction by which the intensity of a systematic failure is reduced by means of corrective modification

3.7**type I test**

time-terminated test

reliability growth test which is terminated at a predetermined time, or test with data available through a time which does not correspond to a failure

3.8**type II test**

failure-terminated test

reliability growth test which is terminated upon the accumulation of a specified number of failures, or test with data available through a time which corresponds to a failure

4 Symbols

For the purposes of this standard, the following symbols apply.

a) For 6.1, clauses A.1 and B.3:

T	product lifetime such as mission, warranty period or operational time
$R_0(T)$	initial product reliability
λ_{a0}	initial average failure rate of product in design period
$d(t)$	number of design modifications at any time during the design period
α_D	reliability growth rate resultant from fault mitigation
D	total number of implemented design improvements
t_D	total duration of the design period available for the design improvements
t	time variable during the design period from 0 to t_D
$\lambda_a(t)$	average failure rate of product as a function of time during the design period

$\lambda_{aG}(t_D)$	goal average failure rate at the end of the design period t_D
$R_G(T)$	reliability goal of the product to be attained during design period
$R(t, T)$	reliability of product as a function of time and design improvements

b) For 6.2, clauses A.2 and B.4:

$R_G(T)$	reliability goal of the product to be attained during design period
t_D	total duration of the design period
α_D	reliability growth rate during design period
λ_{NS}	rate of non-systematic (or residual) failures
D	total number of predicted or implemented design improvements within design period to address weaknesses
K	total number of distinct classes of fault
j, k, i	general purpose indicators
p_{kj}	probability of j -th design weakness in fault class k resulting in failure during the specified life of the product
η_k	expected number of design weaknesses in fault class k resulting in failure during the specified life of the product
D_k	total number of predicted or implemented design improvements within design period to address faults in fault class k
λ_k	failure rate of design weaknesses categorized in fault class k
$R_i(T)$	initial reliability at time T
$R(T)$	reliability of product as a function of T
t_G	expected time to reach reliability goal

c) For 7.1.1, 7.1.2, Clauses 9, A.4, B.1, and B.2:

D	total number of design modifications carried out during product design period to mitigate identified faults
t_D	total duration of the design period available for potential design modifications
t	time variable (during design period $0 \leq t \leq t_D$)
$d(t)$	number of design modifications at any given time t during design period from 0 to t_D
α_D	reliability growth rate during the design period
λ_{a0}	initial average failure rate of a product in design

$\lambda_a(t)$	product average failure rate variable as a function of time during the design period (0 to t_D)
$R_0(T)$	initial product reliability calculated for a time T (mission or other predetermined time)
$R_G(T)$	product reliability goal to be attained through design improvement, calculated for a predetermined time
$R(t)$	product reliability increase as a function of time and design improvements
λ_{aG}	goal average failure rate
T	predetermined time during a product life (mission, warranty, life)
λ	scale parameter for the power law model
β	shape parameter for the power law model
CV	critical value for hypothesis test
d	number of intervals for grouped data analysis
$\bar{E}, E_i, E_j$	mean and individual improvement effectiveness factors
I	number of distinct types of category B failures observed
i, j	general purpose indices
K_A	number of category A failures
K_B	number of category B failures
K_i	number of i -th type category B failures observed: $K_B = \sum_{i=1}^{\ell} K_i$
M	parameter of the Cramér-von Mises test (statistical)
N	number of relevant failures
N_i	number of relevant failures in i -th interval
$N(T)$	accumulated number of failures up to test time T
$E[N(T)]$	expected accumulated number of failures up to test time T
$t(i-1); t(i)$	endpoints of i -th interval of test time for grouped data analysis
T	current accumulated relevant test time
T_i	accumulated relevant test time at the i -th failure
T_N	total accumulated relevant test times for type II test
T^*	total accumulated relevant test times for type I test
$\chi^2_{\gamma}(\nu)$	γ fractile of the χ^2 distribution with ν degrees of freedom
z	general symbol for failure intensity
u_{γ}	γ fractile of the standard normal distribution
z_p	projected failure intensity
$z(T)$	current failure intensity at time T (relevant test time)
$\theta(T)$	current instantaneous mean time between failures
θ_p	projected mean time between failures
p_j	probability of success at the stage i of product modification in design phase described in Barlow, Proschan and Scheuer discrete reliability growth model

$N(t)$	number of non-random type faults remaining at time $t > 0$ in IBM/Rosner continuous reliability growth model
g	fraction that a product/equipment is debugged as given in the IBM/Rosner reliability growth model
E	exposure time of item
λ_{NS}	failure rate of non-systematic (residual) failures
λ_S	failure rate of systematic failures
μ_k	failure rate of the k -th failure class
D_k	number of potential design weakness in failure class k
$p_{k,j}$	probability that the j -th potential design weakness associated with failure class k will result in failure
t_E	expected design phase time to achieve goal reliability, $R_G(T)$
t_D	duration of design phase
$R_i(T)$	initial reliability at time T
α	parameter to represent the expected growth rate
λ_k	the expected number of design weaknesses associated with failure class k that will result in failure
K_2	proportion of systematic (non-random) faults in product design at start of test
K_1	number of faults in the product design at start of test
q	fraction of faults removed through debugging on reliability growth test
$q(T)$	fraction of original faults removed by time t
t_q	Expected time for removing fraction q of systematic faults in test
$\theta(T)$	Cumulative time between failures

d) Symbols used in the discrete reliability growth model, 7.2:

R_i	reliability, or success probability, of the i -th configuration
$f_i = 1 - R_i$	unreliability, or failure probability, of the i -th configuration
k	number of stages and configurations
n_i	number of trials for stage i
m_i	number of failures for stage i
$t_i = \sum_{j=1}^i n_j$	the cumulative number of trials through stage i
$N(t_i)$	the accumulated number of failures up through trial t_i
$E[N(t_i)]$	the expected accumulated number of failure up through trial t_i
λ, β	scale and shape parameters for power law and discrete models

5 Reliability growth models in design and test

The basic principles of reliability growth of a product are the same during design and test. This is because both involve identifying and removing weaknesses to improve the product and both measure that improvement by comparing the estimated reliability with the reliability goal. The difference lies in the tools used to conduct design and test analysis and the models used to measure reliability growth. IEC 61064 provides guidance on the construction of reliability growth programmes and the analysis tools used in design and test. This standard provides details about the models that can be used to measure reliability growth in different stages of the product life cycle and for different types of items, such as repairable or one-shot items.

The mathematical models for reliability growth are constructed to estimate the growth achieved and the projected reliability. Reliability growth models aim to support the planning of reliability improvement programmes by estimating the number and the magnitude of the changes during the design and development process or the test time required to reach a specified reliability goal.

The reliability growth models can be formulated in terms of the failure rate (or intensity) or probability of survival to a specified time (the reliability) as shown in Figure 1.

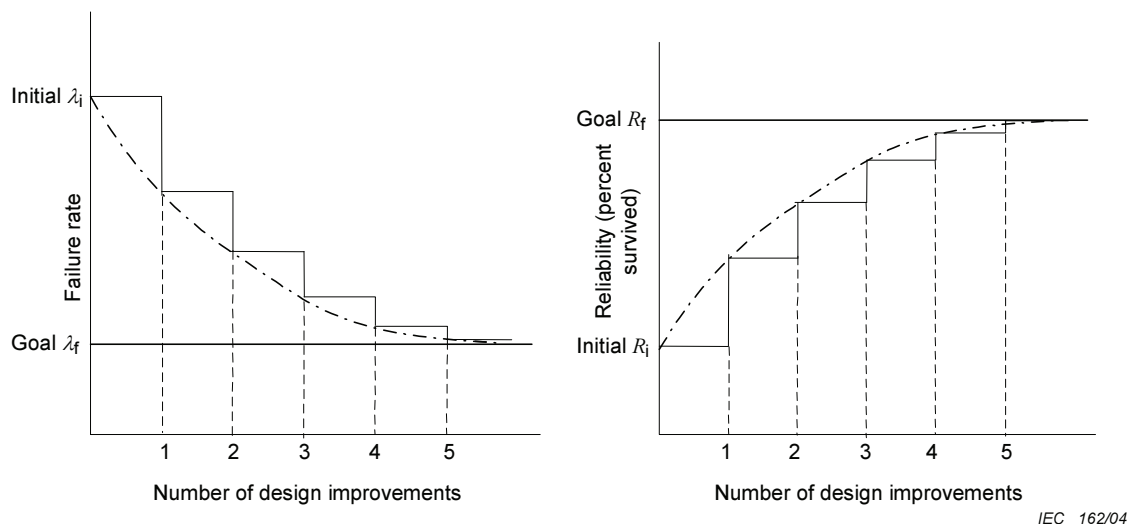


Figure 1 – Planned improvement of the average failure rate or reliability

Within this general framework many models for reliability growth exist. Table 1 provides a summary of the main categories. As well as the distinction between design and test, the type of data available will influence model selection. The continuous category refers to items that operate through time, for example, repaired items. The discrete category refers to data that are collected as if for a success/failure of a trial, for example, one-shot items. The procedures used to estimate reliability growth are labelled classical or Bayesian. The former uses the observed data only, while the latter uses both empirical data from design and test as well as engineering knowledge, for example, regarding the anticipated number of failure modes of concern.

Table 1 – Categories of reliability growth models with clause references

Model type			Time	
			Continuous (time)	Discrete (number of trials)
	Design	Classical	6.1	–
		Bayesian	6.2	–
	Test	Classical	7.1	7.2
		Bayesian	–	–

Many reliability models have been developed for analysing test data. This standard presents one of the most popular growth models, the power law (also known as the AMSAA or the Crow model) in both its continuous and discrete forms. This model is a generalization of the Duane reliability growth model due to Crow [1]¹. Although Bayesian variants of these models exist, they are not presented here. A review of the variety of reliability growth models available for analysing test data can be found in Jewell [2, 3] and Xie [4].

There are fewer documented reports of reliability growth models being used in design. Therefore a reliability growth planning model that is a modification of the power law for use in design and a Bayesian variant of the IBM-Rosner model adapted for design have been introduced. However, these are only given for products operating through continuous time.

In general, the choice of a reliability growth model involves a compromise between simplicity and realism. Selection should be made according to the aforementioned criteria such as stage of lifecycle and type of data, as well as by evaluating the validity of the assumptions underpinning a specific model for the context to which it is to be applied. Further details about the assumptions for the models described in this standard are given in Clauses 6 and 7. Note that reliability growth models should not be regarded as infallible nor should they be applied without discretion but used as statistical tools to aid engineering judgement.

6 Reliability growth models used for systems/products in design phase

6.1 Modified power law model for planning of reliability growth in product design phase

6.1.1 General

The statistical procedure for the modified power law model for the planned reliability growth in the product design phase concerns the necessary implementation of the design reliability improvements by mitigation of a failure mode, or by reduction of its probability of occurrence, and the time from the beginning of design to that improvement.

This model is used for planning purposes (and not for data analysis), to estimate the number or the magnitude of improvements in the original design to increase its reliability from that initially assessed to its goal value. The assumption of a power law for this model is justified by the fact that the early improvements will be those that will contribute the most to the reliability improvement, that is, the failure modes with the highest probability of occurrence will be addressed first, followed by improvements of lesser and lesser reliability contribution. The actual reliability values achieved in the course of the design are then plotted corresponding to the design time when they were realized and compared to the model. This model is thus used to plan the strategies necessary for reliability improvement of a design during the available time period from the initial design revision until the design is completed and released for production.

¹ Figures in square brackets refer to the bibliography.

6.1.2 Planning model for the reliability growth during the product design period

The Krasich reliability growth planning model [5] is derived in the following manner. An example of this model, as well as the spread sheet for its easy determination and plotting, given initial and reliability goal of a product, are given in Annex A.

If the initial product reliability for the predetermined product operational life time T was estimated by analysis or test to be $R_0(T)$, then, assuming that its average failure rate is constant, the initial average failure rate of that product corresponding to the time T is:

$$\lambda_{a0} = -\frac{\ln[R_0(T)]}{T} \quad (1)$$

Assumption of applicability of the power law is justified by the fact that the faults which are found to be the highest unreliability contributors are addressed first, and with design modification (fault mitigation) the product failure rate is continuously improved with a function $d(t)$. The failure rate of the product design at any time during the design period is:

$$\lambda_a(t) = \lambda_{a0} \cdot [1 + d(t)]^{-\alpha_D} \quad (2)$$

where

$d(t)$ is the number of design modifications at any time during the design period;

α_D is the reliability growth rate resultant from fault mitigation;

D is the total number of implemented design improvements;

t_D is the total duration of the design period available for the design improvements.

With a linear approximation, the number of design modifications as a function of time can be linearly distributed over the design period:

$$d(t) = D \cdot \frac{t}{t_D} \quad (3)$$

The average failure rate as a function of time then becomes:

$$\lambda_a(t) = \lambda_{a0} \cdot \left(1 + D \cdot \frac{t}{t_D}\right)^{-\alpha_D} \quad (4)$$

If the goal product average failure rate given the product reliability goal is expressed as $R_G(T)$, then the goal average failure rate at the end of design period t_D , is approximated by:

$$\lambda_{aG}(t_D) = \frac{-\ln[R_G(T)]}{T} \quad (5)$$

At the same time:

$$\begin{aligned} \lambda_{aG}(t_D) &= \lambda_{a0} \cdot \left(1 + D \cdot \frac{t_D}{t_D}\right)^{-\alpha_D} = \lambda_{a0} \cdot (1 + D)^{-\alpha_D} \\ \lambda_{aG}(t_D) &= -\frac{\ln[R_0(T)]}{T} \cdot (1 + D)^{-\alpha_D} \end{aligned} \quad (6)$$

Substituting $\lambda_{aG}(t_D)$ with the expression containing reliability goal and solving for D

$$D = e^{\frac{-\ln\left(\frac{\ln[R_G(T)]}{\ln[R_0(T)]}\right)}{\alpha_D}} - 1 \quad (7)$$

Solving the same equation for the growth rate, expressed as a function of design modifications and initial and reliability goal gives:

$$\alpha_D = \frac{\ln\left(\frac{\ln[R_0(T)]}{\ln[R_G(T)]}\right)}{\ln(1+D)} \quad (8)$$

During the design period, continuous improvement of product reliability that it has to have at the time T is a function of time t (the reliability growth model in the time period from 0 to t_D) can be written as:

$$R(t, T) = \exp(-\lambda_a(t) \cdot T) \quad (9)$$

Substituting the expression for the average failure rate, the Krasich reliability growth model for the design phase $0 < t < t_D$, is derived as follows:

$$R(t, T) = \exp\left(-\lambda_{a0} \cdot \left(1 + D \cdot \frac{t}{t_D}\right)^{-\alpha_D} \cdot T\right) \quad (10)$$

$$R(t, T) = \exp\left(\frac{\ln[R_0(T)]}{T} \cdot \left(1 + D \cdot \frac{t}{t_D}\right)^{-\alpha_D} \cdot T\right) \quad (11)$$

In the above equation, expressing D in terms of initial and reliability goal, the reliability growth as a function of time in design period, available for design improvements becomes:

$$R(t, T) = R_0(T) \left\{ \frac{\left[t_D + t \cdot \left(\frac{\ln[R_G(T)]}{\ln[R_0(T)]} \right)^{-\frac{1}{\alpha_D}} - t \right]}{t_D} \right\}^{-\alpha_D} \quad (12)$$

6.1.3 Tracking the achieved reliability growth

Tracking of the achieved reliability growth means a simple recalculation of the assessed product reliability at the time the design was improved to account for the modifications. The reliability value calculated for the same predetermined life or mission period is simply plotted on the same plot with the reliability growth model for the corresponding design time.

The resultant entries to the graph can then be fitted with a best-fit line (power), or the values on the graph may be simply connected with the straight lines and the resultant achieved reliability is compared to the reliability growth model.

Use of fault tree analysis with commercially available software makes assessment of the reliability improvement easy and quick to accomplish and track as the product reliability is automatically calculated based on the changes.

After completion of the product design and with the introduction of the product validation phase, the planned reliability growth test may further improve product reliability or uncover failure modes that were not accounted for during analytical evaluations. The final reliability assessment of the completed design can then serve as the reliability goal for the reliability growth testing.

An example of practical derivation and application of the planning growth model for reliability improvement in design phase is shown in A.1.1. This real life example shows step by step how the model is constructed and how it is used.

6.2 Modified Bayesian IBM-Rosner model for planning reliability growth in design phase

6.2.1 General

A model is presented to describe the growth of reliability during the design phase of a repairable item prepared by Quigley and Walls [6] to [8] and is based on a Bayesian adaptation of the IBM-Rosner model [9] which was developed for analysing test data and is described in 7.1.2.

It is assumed a design has been developed to a sufficient level of detail to provide an initial estimate of reliability. It is further assumed that the reliability goal is specified. Modifications to the design will be made with a view to improving reliability until the goal is achieved. The model aims to capture the possible timings of the design modifications.

The model assumes that design review and re-assessments result in modifications with the aim of improving reliability and achieving the goal. The rate of growth as measured by advancing the initial reliability to the reliability goal is a function of the removal of aspects of design that contribute to systematic failures. It is assumed through using the model under consideration that there are greater improvements to reliability in the earlier re-design compared with later stages of re-design.

The model can be used in two ways:

- a) to predict the length of time required to achieve reliability goal by forecasting the reliability of the design – this assumes that the expected growth rate can be quantified; or
- b) to estimate the growth rate required to achieve the reliability goal from the initial estimate during a specified design period duration – this assumes that the duration of the design phase is fixed.

Details concerning the mathematical formulation of the model are given in Annex B.

6.2.2 Data requirements

Data are required concerning the reliability goal ($R_G(T)$) and either the duration of the design phase (t_D) or the expected growth rate during design (a_D) according to the purpose of the model application.

The failure rate of non-systematic failures (λ_{NS}) should be specified. This may be estimated from historical data for similar product designs operating in nominally identical environments.

All potential design improvements to mitigate the D potential weaknesses should be identified and may be allocated to one of K fault classes as appropriate.

The probability of each design weakness within each fault class resulting in failure during the specified life of the product should be estimated. This may be based on engineering judgement. The probability for the j -th design weakness in fault class k is given by p_{kj} .

The expected number of design weaknesses in fault class k (η_k) resulting in failure if no modifications are implemented to the product design can be calculated using

$$\eta_k = \sum_{j=1}^{D_k} p_{kj} \quad (13)$$

where D_k is the total number of design weaknesses expected in fault class k .

The failure rate for each fault class is required. These may be estimated from historical data for similar product designs operating in nominally identical environments. The failure rate for fault class k is given by λ_k .

6.2.3 Estimates of reliability growth and related parameters

In this subclause, equations are given to compute the key parameters of the reliability growth model.

The initial reliability of the design at time T is calculated by

$$R_i(T) = \exp \left\{ - \left[\lambda_{NS} T + \sum_{k=1}^K \eta_k (1 - e^{-\lambda_k T}) \right] \right\} \quad (14)$$

The reliability growth of the design at time T is given by

$$R(T) = \exp \left\{ - \left[\lambda_{NS} T + \sum_{k=1}^K \eta_k e^{-\alpha_D T} (1 - e^{-\lambda_k T}) \right] \right\} \quad (15)$$

If a growth rate is to be estimated, then the expected growth rate required to reach the reliability goal, given the reliability goal and the specified duration of the design period, is given by

$$\alpha_D = \frac{\ln \left[\frac{\sum_{k=1}^K \eta_k (1 - e^{-\lambda_k T})}{-\ln[R_G(T)] - \lambda_{NS} T} \right]}{t_D} \quad (16)$$

If a growth rate has been specified, an estimate of the expected time to reach the reliability goal is given by

$$t_G = \frac{\ln \left[\frac{\sum_{k=1}^K \eta_k (1 - e^{-\lambda_k T})}{-\ln[R_G(T)] - \lambda_{NS} T} \right]}{\alpha_D} \quad (17)$$

6.2.4 Tracking reliability growth during design phase

The tracking of the reliability growth with this modified Bayesian IBM-Rosner model is the same as that described in 6.1.3.

7 Reliability growth planning a tracking in the product reliability growth testing

7.1 Continuous reliability growth models

7.1.1 The power law model

The statistical procedures for the power law reliability growth model use the original relevant failure and time data from the test. Except in the projection technique (see 9.6), the model is applied to the complete set of relevant failures (as in IEC 61014, Figure 2 and Figure 4, characteristic 3) without subdivision into categories.

The basic equations for the power law model are given in this subclause. Background information on the model is given in Annex B.

The expected accumulated number of failures up to test time T is given by:

$$E[N(T)] = \lambda T^\beta, \text{ with } \lambda > 0, \beta > 0, T > 0 \quad (18)$$

where

- λ is the scale parameter;
- β is the shape parameter (a function of the general effectiveness of the improvements; $0 < \beta < 1$, corresponds to reliability growth; $\beta = 1$ corresponds to no reliability growth; $\beta > 1$ corresponds to negative reliability growth).

The current failure intensity after T h of testing is given by:

$$z(T) = \frac{d}{dT} E[N(T)] = \lambda \beta T^{\beta-1}, \text{ with } T > 0 \quad (19)$$

Thus, parameters λ and β both affect the failure intensity achieved in a given time. The equation represents in effect the slope of a tangent to the $N(T)$ against T characteristic at time T as shown in IEC 61014, Figure 9.

The current mean time between failures after T h of testing is given by:

$$\theta(T) = \frac{1}{z(T)} \quad (20)$$

Methods are given in 9.1 and 9.2 for maximum likelihood estimation of the parameters λ and β . Subclause 9.3 gives goodness-of-fit tests for the model, and 9.4 and 9.5 discuss confidence interval procedures. An extension of the model for reliability growth projections is given in 9.6.

The model has the following characteristic features:

- It is simple to evaluate.
- When the parameters have been estimated from past programmes, it is a convenient tool for planning future programmes employing similar conditions of testing and equal improvement effectiveness (see Clause 7, and IEC 61014, 6.4.1 to 6.4.7).
- It gives the unrealistic indications that $z(T) = \infty$ at $T = 0$ and that growth can be unending, that is $z(T)$ tends to zero as T tends to infinity. However, these limitations do not generally affect its practical use.

- It is relatively slow and insensitive in indicating growth immediately after a corrective modification, and so may give a low (that is, pessimistic) estimate of the final $\theta(T)$, unless projection is used (see 9.6).
- The normal evaluation method assumes the observed times to be exact times of failure, but an alternative approach is possible for groups of failures within a known time period (see 9.2.2).

7.1.2 The fixed number of faults model

This model, also known as the IBM/Rosner model [9], assumes the following:

- there are random (constant intensity function) failures occurring at a rate z ; and
- there is a fixed, but unknown, number of non-random design, manufacturing and workmanship faults present in the product at the beginning of testing.

The model limitation is the assumption that the effectiveness factor for fault mitigation is equal to unity.

Rate of change of $N(T)$, with respect to time, is proportional to the number of non-random faults remaining at time T :

$$\begin{aligned}\frac{dN(T)}{dT} &= -K_2 \cdot N(T) \\ \text{thus :} & \\ N(T) &= e^{-K_2 \cdot T + c}\end{aligned}\tag{21}$$

If the number of faults at $T = 0$ is K_1 , then:

$$N(T) = K_1 \cdot e^{-K_2 \cdot T}\tag{22}$$

with an assumption that

$$T > 0, K_1, K_2 > 0$$

If $E[N(T)]$ is to be the expected cumulative number of faults up to time T , then:

$$E[N(T)] = z \cdot T + K_1 \cdot (1 - e^{-K_2 \cdot T})\tag{23}$$

The equation above means that by the time T , the total number of faults is equal to the sum of random and non-random faults. Here $E[N(0)] = 0$.

With time approaching infinity:

$$E[N(T)] \rightarrow \infty\tag{24}$$

Since the model is non-linear, the estimate of λ , K_1 , and K_2 has to be accomplished by iterative methods.

The model allows prediction of the time when the product is " q " fraction debugged (fraction of the original non-random faults removed, while $0 < q < 1$).

The number of non-random faults removed by the time t is:

$$N(0) - N(t) = K_1 - K_1 \cdot e^{-K_2 \cdot T}\tag{25}$$