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Statistical interpretation of data—Tests and confidence intervals relating to proportions

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PREFACE

This Standard was prepared by the Australia members of Joint Standards Australia/New Zealand Committee QR-008, Quality Systems. After consultation with stakeholders in both countries, Standards Australia and Standards New Zealand decided to develop this Standard as an Australian Standard rather than an Australia/New Zealand Standard.

This Standard is identical with, and has been reproduced from ISO 11453:1996, *Statistical interpretation of data—Tests and confidence intervals relating to proportions*.

The objective of this Standard is to describe statistical methods to analysts for determining proportions of a population(s) in specified instances that determine what proportion of a population has a particular characteristic.

The terms ‘normative’ and ‘informative’ are used to define the application of the annex to which they apply. A normative annex is an integral part of a standard, whereas an informative annex is only for information and guidance.

As this Standard is reproduced from an international standard, the following applies:

(a) Its number appears on the cover and title page while the international standard number appears only on the cover

(b) In the source text ‘this International Standard’ should read ‘this Australian Standard’.

(c) A full point substitutes for a comma when referring to a decimal marker.

(d) None of the normative references in the source document have been adopted as Australian or Australian/New Zealand Standards.

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CONTENTS

		Page
1	Scope	**1**
2	Normative reference	**1**
3	Definitions	**1**
4	Symbols	**2**
5	Point estimator of the proportion p	**2**
6	Confidence limits for the proportion p	**2**
7	Significance tests on proportions p	**3**
7.1	General	**3**
7.2	Comparison of a proportion with a given value p_0	**3**
7.3	Comparison of two proportions	**4**
8	Forms	**5**
8.1	A Forms: Confidence interval for the proportion p	**6**
8.2	B Forms: Comparison of the proportion p with a given value p_0	**10**
8.3	C Forms: Comparison of two proportions	**16**

Annexes

A	Computation of the operating characteristics of the test according to the B forms	**40**
B	Examples of completed forms	**43**
C	Bibliography	**57**

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AUSTRALIAN STANDARD

Statistical interpretation of data — Tests and confidence intervals relating to proportions

1 Scope

This International Standard describes specific statistical methods for addressing the following questions.

a) Given a population of items from which a sample of n items has been drawn, x of the sample items are found to show a specified characteristic. What proportion of the population has that characteristic? (See A forms, subclause 8.1.)

b) Is the proportion estimated in a) different from a nominal (specified) value? (See B forms, subclause 8.2.)

c) Given two distinct populations, are the proportions with the characteristic in the two populations different? (See C forms, subclause 8.3.)

d) In b) and c) how many items must be sampled in the population(s) to be sufficiently sure that the result of the test is correct? (See 7.2.3 and 7.3.3.)

It is essential that the drawing of samples does not have any appreciable effect on the population. If the sample drawn at random is less than 10 % of the population this is usually satisfactory, but if the sample is greater than this, reliable results can be obtained only by replacing each item sampled before drawing the next item at random from the population.

2 Normative reference

The following standard contains provisions which, through reference in this text, constitute provisions of this International Standard. At the time of publication, the edition indicated was valid. All standards are subject to revision, and parties to agreements based on this International Standard are encouraged to investigate the possibility of applying the most recent edition of the standard indicated below. Members of IEC and ISO maintain registers of currently valid International Standards.

ISO 3534-1:1993, *Statistics — Vocabulary and symbols — Part 1: Probability and general statistical terms.*

3 Definitions

For the purposes of this International Standard, the definitions given in ISO 3534-1 and the following definition apply.

3.1 target item: One in which the specified characteristic is found.

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4 Symbols

α	significance level chosen
α'	achieved significance level
$1-\alpha$	confidence level chosen
β	probability of the error of the second kind
n; n_1; n_2	sample size; sample size of sample 1; sample size of sample 2
X	number of target items in the sample (random variable)
x	value of X
p	proportion of target items in the population
$p_{u,o}$	upper limit of the one-sided confidence interval for p
$p_{l,o}$	lower limit of the one-sided confidence interval for p
$p_{u,t}$	upper limit of the two-sided confidence interval for p
$p_{l,t}$	lower limit of the two-sided confidence interval for p
T	value from table 2 for the determination of confidence limits for $n \leqslant 30$
$C_{l,o}$	critical value of the test of the null hypothesis H_0: $p \geqslant p_0$
$C_{u,o}$	critical value of the test of the null hypothesis H_0: $p \leqslant p_0$
$C_{l,t}$	lower critical value of the test of the null hypothesis H_0: $p = p_0$
$C_{u,t}$	upper critical value of the test of the null hypothesis H_0: $p = p_0$
p_0	given value for p
p'	value of p for which the probability of not rejecting the null hypothesis (P_a) is to be determined
P_a	probability of not rejecting the null hypothesis
f_1, f_2	number of degrees of freedom of the F-distribution
F_1, F_2	test statistics
$F_q(f_1, f_2)$	q-quantile of the F-distribution with f_1 and f_2 degrees of freedom
z_1, z_2	test statistics
u_q	q-quantile of the standard normal distribution
q, η, K	auxiliary values

5 Point estimator of the proportion p

The estimator of p from a sample of n items including x target items is

$$\hat{p} = x/n$$

This estimator is unbiased if the sample is drawn at random, whatever the sample size and population size may be, even if the sample forms an appreciable part of the population.

6 Confidence limits for the proportion p

The calculation of a confidence interval for p is described in forms A-1 to A-3.

The confidence limits will depend on sample size (n), the number of target items in the sample (x), and the desired confidence level ($1-\alpha$). It is not possible in general to achieve the desired confidence level exactly because the probability distribution governing the outcome x is discrete. Thus, the procedure yields the nearest confidence level greater than or equal to ($1-\alpha$).

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The procedure used in this International Standard for determining the two-sided confidence limits for the desired confidence level $(1-\alpha)$ uses the limits for the upper and lower one-sided limits each for the desired confidence level $(1-\alpha/2)$. It thereby guarantees that the error probability is less than or equal to $\alpha/2$ on each side of the interval.

7 Significance tests on proportions p

7.1 General

For practical applications, forms B-1 to B-3 and C-1 to C-3 present the null hypotheses concerning proportions and schemes for carrying out tests. At the beginning of the procedures, the appropriate null hypothesis and the sample size n (the sample sizes n_1 and n_2) are to be determined and the significance level is to be chosen. Because the underlying sampling distributions are discrete, the procedures are designed to achieve the nearest significance level less than or equal to the desired (nominal) level. The alternative hypotheses are not indicated in the forms because in each application the alternative hypothesis is implicitly assumed to be complementary to the null hypothesis.

EXAMPLES

At the beginning of the B forms (procedure for the comparison of a proportion with a given value), one of the following three null hypotheses H_0 (with the complementary alternative hypothesis H_1) is to be chosen:

a) one-sided test with $H_0: p \geq p_0$ $\quad H_1: p < p_0$

b) one-sided test with $H_0: p \leq p_0$ $\quad H_1: p > p_0$

c) two-sided test with $H_0: p = p_0$ $\quad H_1: p \neq p_0$

where p_0 is the given value.

The result of each test is either to reject or not to reject the null hypothesis.

Rejecting the null hypothesis means adopting the alternative hypothesis. Not rejecting the null hypothesis does not necessarily mean accepting the null hypothesis (see 7.2.2).

7.2 Comparison of a proportion with a given value p_0

7.2.1 Test procedure

The test procedures for the null hypotheses

$H_0: p \geq p_0$

$H_0: p \leq p_0$

$H_0: p = p_0$

(where p_0 is the given value) are described in forms B-1 to B-3. They are especially easy to apply if the critical value(s) for the specified values of n, p and α is (are) known. The critical value(s) may have been determined through repeated execution of the test in accordance with the B forms. Otherwise the standard procedure to determine the critical values is the one given in the same forms.

7.2.2 Operating characteristics

The computation of the operating characteristics (including the probability of the error of the first kind, the achieved significance level and the probability of the error of the second kind) is described in annex A. For this purpose the critical value(s) need(s) to be known (see 7.2.1) and the alternative hypothesis, $p = p_1$, for which the probability of the error of the second kind is to be computed has to be chosen.

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7.2.3 Determination of sample size n

If the sample size is not already specified (for example for economic or technical reasons), its minimum value shall be determined such that for a given null hypothesis H_0 (see 7.2.1), the achieved value of the significance level α is not greater than its chosen or given value. In addition, the achieved value of the type II error, probability β, shall be approximately equal to its chosen or given value if p equals a particular chosen or given value p'. For this purpose, p_0 and p' are to be marked on the p-scale and α, $(1-\alpha)$, $\alpha/2$, $(1-\alpha/2)$ on the P-scale and the straight lines 1 and 2 as defined through the procedure shown in table 1 are drawn in the Larson nomograph (figure 2).

Table 1 — Procedure for determining the sample size from the Larson nomograph (figure 2)

Case	Given value	Straight line 1 from p_0 to	Straight line 2 from p' to
$H_0: p \geq p_0$	$p' < p_0$	α	$1-\beta$
$H_0: p \leq p_0$	$p' > p_0$	$1-\alpha$	β
$H_0: p = p_0$	$p' > p_0$	$1-\alpha/2$	β
$H_0: p = p_0$	$p' < p_0$	$\alpha/2$	$1-\beta$

The point of intersection of the two lines leads to the values $C_{l,o}(C_{u,o})$ on the x-scale. If x is not an integer, round up or down to the next integer.

7.3 Comparison of two proportions

7.3.1 Test procedure

The test procedures for the null hypothesis

$H_0: p_1 \geq p_2$

$H_0: p_1 \leq p_2$

$H_0: p_1 = p_2$

(where p_1 is the proportion of target items in population 1 and p_2 is the proportion of target items in population 2) are described in forms C-1 to C-3. These procedures are also suitable for testing the independence of two attributes (dichotomous characteristics) of items in a population.

7.3.2 Operating characteristics

It is assumed:

a) that for a one-sided test of H_0: $p_1 \leq p_2$, the power $(1-\beta)$ must be determined for a given pair of proportions p_1 and p_2, with $p_1 > p_2$;

b) that the test is carried out with two samples of the same size, i.e. $n_1 = n_2 = n$.

The significance level is α. Then a very accurate approximate value of the power can be obtained by the arc sine transformation (proposed by Walters [1]) as follows:

$$1-\beta = \Phi\left(z - u_{1-\alpha}\right)$$

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where

Φ is the distribution function of the standard normal distribution,

$u_{1-\alpha}$ is the $(1-\alpha)$-quantile of that normal distribution, and

$$z = \sqrt{2n}\left[\arcsin\sqrt{p_1 - (1/2n)} - \arcsin\sqrt{p_2 - (1/2n)}\right]$$

This approximation can also be used for the two-sided case: $H_0: p_1 = p_2$ with the alternative hypothesis $H_1: p_1 > p_2$ if α is replaced by $\alpha/2$ in the formula.

7.3.3 Determination of sample size n

If the sample sizes n_1 and n_2 are not predetermined, their minimum values shall be determined such that the power of the test is at least $(1 - \beta)$ while the significance level is α.

It is assumed that the null hypothesis is $H_0: p_1 \leqslant p_2$. The following procedures, however, also apply in the two-sided case $H_0: p_1 = p_2$, with the restricted alternative hypothesis $H_1: p_1 > p_2$ if α is replaced by $\alpha/2$.

Exact values of the sample size are given in tables 5 and 6 (originally published by Haseman[2]) for selected values of α and β. These tables assume a common sample size $n = n_1 = n_2$.

For configurations of α, p_1, p_2 and $(1-\beta)$ not covered by these tables, the following approximation can be used which also allows for unequal sample sizes. It requires that the ratio r of sample sizes n_1/n_2 has been chosen in advance.

$$n_1 = \frac{n'}{4}\left[1 + \sqrt{1 + \frac{2(r+1)}{rn'(p_1 - p_2)}}\right]^2$$

$$n_2 = n_1 / r$$

where

$$n' = \frac{\left\{u_{1-\alpha}\sqrt{(r+1)\,\bar{p}\,\bar{q}} + u_{1-\beta}\sqrt{\left[rp_1(1-p_1) + p_2(1-p_2)\right]}\right\}^2}{r(p_1 - p_2)^2}$$

$$\bar{p} = \frac{rp_1 + p_2}{r+1}$$

$$\bar{q} = 1 - \bar{p}$$

8 Forms

For ease of application, make a tick in the box representing the activated part of the form. (The horizontal position of the box symbolizes the position of the respective part in the hierarchy of the form, decreasing from the right to the left.) Then follow the procedure by entering the necessary data and carrying out the actions required.

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8.1 A forms: Confidence interval for the proportion p

8.1.1 Form A-1: One-sided, with upper limit confidence interval for the proportion p

Characteristic:
Determination procedure:
Items:
Criterion for the identification of target items:
Notes:

Confidence level chosen: $1-\alpha =$
Sample size: $n =$
Number of target items in the sample: $x =$

Determination of confidence limits:

a) Procedure for $n \leqslant 30$ ☐

1) Case $x = n$ ☐
$p_{u,o} = 1$

2) Case $x < n$ ☐

Read the value from table 2 for the known values n, $X = x$ and $q = 1 - \alpha$ (this value is the confidence limit):
$T_{(1-\alpha)}(n, x) = p_{u,o} =$

b) Procedure for $n > 30$ ☐

1) Case $x = 0$ ☐
Computation:
$p_{u,o} = 1 - \alpha^{1/n} =$

2) Case $x = n$ ☐
$p_{u,o} = 1$

3) Case $0 < x < n$ ☐

Read the value from table 3 for $q = 1-\alpha$: $u_{1-\alpha} =$
Read the value d corresponding to the confidence level chosen:

$1-\alpha$	0,90	0,95	0,99
d	0,411	0,677	1,353

Computation:
$p_{u,o} = p_* + (1-2p_*)\,d/(n+1) + u_{1-\alpha}\sqrt{p_*(1-p_*)\left[1-d/(n+1)\right]/(n+1)} =$
with $p_* = (x+1)/(n+1)$

Result:
$p \leqslant p_{u,o} =$

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8.1.2 Form A-2: One-sided, with lower limit confidence interval for the proportion p

Characteristic:
Determination procedure:
Items:
Criterion for the identification of target items:
Notes:

Confidence level chosen: $1-\alpha =$
Sample size: $n =$
Number of target items in the sample: $x =$

Determination of confidence limits:

a) Procedure for $n \leq 30$ ☐

1) Case $x = 0$ ☐
$p_{l,o} = 0$

2) Case $x > 0$ ☐

Read the value from table 2 for the known values n, $X = n - x$ and $q = 1-\alpha$:

$T_{(1-\alpha)}(n, n-x) =$

Computation:
$p_{l,o} = 1 - T_{(1-\alpha)}(n, n-x) =$

b) Procedure for $n > 30$ ☐

1) Case $x = 0$ ☐
$p_{l,o} = 0$

2) Case $x = n$ ☐
Computation:
$p_{l,o} = \alpha^{1/n} =$

3) Case $0 < x < n$ ☐

Read the value from table 3 for $q = 1-\alpha$: $u_{1-\alpha} =$
Read the value of d corresponding to the confidence level chosen:

$1-\alpha$	0,90	0,95	0,99
d	0,411	0,677	1,353

Computation:
$p_{l,o} = p_* + (1-2p_*)\,d/(n+1) - u_{1-\alpha}\sqrt{p_*(1-p_*)\left[1-d/(n+1)\right]/(n+1)} =$
with $p_* = x/(n+1)$

Result:
$p_{l,o} = \quad \leq p$

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8.1.3 Form A-3: Two-sided confidence interval for the proportion p

Characteristic:
Determination procedure:
Items:
Criterion for the identification of target items:
Notes:

Confidence level chosen: $1-\alpha =$
Sample size: $n =$
Number of target items in the sample: $x =$

Determination of confidence limits:

a) Procedure for $n \leqslant 30$ ☐

1) Upper confidence limit

— Case $x = n$ ☐

$p_{u,t} = 1$

— Case $x < n$ ☐

Read the value from table 2 for the known values n, $X = x$ and $q = 1-\alpha/2$ (this value is the confidence limit):

$T_{(1-\alpha/2)}(n, x) = p_{u,t} =$

2) Lower confidence limit

— Case $x = 0$ ☐

— Case $x > 0$ ☐

Read the value from table 2 for the known values n, $X = n - x$ and $q = 1-\alpha/2$:

$T_{(1-\alpha/2)}(n, n-x) =$

Computation:

$p_{l,t} = 1 - T_{(1-\alpha/2)}(n, n-x) =$

b) Procedure for $n > 30$ ☐

1) Upper confidence limit

— Case $x = 0$ ☐

Computation:

$p_{u,t} = 1-(\alpha/2)^{1/n} =$

— Case $x = n$ ☐

$p_{u,t} = 1$

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— Case $0 < x < n$ ☐

Read the value from table 3 for $q = 1-\alpha/2$: $u_{1-\alpha/2}$ =

Read the value of d corresponding to the confidence level chosen:

$1-\alpha$	0,90	0,95	0,99
d	0,677	0,960	1,659

Computation:

$$p_{u,t} = p_* + (1-2p_*)\,d/(n+1) + u_{1-\alpha/2}\sqrt{p_*(1-p_*)\left[1-d/(n+1)\right]/(n+1)} =$$

with $p_* = (x+1)/(n+1)$

2) Lower confidence limit

— Case $x = 0$ ☐

$p_{l,t} = 0$

— Case $x = n$ ☐

Computation:

$$p_{l,t} = (\alpha/2)^{1/n} =$$

— Case $0 < x < n$ ☐

Read the value from table 3 for $q = 1-\alpha/2$: $u_{1-\alpha/2}$ =

Read the value of d corresponding to the confidence level chosen:

$1-\alpha$	0,90	0,95	0,99
d	0,677	0,960	1,659

Computation:

$$p_{l,t} = p_* + (1-2p_*)\,d/(n+1) - u_{1-\alpha/2}\sqrt{p_*(1-p_*)\left[1-d/(n+1)\right]/(n+1)} =$$

with $p_* = x/(n+1)$

Results:

$p_{l,t}$ = ; $p_{u,t}$ = ; $p_{l,t} \leq p \leq p_{u,t}$

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8.2 B Forms: Comparison of the proportion p with a given value p_0

8.2.1 Form B-1: Comparison of the proportion p with a given value p_0 and with one-sided test with H_0: $p \geq p_0$

Characteristic:
Determination procedure:
Items:
Criterion for the identification of target items:
Notes:

Given value p_0 =
Significance level chosen: α =
Sample size: n =
Number of target items found in the sample: x =

Test procedure:

I The critical value(s) is (are) already known (see 7.2.1 and, if applicable, the determination of the critical values below): ☐
$C_{l,o}$ =
H_0 is rejected if $x < C_{l,o}$; otherwise it is not rejected.

II The critical value(s) is(are) not known: ☐

a) Case $x \geq p_0 n$ ☐
H_0 is not rejected

b) Case $x < p_0 n$ ☐

1) Procedure for $n \leq 30$ ☐
Determine according to form A-1 the one-sided upper confidence limit for n, x and the confidence level $(1-\alpha)$:
$p_{u,o}$ =
H_0 is rejected if $p_{u,o} < p_0$; otherwise it is not rejected.

2) Procedure for $n > 30$ ☐

— Case $x = 0$ ☐
Computation:
$p_{u,o} = 1-\alpha^{1/n}$ = [see form A-1 b) 1)]
H_0 is rejected if $p_{u,o} < p_0$; otherwise it is not rejected.

— Case $0 < x < n$ ☐
Read the value from table 3 for $q = 1-\alpha$: $u_{1-\alpha}$ =
Computation:
$u_1 = 2\left[\sqrt{(n-x)p_0} - \sqrt{(x+1)(1-p_0)}\right]$ =
H_0 is rejected if $u_1 > u_{1-\alpha}$; otherwise it is not rejected.

Test result:
H_0 is rejected ☐ H_0 is not rejected ☐

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Determination of the critical values:

$C_{l,o}$ is the smallest non-negative integer x for which the test according to form B-1-II does not lead to the rejection of H_0. $C_{l,o}$ is to be determined iteratively through repeated application of form B-1-II with different values of x [1]. Thereby those values of x are to be determined which differ from each other by 1 and one of which leads to the rejection of the null hypothesis while the other does not. If desired a start value for x, x_{start} can be obtained as follows.

Computations:

np_0, rounded to the next integer, is $x^* =$

$p_{l,o}|_{x=x^*} =$ ($p_{l,o}|_{x=x^*}$ from form A-2)

$np_{l,o}|_{x=x^*}$, rounded to the next integer, is $x_{start} =$

Interpretation of the test results from form B-1-II:

for $x \leq C_{l,o} - 1 =$ H_0 is rejected

for $x \geq C_{l,o} =$ H_0 is rejected

Result:

$C_{l,o} =$

1) The critical value or one of the critical values, respectively, may not exist for extreme values of p_0 and/or for very small sample sizes n.

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8.2.2 Form B-2: Comparison of the proportion p with a given value p_0 and with one-sided test with $H_0: p \leq p_0$

Characteristic:
Determination procedure:
Items:
Criterion for the identification of target items:
Notes:

Given value $p_0 =$
Significance level chosen: $\alpha =$
Sample size: $n =$
Number of target items found in the sample: $x =$

Test procedure:

I The critical value(s) is (are) already known (see 7.2.1 and, if applicable, the determination of the critical values below): ☐
$C_{u,o} =$

H_0 is rejected, if $x > C_{u,o}$; otherwise it is not rejected.

II The critical value(s) is (are) not known: ☐

a) Case $x \leq p_0 n$ ☐
H_0 is not rejected.

b) Case $x > p_0 n$ ☐

1) Procedure for $n \leq 30$ ☐
Determine according to form A-2 the one-sided lower confidence limit for n, x and the confidence level $(1-\alpha)$:
$p_{l,o} =$
H_0 is rejected if $p_{l,o} > p_0$; otherwise it is not rejected.

2) Procedure for $n > 30$ ☐

— Case $x = n$ ☐
Computation :
$p_{l,o} = \alpha^{1/n} =$ [see form A-2 b) 2)]
H_0 is rejected if $p_{l,o} > p_0$; otherwise it is not rejected.

— Case $0 < x < n$ ☐

Read the value from table 3 for $q = 1-\alpha$: $u_{1-\alpha} =$
Computation:
$u_2 = 2\left[\sqrt{x(1-p_0)} - \sqrt{(n-x+1)p_0}\right] =$
H_0 is rejected if $u_2 > u_{1-\alpha}$; otherwise it is not rejected.

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Test result:

H_0 is rejected ☐

H_0 is not rejected ☐

Determination of the critical values:

$C_{u,o}$ is the largest integer x for which the test according to form B-2-II does not lead to the rejection of the null hypothesis. $C_{u,o}$ is to be determined iteratively through repeated application of form B-2-II with different values of x [1]. Thereby those values of x are to be determined which differ from each other by 1 and one of which leads to the rejection of the null hypothesis while the other does not. If desired, a start value for x, x_{start} can be obtained as follows.

Computations:

np_0, rounded to the next integer, is $x^* =$

$p_{u,o}|_{x=x^*} =$ ($p_{u,o}|_{x=x^*}$ from form A-1)

$np_{u,o}|_{x=x^*}$, rounded to the next integer, is $x_{start} =$

Interpretation of the test results from form B-2-II:

for $x \leq C_{u,o} =$ H_0 is not rejected

for $x \geq C_{u,o}+1 =$ H_0 is rejected

Result:

$C_{u,o} =$

1) The critical value or one of the critical values, respectively, may not exist for extreme values of p_0 and/or for very small sample sizes n.

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8.2.3 Form B-3: Comparison of the proportion p with a given value p_0 and with two-sided test with H_0: $p = p_0$

Characteristic:
Determination procedure:
Items:
Criterion for the identification of target items:
Notes:

Given value p_0 =
Significance level chosen: α =
Sample size: n =
Number of target items found in the sample: x =

Test procedure:

I The critical value(s) is (are) already known (see 7.2.1 and, if applicable, the determination of the critical values below): ☐

$C_{l,t}$ = $C_{u,t}$ =

H_0 is rejected if $x < C_{l,t}$ or $x > C_{u,t}$; otherwise, it is not rejected.

II The critical value(s) is (are) not known: ☐

a) Procedure for $n \leq 30$ ☐

Determine according to form A-3 the two-sided confidence limits for n, x, and the confidence level $(1-\alpha)$:
$p_{l,t}$ = and
$p_{u,t}$ =

H_0 is rejected if $p_{l,t} > p_0$ or $p_{u,t} < p_0$; otherwise, it is not rejected.

b) Procedure for $n > 30$ ☐

1) Case $x = 0$ ☐

Computation:
$p_{u,t} = 1-(\alpha/2)^{1/n}$ =
H_0 is rejected if $p_{u,t} < p_0$; otherwise, it is not rejected.

2) Case $x = n$ ☐

Computation:
$p_{l,t} = (\alpha/2)^{1/n}$ =
H_0 is rejected if $p_{l,t} > p_0$; otherwise, it is not rejected.

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3) Case $0 < x < n$ ☐

Read the value from table 3 for $q = 1-\alpha/2$: $u_{1-\alpha/2} =$

Computations:

$u_1 = 2\left[\sqrt{(n-x)p_0} - \sqrt{(x+1)(1-p_0)}\right] =$

$u_2 = 2\left[\sqrt{x(1-p_0)} - \sqrt{(n-x+1)p_0}\right] =$

H_0 is rejected if $u_1 > u_{1-\alpha/2}$ or $u_2 > u_{1-\alpha/2}$; otherwise, it is not rejected.

Test result:

H_0 is rejected ☐

H_0 is not rejected ☐

Determination of the critical values:

$C_{l,t}$ is the smallest non-negative integer x and $C_{u,t}$ is the largest integer x for which the test according to form B-3-II does not lead to the rejection of H_0. $C_{l,t}$ and $C_{u,t}$ are to be determined iteratively through repeated application of form B-3-II with different values of x [1]. Thereby two pairs of values are to be determined such that in each pair the values differ from each other by 1 and one of the values leads to the rejection of the null hypothesis while the other does not. If desired, start values for x, x_{start} can be obtained as follows.

Computations:

np_0, rounded to the next integer, is $x^* =$

$p_{l,t}|_{x=x^*} =$ $p_{u,t}|_{x=x^*} =$

$p_{l,t}|_{x=x^*}$ and $p_{u,t}|_{x=x^*}$ from form A-3

$np_{l,t}|_{x=x^*}$, rounded to the next integer, is x_{start} (lower) =

$np_{u,t}|_{x=x^*}$, rounded to the next integer, is x_{start} (upper) =

Interpretation of the test results from form B-3-II:

for $x \leq C_{l,t} - 1 =$		H_0 is rejected
for $x = C_{l,t} =$	to $x = C_{u,t} =$	H_0 is not rejected
for $x \geq C_{u,t} + 1 =$		H_0 is rejected

Results:

$C_{l,t} =$ $C_{u,t} =$

1) The critical value or one of the critical values, respectively, may not exist for extreme values of p_0 and/or for very small sample sizes n.

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