

Standard Guideline for Fitting Saturated Hydraulic Conductivity Using Probability Density Functions

ASCE/EWRI 50-08

Standard Guideline for Calculating the Effective Saturated Hydraulic Conductivity

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This document uses both the
International System of Units (SI)
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American Society of Civil Engineers

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Fitting of Hydraulic Conductivity Using Statistical Spatial Estimation
Standards Committee of the
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The following standards have been issued:

- ANSI/ASCE 1-82 N-725 Guideline for Design and Analysis of Nuclear Safety Related Earth Structures
- ASCE/EWRI 2-06 Measurement of Oxygen Transfer in Clean Water
- ANSI/ASCE 3-91 Standard for the Structural Design of Composite Slabs and ANSI/ASCE 9-91 Standard Practice for the Construction and Inspection of Composite Slabs
- ASCE 4-98 Seismic Analysis of Safety-Related Nuclear Structures
- Building Code Requirements for Masonry Structures (ACI 530-02/ASCE 5-02/TMS 402-02) and Specifications for Masonry Structures (ACI 530.1-02/ASCE 6-02/TMS 602-02)
- ASCE/SEI 7-05 Minimum Design Loads for Buildings and Other Structures
- SEI/ASCE 8-02 Standard Specification for the Design of Cold-Formed Stainless Steel Structural Members
- ANSI/ASCE 9-91 listed with ASCE 3-91
- ASCE 10-97 Design of Latticed Steel Transmission Structures
- SEI/ASCE 11-99 Guideline for Structural Condition Assessment of Existing Buildings
- ASCE/EWRI 12-05 Guideline for the Design of Urban Subsurface Drainage
- ASCE/EWRI 13-05 Standard Guidelines for Installation of Urban Subsurface Drainage
- ASCE/EWRI 14-05 Standard Guidelines for Operation and Maintenance of Urban Subsurface Drainage
- ASCE 15-98 Standard Practice for Direct Design of Buried Precast Concrete Pipe Using Standard Installations (SIDD)
- ASCE 16-95 Standard for Load Resistance Factor Design (LRFD) of Engineered Wood Construction
- ASCE 17-96 Air-Supported Structures
- ASCE 18-96 Standard Guidelines for In-Process Oxygen Transfer Testing
- ASCE 19-96 Structural Applications of Steel Cables for Buildings
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- ANSI/ASCE/T&DI 21.3-08 Automated People Mover Standards—Part 3
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- CI/ASCE 38-02 Standard Guideline for the Collection and Depiction of Existing Subsurface Utility Data
- EWRI/ASCE 39-03 Standard Practice for the Design and Operation of Hail Suppression Projects
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- ASCE/SEI 41-06 Seismic Rehabilitation of Existing Buildings
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ASCE/EWRI 47-05 Standard Guidelines for the Operation and Maintenance of Urban Stormwater Systems
ASCE/SEI 48-05 Design of Steel Transmission Pole Structures

ASCE/EWRI 50-08 Standard Guideline for Fitting Saturated Hydraulic Conductivity Using Probability Density Functions
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FOREWORD

The Board of Direction approved revisions to the American Society of Civil Engineers' (ASCE's) Rules for Standards Committees to govern the writing and maintenance of standards developed by ASCE. All such standards are developed by a consensus standards process managed by the ASCE Codes and Standards Committee. The consensus process includes balloting by a balanced standards committee and reviewing during a public comment period. All standards are updated or reaffirmed by the same process at intervals of between five and ten years.

This is a standard guideline for fitting saturated hydraulic conductivity using probability density functions. It represents the consensus of the Standards Committee on Fitting of Hydraulic Conductivity Using Statistical Spatial Estimation (called KSTAT) of the Standards Development Council of the Environmental and Water Resources Institute of the American Society of Civil Engineers. This standard guideline is the first in an expected series of standards that seeks to enhance the probabilistic characterization and understanding of the behavior of a key groundwater parameter, the saturated hydraulic conductivity (K). KSTAT's companion standard guideline, ASCE/EWRI 51-08 (in this volume), addresses calculating the effective saturated hydraulic conductivity (K_e or K_{ew}) in local-scale groundwater flow whether or not it is isotropic.

The provisions of this document are written in permissive language and, as such, offer to the user a series of options or instructions but do not prescribe a specific course of action. Significant judgment is left to the user of this document.

The saturated hydraulic conductivity is central in quantifying groundwater flow and contaminant transport phenomena in the subsurface. Although seemingly a deterministic quantity from the original empirical meaning appended to it by Henri Darcy in the mid-19th century, saturated hydraulic conductivity measurements exhibit variability even in the most geologically homogeneous aquifers. This variability arises from local-scale fluctuations in the textural properties of an aquifer. When these fluctuations are averaged by a measuring device, such as in a slug test, in a pumping test, or in a permeameter, the resulting

measurements of saturated hydraulic conductivity made at different locations in an aquifer constitute a set of values that can be treated as a sample of realizations of a random variable. The sample exhibits traits of central tendency, of dispersion about its center, and of asymmetry, to cite some such key traits. These statistical parameters can be subsumed into a more general probabilistic description of the sample of measurements of saturated hydraulic conductivity by means of a probability density function (pdf). The present standard outlines a methodology to fit saturated hydraulic conductivity with a pdf. Once this fit is achieved, variables dependent on the saturated hydraulic conductivity, such as groundwater velocity, or the concentrations of chemicals advected and dispersed in groundwater, can themselves be interpreted and treated as realizations of statistical processes prone to probabilistic description.

The formulas in this standard guideline involving saturated hydraulic conductivity values require that all the values in a sample be expressed in the same system of units, be it the International System of Units (SI) (say, cm/s) or the common system of units in the United States (say, ft/s). The data presented in this standard guideline are in SI units.

ASCE does not endorse commercial spreadsheets or numerical software cited in this standard guideline. Any such registered products are cited in this standard guideline to illustrate one possible way of calculating statistical parameters and special numerical functions that appear as part of this standard guideline's methodology. It is left to the users' discretion to choose and verify the accuracy of whichever computational technique they apply in the calculations needed to implement this standard guideline's methodology.

This standard has been prepared in accordance with recognized engineering principles and should not be used without the user's competent knowledge for a given application. The publication of this standard by ASCE is not intended to warrant that the information contained therein is suitable for any general or specific use, and ASCE takes no position respecting the validity of patent rights. The user is advised that the determination of patent rights or risk of infringement is entirely their own responsibility.

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The American Society of Civil Engineers and the Environmental and Water Resources Institute (ASCE/EWRI) acknowledge the efforts of the Standards Committee on Fitting of Hydraulic Conductivity Using Statistical Spatial Estimation (called KSTAT). The committee members constitute a wide spectrum of professionals from academia, government, and the private sector. Seminal contributions

to the contents of this standard guideline were made by the Task Committee on Effective Parameters in Groundwater Management of the Groundwater Management Committee of the Groundwater Council of EWRI. The current members of the Standards Committee on Fitting of Hydraulic Conductivity Using Statistical Spatial Estimation are the following:

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Standard Guideline for Fitting Saturated Hydraulic Conductivity Using Probability Density Functions

1.0 SCOPE

This standard guideline outlines a procedure to optimize the fitting and goodness-of-fit testing of a probability density function (pdf) to a sample of saturated hydraulic conductivity (K) measurements. The procedure assumes a uniform scale of observation (similar measuring device) and statistically homogeneous and independent hydraulic conductivity measurements. Two pdfs are recommended for fitting purposes when such a sample of saturated hydraulic conductivity measurements is available. The first is the lognormal pdf, recommended when logarithmically transformed K data have coefficient of skew (C_{sy}) in the range $-0.05 \leq C_{sy} \leq 0.05$. The second is the (three-parameter) log-gamma pdf, recommended when logarithmically transformed K data have coefficient of skew $|C_{sy}| > 0.05$. Testing the goodness-of-fit to K data achieved by the lognormal or the log-gamma pdfs is performed with the chi-squared method.

2.0 PURPOSE

Saturated hydraulic conductivity measurements made at different locations in an aquifer exhibit substantial variability. Those measurements can be treated using the laws of probability to obtain a proper description of the characteristics of saturated hydraulic conductivity that goes beyond the calculation of its average, standard deviation, or other indicators of central tendency and asymmetry. The fitting of saturated hydraulic conductivity with a proper pdf is a necessary step—after its measurement in the field or in the laboratory—to arrive at a complete description of its probabilistic characteristics. Analysts can then use the fitted pdf in a variety of analyses and design modes that provide a wider range of options than those available when saturated hydraulic conductivity is treated deterministically, i.e., as a nonrandom entity.

3.0 DEFINITIONS

The following definitions involve several variables for which notation and symbolism are found in Sections 5.0 and 12.0, respectively. The Symbols Used section (12.0) covers all variables introduced in other sections of this document.

3.1 SPATIAL CORRELATION

Spatial correlation is a measure of the degree of statistical association among saturated hydraulic conductivity measurements made at different locations in an aquifer. This definition also applies to spatial variables other than saturated hydraulic conductivity. This standard is concerned with positively correlated saturated hydraulic conductivity measurements, in which case the spatial correlation between two measurements of saturated hydraulic conductivity K_1 and K_2 made at locations x_1 and x_2 , respectively, ranges between 0 and 1. The closer the spatial correlation is to 1, the greater the degree of statistical association between the saturated hydraulic conductivity measurements K_1 and K_2 .

3.2 CORRELATION SCALE

Correlation scale is the distance between two points x_1 and x_2 beyond which the saturated hydraulic conductivities K_1 (at x_1) and K_2 (at x_2) cease to be spatially correlated. It is also referred to as range.

3.3 SATURATED HYDRAULIC CONDUCTIVITY (K)

Saturated hydraulic conductivity (K) is the ability of a porous material to transmit groundwater. It is equal to the groundwater flow (volume/time) per unit area of aquifer perpendicular to the groundwater flow, when the flow is driven by a hydraulic gradient equal to 1. It has units of length over time.

3.4 HYDRAULIC GRADIENT

Hydraulic gradient is the change of hydraulic head per unit distance along the path traveled by groundwater. It is dimensionless.

3.5 PROBABILITY DENSITY FUNCTION (PDF)

Probability density function (pdf) is a mathematical formula that assigns a nonnegative value to any number that is contained in the domain of the pdf. They are functions of the form $f(x)$, in which x denotes any

value at which the function f is calculated. The set of x values over which the function f is defined is called the domain of the pdf. The pdf integrated over its entire domain yields a value of 1. When integrated over part of its domain, it produces a probability between 0 and 1. The mathematical formula of a pdf may take many forms, giving rise to a variety of pdfs. Among the best known ones are the normal (or Gaussian), the lognormal, the gamma and log-gamma, the beta, and the chi-squared pdfs, to cite a few (Loáiciga et al. 2006). There are pdfs in which the x values are strictly integer values. These pdfs are more commonly referred to as probability distributions. The binomial, Poisson, and geometric probability distributions are widely used.

3.6 SLUG TEST

The slug test is used to measure the saturated hydraulic conductivity in the vicinity of a well's screen by suddenly dropping a solid volume into a well or by removing a small volume of water from a well and then recording the decline (or rise, if the volume is removed) of the water level in the well over time until it returns to its original position. The saturated hydraulic conductivity is estimated from the water level vs. time data and from other well and aquifer characteristics (Fetter 2001, pp. 190–204).

4.0 RANGE OF APPLICABILITY

4.1 STATISTICALLY HOMOGENEOUS AND INDEPENDENT K MEASUREMENTS

Statistical homogeneity implies that the pdf of K is the same everywhere in the aquifer in which K measurements are made with a similar device (deployed in the field or applied in the laboratory to core samples). In this case, the saturated hydraulic conductivity measurements exhibit a constant average and a spread of K values about the average that are devoid of spatial trends or spatial periodic patterns. Independence of K measurements implies that the value of K measured at any location in an aquifer is not related in a probabilistic sense to any other values of K measured at other locations in the same aquifer. Independent K measurements are uncorrelated. K measurements can be statistically homogeneous and correlated simultaneously (for example, Dagan 1989, p. 12). In the latter instance, one must resort to geostatistics, a field of statistics concerned with the study of spatially correlated variables (Journel and Huijbregts 1978).

Phoon et al. (2003) have proposed a procedure to identify statistically homogeneous soil layers in the presence of spatial correlation.

Commentary. From a physical standpoint, statistical homogeneity is approximated in the field when geological processes produce unconsolidated deposits (clays, silts, sands, gravels, or combinations of these) or consolidated deposits (bedrock aquifers, in the vernacular) of similar texture, porosity characteristics, and mineral composition.

Independence requires physical separations among K measurement locations that ensure the vanishing of any statistical dependence among K values. Measurement locations so chosen produce uncorrelated K samples as well. The minimal spatial separation among measurements must exceed the correlation scale of the saturated hydraulic conductivity. The correlation scale can be estimated using geostatistical procedures.

A recommended minimum sample size needed for the implementation of this standard's method is between 20 and 30 measurements of saturated hydraulic conductivity. The quality of the K -fitting results improves with the sample size, but the measurement costs rise.

4.2 GRAPHICAL EXAMPLE

Figure 4-1 shows a plot of 201 slug-test measurements of K made in a clayey aquifer (from Loáiciga et al. 2006). The K values shown in Fig. 4-1 are representative of statistically homogeneous and independent measurements. Figure 4-2 shows a plot of the values of the natural logarithm of the K data in Fig. 4-1. The assumption that $Y = \ln K$ is normally distributed is widely used in the statistical analysis of saturated hydraulic conductivity data (see Freeze 1975 or Sudicky 1986, to cite two examples). Figure 4-3 shows a histogram of the $\ln K$ data plotted in Fig. 4-2. The histogram shows that the $\ln K$ data are asymmetrical and, therefore, nonlognormal. This standard's method addresses the fitting of lognormal and log-gamma pdfs to K data, the former being a subcase of the latter.

5.0 NOTATION

It is assumed in the remainder of this standard that a sample of n measurements of saturated hydraulic conductivity is available. The individual measurements are denoted by $K_1, K_2, \dots, K_n$, or, symbolically, by, K_j , where $j = 1, 2, \dots, n$. The natural logarithm of

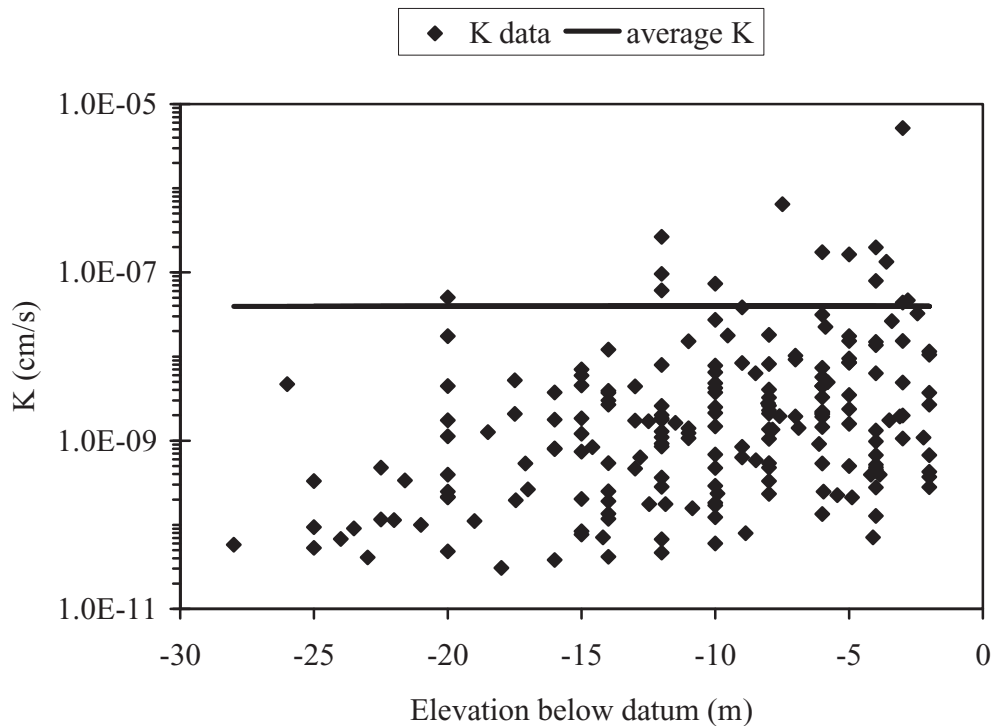


FIGURE 4-1. Scattergram of 201 statistically homogeneous K values (after Loáiciga et al. 2006). A preponderance of the data were measured at shallower elevations. No spatial trend or structure is apparent. The horizontal line represents the average saturated hydraulic conductivity, approximately 3.94×10^{-8} cm/s.

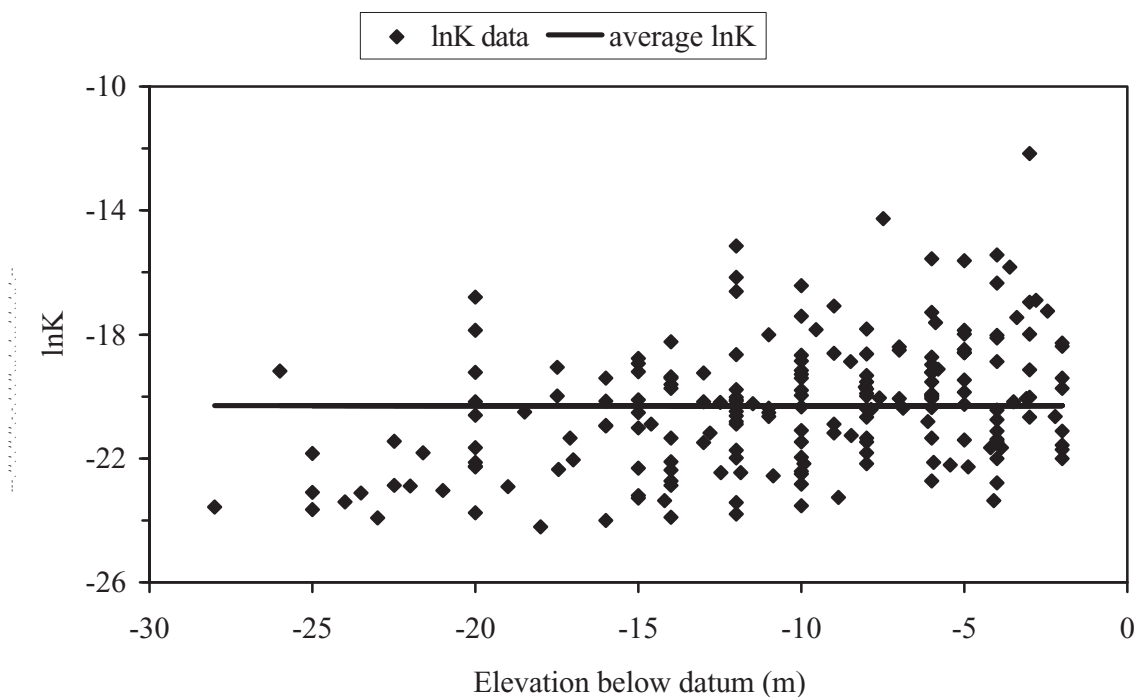


FIGURE 4-2. Scattergram of 201 statistically homogeneous $\ln K$ values. The K values were plotted in Figure 4-1. The horizontal line represents the average log conductivity of $\ln K$, approximately -20.3. Data are from Loáiciga et al. (2006).