

Reliability growth—statistical test and estimation methods

ANSI/IEC/ASQ D61164-1997

Approved as an American National Standard by:

American Society for Quality

AMERICAN NATIONAL STANDARD

Reliability Growth—Statistical Test and Estimation Methods

Approved as an American National Standard by:
American Society for Quality

An American National Standard Approved on September 16, 1997

American National Standards: An American National Standard implies a consensus of those substantially concerned with its scope and provisions. An American National Standard is intended as a guide to aid the manufacturer, the consumer, and the general public. The existence of an American National Standard does not in any respect preclude anyone, whether he or she has approved the standard or not, from manufacturing, purchasing, or using products, processes, or procedures not conforming to the standard. American National Standards are subject to periodic review and users are cautioned to obtain the latest edition.

Caution Notice: This American National Standard may be revised or withdrawn at any time. The procedures of the American National Standards Institute require that action be taken to reaffirm, revise, or withdraw this standard no later than five years from the date of publication. Purchasers of American National Standards may receive current information on all standards by calling or writing the American National Standards Institute.

© 1995 by IEC

Copyright Protection Notice for the ANSI/IEC/ASQ D61164-1997 Standard.

This standard is subject to copyright claims of IEC, ANSI and ASQ. Not for resale. No part of this publication may be reproduced in any form, including an electronic retrieval system, without the prior written permission of ASQ. All requests pertaining to the ANSI/IEC/ASQ D1164-1997 standard should be submitted to ASQ.

Note: As used in the document, the term "International Standard" refers to the American National Standard adoption of this and other International Standards.

ASQ Mission: To facilitate continuous improvement and increase customer satisfaction by identifying, communicating, and promoting the use of quality principles, concepts, and technologies; and thereby be recognized throughout the world as the leading authority on, and champion for, quality.

10 9 8 7 6 5 4 3 2 1

Printed in the United States of America

♾ Printed on acid-free paper

Published by:

American Society for Quality



Quality Press
611 East Wisconsin Avenue
Milwaukee, Wisconsin 53201-3005
800-248-1946

Website: <http://www.asq.org>

Contents

	Page
Foreword.....	1
Introduction.....	2
Clause	
1 Scope	3
2 Normative references	3
3 Definitions	3
4 Symbols	4
5 The power law model	5
6 Use of the model in planning reliability improvement programmes	6
7 Statistical test and estimation procedures	6
Tables	17
Annexes	
A Numerical examples	20
B The power law reliability growth model—Background information	28

Reliability Growth— Statistical Test and Estimation Methods

Foreword

- 1) The IEC (International Electrotechnical Commission) is a worldwide organization for standardization comprising all national electrotechnical committees (IEC National Committee). The object of the IEC is to promote international cooperation on all questions concerning standardization in the electrical and electronic fields. To this end and in addition to other activities, the IEC publishes International Standards. Their preparation is entrusted to technical committees; any IEC National Committee interested in the subject dealt with may participate in this preparatory work. International, governmental and non-governmental organizations liaising with the IEC also participate in this preparation. The IEC collaborates closely with the International Organization for Standardization (ISO) in accordance with conditions determined by agreement between the two organizations.
- 2) The formal decisions or agreements of the IEC on technical matters, prepared by technical committees on which all the National Committees having a special interest therein are represented, express, as nearly as possible, an international consensus of opinion on the subjects dealt with.
- 3) They have the form of recommendations for international use published in the form of standards, technical reports or guides and they are accepted by the National Committees in that sense.
- 4) In order to promote international unification, IEC National Committees undertake to apply IEC International Standards transparently to the maximum extent possible in their national and regional standards. Any divergence between the IEC Standard and the corresponding national or regional standard shall be clearly indicated in the latter.

International Standard IEC 1164 has been prepared by IEC Technical Committee No. 56: Dependability.

The text of this standard is based on the following documents:

DIS	Report on voting
56(CO)150	56(CO)159

Full information on the voting for the approval of this standard can be found in the report on voting indicated in the above table.

This standard shall be used in conjunction with IEC 1014.

Annexes A and B are for information only.

Introduction

This International Standard describes the power law reliability growth model and related projection model and gives step-by-step directions for their use. There are several reliability growth models available, the power law being one of the most widely used. This standard provides procedures to estimate some or all of the quantities listed in clause 9 of IEC 1014. An input is required consisting of a data set of accumulated test times at which relevant failures occurred, or were observed, for a single system, and the time of termination of the test, if different from the time of the final failure. It is assumed that the collection of data as input for the model beings after the completion of any preliminary tests, such as environmental stress screening, intended to stabilize the system's initial failure intensity.

Model parameters estimated from previous results may be used to plan and predict the course of future reliability growth programmes, if the conditions are similar.

Some of the procedures may require computer programs, but these are not unduly complex. This standard presents algorithms for which computer programs should be easy to construct.

Reliability Growth— Statistical Test and Estimation Methods

1 Scope

This International Standard gives models and numerical methods for reliability growth assessments based on failure data from a single system which were generated in a reliability improvement programme. These procedures deal with growth, estimation, confidence intervals for system reliability and goodness-of-fit tests.

2 Normative references

The following normative documents contain provisions which, through reference in this text, constitute provisions of this International Standard. At the time of publication, the editions indicated were valid. All normative documents are subject to revision, and parties to agreements based on this International Standard are encouraged to investigate the possibility of applying the most recent editions of the normative documents indicated below. Members of IEC and ISO maintain registers of currently valid International Standards.

IEC 60050(191): 1990, *International Electrotechnical Vocabulary (IEV)—Chapter 191: Dependability and quality of service*

IEC 60605-1: 1978, *Equipment reliability testing—Part 1: General requirements*

IEC 60605-4: 1986, *Equipment reliability testing—Part 4: Procedures for determining point estimates and confidence limits from equipment reliability determination tests*

IEC 60605-6: 1986, *Equipment reliability testing—Part 6: Tests for the validity of a constant failure rate assumption*

IEC 61014: 1989, *Programmes for reliability growth*

3 Definitions

For the purposes of this standard the terms and definitions of IEC 60050(191) and IEC 61014 apply, together with the following additional terms and definitions:

3.1 delayed modification: A corrective modification which is incorporated into the system at the end of a test.

NOTE—A delayed modification is not incorporated during the test.

3.2 improvement effectiveness factor: The fraction by which the intensity of a systematic failure is reduced by means of corrective modification.

3.3 type I test: A test which is terminated at a predetermined time or test with data available through a time which does not correspond to a failure.

NOTE—Type I test is sometimes called time terminated test.

3.4 type II test: A reliability growth test which is terminated upon the accumulation of a specified number of failures, or test with data available through a time which corresponds to a failure.

NOTE—Type II test is sometimes called failure terminated test.

4 Symbols

For the purpose of this international standard, the following symbols apply:

λ, β	scale and shape parameters for the power law model
CV	critical value for hypothesis test
d	number of intervals for grouped data analysis
$\bar{E}, E_i, E_j$	mean and individual improvement effectiveness factors
I	number of distinct types of category B failures observed
i, j	general purpose indices
K_A	number of category A failures
K_B	number of category B failures
K_i	number of i -th type category B failures observed; $K_B = \sum_{i=1}^k K_i$
M	parameter of the Cramér-von Mises test (statistical)
N	number of relevant failures
N_i	number of relevant failures in i -th interval
$N(T)$	accumulated number of failures up to test time T
$E[N(T)]$	expected accumulated number of failures up to test time T
$t(i-1); t(i)$	endpoints of i -th interval of test time for grouped data analysis
T	current accumulated relevant test time
T_i	accumulated relevant test time at the i -th failure
T_N	total accumulated relevant test times for type II test
T^*	total accumulated relevant test times for type I test
$\chi^2_\gamma(\nu)$	γ fractile of the χ^2 distribution with ν degrees of freedom
z	general symbol for failure intensity
u_γ	γ fractile of the standard normal distribution
$z(T)$	current failure intensity at time T
$\theta(T)$	current instantaneous mean time between failures
θ_p	projected mean time between failures

5 The power law model

The statistical procedures for the power law reliability growth model use the original relevant failure and time data from the test. Except in the projection technique (see 7.6), the model is applied to the complete set of relevant failures (as in IEC 61014, figure 2, characteristic (3)) without subdivision into categories.

The basic questions for the power law model are given in this clause. Background information on the model is given in annex B.

The expected accumulated number of failures up to test time T is given by:

$$E[N(T)] = \lambda T^\beta, \text{ with } \lambda > 0, \beta > 0, T > 0$$

where

λ is the scale parameter

β is the shape parameter (a function of the general effectiveness of the improvements; $0 < \beta < 1$, corresponds to reliability growth; $\beta = 1$ corresponds to no reliability growth; $\beta > 1$ corresponds to negative reliability growth).

The current failure intensity after T h of testing is given by:

$$z(T) = \frac{d}{dT} E[N(T)] = \lambda \beta T^{\beta-1}, \text{ with } T > 0$$

Thus, parameters λ and β both affect the failure intensity achieved in a given time. The equation represents in effect the slope of a tangent to the $N(T)$ vs. T characteristic at time T as shown in IEC 61014, figure 6.

The current mean time between failures after T h of testing is given by:

$$\theta(T) = \frac{1}{z(T)}$$

Methods are given in 7.1 and 7.2 for maximum likelihood estimation of the parameters λ and β . Subclause 7.3 gives goodness-of-fit tests for the model, and 7.4 and 7.5 discuss confidence interval procedures. An extension of the model for reliability growth projections is given in 7.6.

The model has the following characteristic features:

- it is simple to evaluate;
- when the parameters have been estimated from past programmes it is a convenient tool for planning future programmes employing similar conditions of testing and equal improvement effectiveness (see clause 5, and IEC 61014, clause 6);
- it gives the unrealistic indications that $z(T) = \infty$ at $T = 0$ and that growth can be unending, that is $z(T)$ tends to zero as T tends to infinity; however, these limitations do not generally affect its practical use;
- it is relatively slow and insensitive in indicating growth immediately after a corrective modification, and so may give a low (that is, pessimistic) estimate of the final $\theta(T)$, unless projection is used (see 7.6);

—the normal evaluation method assumes the observed times to be exact times of failure, but an alternative approach is possible for groups of failures within a known time period (see 7.2.2).

6 Use of the model in planning reliability improvement programmes

As inputs to the procedure described in 6.3 of IEC 1014, two quantities have to be predicted by means of reliability growth models:

- the accumulated relevant test time in hours expected to be necessary to meet the aims of the programme;
- the number of relevant failures expected to occur during this time period.

The accumulated relevant test time is then converted to calendar time from the planned test time per week or month, making allowance for the predicted total downtime (see below) and other contingencies, and the number of relevant failures is increased by judgment to include non-relevant failures and used to predict total downtime.

The inputs to the model for these calculations will be the assumed parameters for the model, as already estimated from one or more previous programmes, and judged to be valid for the future application by similarity of the test items, test environment, management procedures and other significant influences.

7 Statistical test and estimation procedures

7.1 Overview

The procedures in 7.2 utilize system failure data during a test programme to estimate the progress of reliability growth, and to estimate, in particular, the final system reliability at the end of the test. The reliability growth which is assessed in the result of corrective modifications incorporated into the system during test. The procedures discussed in 7.2.1 assume that the accumulated test time to each relevant failure is known. Subclause 7.2.2 addresses the situation where actual failure times are not known and failures are grouped in intervals of test time.

Type I tests, which are concluded at T^* , which is not a failure time, and type II tests, which are concluded at failure time T_N , use slightly different formulae, as indicated in 7.2.1.

An appropriate goodness-of-fit test, as described in 7.3, shall be performed after the growth test procedures of 7.2.1 and 7.2.2.

Subclause 7.6 addresses the situation where the corrective modifications are incorporated into the system at the end of the test as delayed modifications. The projection technique estimates the system reliability resulting from these corrective modifications.

7.2 Growth tests and parameter estimation

7.2.1 Case 1—Time data for every relevant failure

This method applies only where the time of failure has been logged for every failure.

- Step 1: exclude non-relevant failures by reference to 7.1 of IEC 61014, and/or other appropriate documentation.
- Step 2: assemble into a data set the accumulated relevant test times (as defined in 9.5 of IEC 60605-1) at which each relevant failure occurred. For type I tests, note also the time of termination of the test.
- Step 3: Calculate the test statistic.

$$U = \frac{\sum_{i=1}^N T_i - N \frac{T^*}{z}}{T^* \sqrt{\frac{N}{12}}} \quad [\text{type I}]$$

or

$$U = \frac{\sum_{i=1}^{N-1} T_i - (N-1) \frac{T_N}{2}}{T_N \sqrt{\frac{N-1}{12}}} \quad [\text{type II}]$$

where

- N is the total number of relevant failures;
- T^* is the total accumulated relevant test times for type I test;
- T_N is the total accumulated relevant test times for type II test;
- T_i is the accumulated relevant test time at the i -th failure.

Under the hypothesis of zero growth, that is, the failure times follow a homogeneous Poisson process, the statistic U is approximately distributed as a standard normal random variable with mean 0 and standard deviation 1. The statistic U can be used to test if there is evidence of reliability growth, positive or negative, independent of the reliability growth model.

A two-sided test for positive or negative growth at the α significance level has critical values $u_{1-\alpha/2}$ and $-u_{1-\alpha/2}$, where $u_{1-\alpha/2}$ is the $(1-\alpha/2) \times 100$ -th fractile of the standard normal distribution.

If

$$U < -u_{1-\alpha/2} \text{ or } U > u_{1-\alpha/2}$$

then there is evidence of positive or negative reliability growth, respectively, and the analysis is continued with Step 4.

If, however,

$$-u_{1-\alpha/2} < U < u_{1-\alpha/2}$$

then there is not evidence of positive or negative reliability growth at the α significance level and the growth analysis is terminated. In this case, the hypothesis of exponential times between successive failures (or a homogeneous Poisson process) is accepted at the α significance level. The critical values $-u_{1-\alpha/2}$ and $u_{1-\alpha/2}$ correspond to a one-sided test for positive or negative growth, respectively, at the $\alpha/2$ significance level.