

AMERICAN NATIONAL STANDARD

An attribute chain sampling program



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An attribute chain sampling program

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An attribute chain sampling program provides procedures and properties for plans that can contribute to reducing inspection and improving performance characteristics of sampling inspection. The standard can be used alone or in conjunction with ANSI/ASQ Z 1.4 and ANSI/ISO/ASQ 2859-1.

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Contents

Introduction	viii
1 Scope and purpose	1
1.1 Field of application	1
2 References	2
3 Terms and definitions	2
3.1 Chain sampling inspection	2
3.2 Lot-by-lot inspection	2
3.3 Essentially continuous production	3
4 Supplier and product qualification	3
4.1 Supplier qualification	3
4.2 Product qualification	3
5 Chain sampling procedures	3
6 Chain sampling evaluation	6
6.1 OC curves and tables of operating characteristics (performance)	6
6.2 Table useful for AQL and LQL matching	7
6.3 Determination of AOQL for chain sampling plans	7
6.4 Fractional acceptance numbers	7
7 Supplier responsibilities	9
8 Inspection agency and responsible authority responsibilities	9
Annex A	
Operating characteristics of chain sampling plan	10
Bibliography	39

Introduction

This publication is intended as a stand-alone standard to provide a source of a limited selection of chain sampling plans and their operating characteristics (OC), and to assist in the application of the chain sampling procedures while performing attribute, lot-by-lot, single sampling inspection. It is also intended to serve as a further explanation of and to provide the operating characteristics for the *fractional acceptance number* plans for single sampling that are included in ANSI/ISO/ASQ 2859-1: *Sampling procedures for inspection by attributes — Part 1: Sampling schemes indexed by acceptance quality limit (AQL) for lot-by-lot inspection*. Fractional acceptance number plans are essentially chain sampling procedures and are provided in ISO 2859-1 as an optional procedure in cases where the combination of sample size code letters and AQLs result in plans between the 0 and 1 acceptance numbers. It is expected that this optional procedure will be included in ANSI/ASQ Z1.4 in a subsequent revision. Chain sampling plans are ideally suited to replace zero (0) acceptance number plans for single sampling during essentially continuous lot submissions under steady production. As shown herein their operating characteristics fall between those of $c = 0$ and $c = 1$ sampling plans, hence the name used in ANSI/ISO/ASQ 2859-1 plans.

1 Scope and purpose

This standard describes the process of chain sampling, the theory, applications, plans for use, operating (performance) characteristics, and the utility of chain sampling as compared to single sampling via the operating characteristics. The purpose of this standard is to provide the procedures for a collection of sample sizes as well as to present the static operating characteristics for the plans presented, for use of the chain sampling technique. There is no direct indexing of the plans such as that of acceptance quality limit (AQL) used in ANSI/ASQ Z1.4 and ANSI/ISO/ASQ 2859-1. However, the complete OC are presented in both table and OC curve form for every sample size included. Furthermore, formulations are provided to compute the average outgoing quality limit (AOQL) for any of the chain sampling plans presented, related to the five values of the parameter, i , the sample size, n , and the lot size N , being applied. Those interested in using chain sampling to replace attribute sampling plans indexed either by AQL or limiting quality level (LQL) as given in ANSI/ASQ Z1.4 and ANSI/ISO/ASQ 2859-1, may select an appropriate plan from this standard by consulting the table of operating characteristics for the various sample sizes to find a plan with p at the desired AQL with probability of acceptance (P_a) equal to 0.95, 0.98, etc and/or the desired LQL at P_a equal to 0.10, Table 1 is also given herein that summarizes the $p_{0.95}$ and the $p_{0.10}$ values for combinations of the parameters, n and i , to assist with this possible interest.

1.1 Field of application

Sampling plans designated in this publication are applicable, but not limited to, inspection of the following:

- a) end items such as complete units or sub-assemblies;
- b) components and raw materials;
- c) operations or services;
- d) materials in process;
- e) maintenance operations;
- f) data or records;
- g) administrative procedures.

These plans are intended primarily to be used for a continuing series of lots or batches (operations), essentially a series of continuous supply, preferably in order of production (service). It is recommended that they not be used for isolated lots. Normally, lots or batches (operations) are expected to be of essentially the same satisfactory quality. They are particularly applicable to situations where the customer (receiving the inspected lots) has some degree of confidence in the integrity of the producer/supplier. Chain sampling plans are particularly suited to situations satisfying the preceding conditions and involving destructive, complex, and/or costly tests for which a relatively small sample is desired yielding good quality discrimination. They are additionally useful for achieving better discrimination (than single sampling plans in particular) in situations associated with assuring tighter quality levels (with comparably larger sample sizes). A special purpose of chain sampling is to achieve a more favorable

operating (performance) characteristic than that of single sampling, especially with an acceptance number, c , of zero for relatively small samples as mentioned above. The situations of supply should be suitable to the use of cumulative results of two or more samples from consecutive lots in making an acceptance/rejection decision on the current lot.

This publication shall be used only for characteristics inspected by attributes as designated in publication ANSI/ASQ Z1.4. With respect to the inspection of multiple characteristics, the chain sampling procedures will follow the same principles used in ANSI/ASQ Z1.4.

This standard may be referenced in a purchasing or specification contract, inspection instructions, or other contractual and/or operational documents.

2 References

Basic definitions may be found in the following standards:

1. ANSI/ISO/ASQ 3534-2: *Statistics – Vocabulary and symbols – Part 2: Applied Statistics*
2. ANSI/ASQ S2: *Introduction to Attribute Sampling*
3. ANSI/ASQ Z1.4: *Sampling Procedures and Tables for Inspection by Attributes*
4. ANSI/ISO/ASQ 2859-1: *Sampling procedures for inspection by attributes – Part 1: Sampling schemes indexed by acceptance quality limit (AQL) for lot-by-lot inspection*

3 Terms and definitions

The terminology and definitions are in accordance with ANSI/ISO/ASQ 3534-2, ANSI/ASQ Z1.4, and ANSI/ISO/ASQ 2859-1.

3.1 Chain sampling inspection

An acceptance sampling procedure in which the criteria for acceptance of the current lot, in lot-by-lot inspection of an essentially continuous production sequence, is governed by the sampling results of the lot sampled and those of a specified number of preceding consecutive lot samples. It is a member of a class of procedures referred to as *cumulative results* sampling inspection. Chain sampling, in its simplest form, is designated ChSP-1, and this standard is limited to plans of this form. Individual lots and samples are considered as links in a chain, hence, the name, *chain sampling*. Considerable extensions and variations have been developed to form a larger family of chain sampling and cumulative-results inspection plans and are available in the literature.

3.2 Lot-by-lot inspection

Inspection of product submitted in a series of lots. A sample is selected from each lot and inspected for the designated characteristic(s) using attribute inspection procedures.

3.3 Essentially continuous production

Production that is at a steady rate. Production shall be considered essentially continuous if at least one lot of product is submitted for inspection at a production frequency as agreed to by both the supplier and the responsible authority (see ANSI/ASQ Z1.4, Section 1.1). If no production frequency is specified, at least one lot shall be submitted each month. Product shipped to other parties or products of a similar nature shall be considered in the determination of essentially continuous production unless excluded by both the supplier and responsible authority.

4 Supplier and product qualification

4.1 Supplier qualification

The supplier shall:

- a) Have implemented and maintained a documented system for controlling product quality and design changes so as to assure that the quality of the product is of an acceptable level and that this level is stable. Using cumulative results such as in chain sampling requires that the quality of lots from which sample results are cumulated to make an accept/reject decision on the current lot shall be stable.
- b) Have instituted a system which is capable of detecting and correcting shifts in quality levels, and monitoring changes which may adversely affect quality and acceptance/rejection decisions using chain sampling.
- c) Not have experienced an organizational change that might adversely affect quality.

4.2 Product qualification

The product shall have met all of the following requirements:

- a) Be of stable design.
- b) Have been manufactured on an essentially continuous basis for a period mutually agreed to by both the supplier and responsible authority. If no period is specified, the period shall be six months. Whenever production is held pending sample approval, only the time period after approval and resumption of product shall be included.

5 Chain sampling procedures

The main purpose of chain sampling procedures (ChSP) is to achieve a more favorable operating characteristic than that of single sampling, especially with an acceptance number c of zero for relatively small samples motivated by situations of destructive or costly tests, satisfying conditions suitable to the use of cumulative results of samples from two or more lots in making an acceptance decision on the current lot. Individual lots and samples are considered as links in a chain; hence, the name *chain sampling*. Considerable extensions and variations have been developed from the basic procedures.

An interesting application of chain sampling is replacement of $c = 0$ sampling plans in sampling tables in which there is an otherwise large disparity of quality protection/discrimination between the $c = 1$ sampling plans and the $c = 0$ sampling plans. For this application, chain sampling is seen to achieve the effect of *fractional acceptance numbers* between 0 and 1. This application is an optional procedure in the international acceptance sampling table, ANSI/ISO/ASQ 2859-1, which standard is similar to ANSI/ASQ Z1.4, though the latter does not include this option at this time. This application is also feasible for stand alone sampling plans not taken from tables. The operating characteristic (OC) curves presented herein as part of the evaluation of the plans included here vividly display this effect of protection/discrimination between that of the single sampling plans with $c = 0$ and 1 and the ChSP-1 sampling plans.

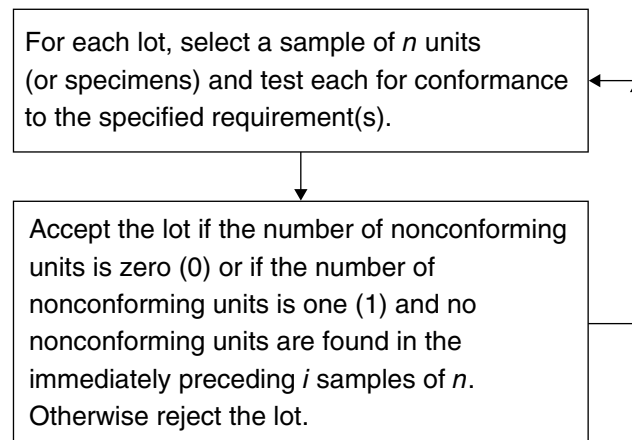
Chain sampling, of the ChSP-1 form provided in this standard, specifies two operating parameters: namely,

n : the sample size to be taken from each lot

i : the number of preceding sample results that, if clear of nonconforming (defective) units or nonconformities (defects), allows acceptance of the current lot, when one nonconforming unit is found in its sample, n .

Acceptance of the current lot occurs immediately when no nonconforming (defective) units or no nonconformities (defects) are found in its sample. Rejection of the current lot occurs immediately when two or more nonconforming (defective) units or nonconformities (defects) are found in its sample, n . When one nonconforming (defective) unit or one nonconformity (defect) is found in its sample, n , the chain sampling feature applies which allows acceptance of the current lot, if the preceding, i , samples (taken from their respective lots) contain no nonconforming (defective) units or no nonconformities (defects). For this standard, plans with $i = 1, 2, 3, 4,$ and 5 are provided for a range of sample sizes from 2 to 2000.0,

Figure 1 – Operation schematic for ChSP-1



The acceptance criteria are fixed at zero (0) or one (1) as noted above. The lot size, N , is not designated as an explicit parameter, being of insignificant consequence on the operating characteristics of the plans under binomial or Poisson probabilities as used for evaluation. However, the table of Sample Size Code Letters as given in

ANSI/ASQ Z1.4 and ANSI/ISO/ASQ 2859-1 can serve as a guide in selecting an appropriate sample size for a given lot or batch size range. All 16 of the normal inspection single sampling sample sizes used in these standards are provided in this standard along with 13 additional sample sizes for convenience.

EXAMPLE 1: This example is taken from ANSI/ASQC S2: *Introduction to Attribute Sampling*, Example 1. “Vitrified clay pipes, with a nominal length of 3 m and nominal diameter 150 mm, are due to be delivered as a lot of 250 pipes. Before delivery, a random sample of eight pipes is drawn from the lot and these eight pipes are tested for crushing strength, bending moment resistance, and impermeability. The lot is delivered if, and only if, there are at least seven pipes, each conforming to the specified tolerances of all three of these characteristics.” Referring to Figure A.4 (SS, $c = 1$) it can be seen that with a sample size of $n = 8$, and an acceptance number $c = 1$, product with as much as 10% nonconforming for the three quality characteristics can be accepted with a probability in excess of 80%. It is entirely possible that for this application the $c = 0$ sampling plan (with $n = 8$) was not chosen because product of 2% nonconforming for the three quality characteristics has an approximate 15% chance of being rejected (see SS, $c = 0$). By using chain sampling as given in this standard, it would be possible to reduce the chance of acceptance of 10% nonconforming product to 50%, while increasing the chance of rejection of 2% nonconforming product to 5% or less. A chain sampling plan with $n = 8$, $i = 2$ will achieve this compromise

EXAMPLE 2: This example is taken from ANSI/ASQC S2: *Introduction to Attribute Sampling*, Example 16. A product is to be inspected under the following conditions: lot size 500, inspection level II, normal inspection, and single sampling. The AQL for the Class A characteristics is 0.065%. The applicable sampling plan is found in ANSI/ASQ Z1.4 or ANSI/ISO/ASQ 2859-1 to be $n = 200$, $c = 0$. The OC curve for this sampling plan can be seen in Figure A.22 for $c = 0$, SS. It shows the peculiar performance characteristic of all “0” acceptance number plans, namely, no inflection point in the curve, leading to a sharp drop in the probability of acceptance, P_a , even for low fraction of nonconforming product. In fact, at the specified AQL value (0.065%), the probability of acceptance is only 0.878, for a probability of rejection at the AQL of 12.2%. Now, a more favorable OC Curve (performance) for this situation can be achieved by using a chain sampling plan. Table 1 reveals that for $n = 200$ and for $i = 1$, $p_{0.0,98} = 0.062\%$, relatively close to the specified AQL and by definition and nomenclature this level of percent nonconforming has a P_a of 0.98% or a probability of rejection of only 2.0%. Table 1 for $n = 200$ also reveals that $p_{0.0,10} = 1.238\%$ which is the percent nonconforming that would be accepted only 10% of the time (rejected 90% of the time), labeled LQ in ANSI/ASQ Z1.4 and ANSI/ISO/ASQ 2859-1. On further analysis of the $c = 0$ sampling plan, its LQ is around 1.114%. It can be seen on the OC Curves of Figure A.22 for high percent nonconforming the OC Curves for the chain sampling plans converge with the OC Curve for the SS, $c = 0$ plan. This is also the proportion of the percent conforming scale not expected to be applicable a large majority of the time, as these levels of poor quality are not expected to occur. Table A.22 is also useful for these types of comparisons.

EXAMPLE 3: In using either ANSI/ASQ Z-1.4 or ANSI/ISO/ASQ 2859-1, an AQL of 1.0 % is required; lot sizes, N , run around 200; inspection level II, single sampling, and beginning on normal inspection are specified. This leads to Code Letter G to obtain the sampling plan. When entering Table II-A of Z1.4 or Table 2-A of ANSI/ISO/ASQ 2859-1, Code Letter G denotes a sample size of $n = 32$. However, in determining the acceptance number with these tables a downward arrow is encountered. As explained in Section 6.4 of this standard, ANSI/ISO/ASQ 2859-1 provides an option under these circumstances that allow for the use of “fractional acceptance numbers” which are essentially chain sampling. The fractional acceptance number specified in ANSI/ISO/ASQ 2859-1, Table 11-A is $\frac{1}{2}$. (ANSI/ASQ Z1.4 does not presently have this option, but based on the inclusion in ANSI/ISO/ASQ 2859-1, the responsible authority can always be petitioned for its use when sampling according to ANSI/ASQ Z1.4.) The

applicable chain sampling plan is $n = 32$, $i = 1$. The properties of this plan are found in Figure A.9 of this standard. If switching to tightened inspection is necessary, the chain sampling plan is $n = 32$, $i = 2$, based on a fractional acceptance number of $1/3$ in Table 11-B of ANSI/ISO/ASQ 2859-1. If reduced inspection becomes possible, the applicable chain sampling plan is $n = 13$, $i = 2$, based on Table 11-C of ANSI/ISO/ASQ 2859-1. The properties of this plan are found in Figure A.6 of this standard.

6 Chain sampling evaluation

Evaluation of chain sampling of the ChSP-1 form is by means of the probability of acceptance, P_a , the operating characteristic that is readily computed from results derived by Dodge (see Bibliography) as follows.

$$P_a = P_{0,n} + P_{i,n} (P_{0,n})^i, \text{ where}$$

$P_{0,n}$ = the probability of finding no nonconforming units (nonconformities) in a sample of n units from product of fraction nonconforming, p .

$P_{i,n}$ = the probability of finding one nonconforming unit (nonconformity) in such a sample.

6.1 OC curves and tables of operating characteristics (performance)

The operating characteristics for the ChSP-1 chain sampling plans provided in this standard are given in Table 1, A through CC, for each of the 29 sample sizes provided. These tables contain both the OC curves and a table of p values for a selected set of P_a values, ranging from 0.99 to 0.01. Binomial probabilities are used.

For each of the sample sizes, $n = 2$ to $n = 2000$, the OC are provided for five ChSP-1 plans for $i = 1$ to 5. For comparison, the OC are provided for two single sampling plans with the same sample size and for $c = 0$, and $c = 1$. As can be seen on the OC curves, in particular, the OC curves for the five ChSP-1 plans fall on or between the OC curves for the two single sampling plans. Under ordinary lot-by-lot single sampling with integer acceptance numbers, one would have to choose between the plans with $c = 0$ and $c = 1$. As can be seen, the ChSP-1 chain sampling plans provide operating characteristics falling between those of $c = 0$, and $c = 1$. Hence, they give the appearance of having effective acceptance numbers between 0 and 1, leading to the use of the term, *fractional acceptance numbers*, as used in ANSI/ISO/ASQ 2859-1 as an optional procedure. Adding the provision for using cumulative results (over i lot samples) has the same effect on the operating characteristics as taking a second sample (from the same lot). It increases the chances of acceptance in the region of principal interest – where the product percent defective is reasonably small. By calling for rejection provided only that two defects are fairly close together, it adjusts for the undesirable features of $c = 0$ plans with poor discrimination between good and bad product quality.

The OC curves contain limited labels such as $c = 0$, SS; $c = 1$, SS; and $i = 1$. As can be seen in the figures of the Appendix, the solid line curve to the far right is always that for the single sampling (SS) plan with $c = 1$; the solid line curve to the far left is always that for the SS plan with $c = 0$; and the dashed line curves between these curves are those for the ChSP-1 plans for $i = 1$ to 5, with the far right dashed curve being for $i = 1$ and with i increasing from 2 to 5 for the curves moving from right to left.

6.2 Table useful for AQL and LQL matching

In addition to Annex A, Table 1 provides a summary of two selected P_a values, namely, 0.95 and 0.10, for each of the sample sizes provided. This information is presented in summary form (otherwise available from Annex A) to provide possible matching of the chain sampling plans with the sampling plans of ANSI/ASQ Z1.4 and ANSI/ISO/ASQ 2859-1, that are indexed by AQL and/or LQL. The P_a values used in Table 1, with the corresponding p values for each of these P_a values, are often associated with the AQL and LQL values, namely, $p_{0,0,95}$ for AQL and $p_{0,0,10}$ for LQL.

6.3 Determination of AOQL for chain sampling plans

One further table, Table 2, provides factors (k) for determining the approximate AOQL of the ChSP-1 chain sampling plans when used under a system of rectification inspection/sampling. Under conditions suitable to the use of Poisson probabilities (in its own right or as approximations to binomial and hypergeometric probabilities), the following expression can be used along with the k factors of Table 2 to determine the AOQL of the ChSP-1 plans, as follows.

$$\text{AOQL} = k/n - k/N$$

6.4 Fractional acceptance numbers

As noted earlier, ANSI/ISO/ASQ 2859-1 allows an optional procedure for using the chain sampling technique for sampling plans in the tables for single sampling under tightened, normal and reduced inspection. This procedure is allowed to replace the arrows on the diagonal of their tables between $c = 0$ and $c = 1$ sampling plans. For tightened and normal inspection, fractions of $\frac{1}{2}$ and $\frac{1}{3}$, respectively, are used. For reduced inspection, fractions of $\frac{1}{5}$, $\frac{1}{3}$, and $\frac{1}{2}$ are used. These fractional acceptance numbers are used in the following way: For a fraction of $\frac{1}{2}$, a given lot is considered acceptable (1st) if no nonconforming units or nonconformities are found in the sample from the lot, or (2nd) if one nonconforming unit or nonconformity is found in the sample from the lot, but no nonconforming units or nonconformities were found in the sample from the immediately previous lot. For a fraction of $\frac{1}{3}$, the acceptance criterion is 0, or 1 with 0 in the immediately previous 2 samples from their corresponding lots. Essentially then, it can be seen from the preceding discussion of chain sampling that an acceptance number of $\frac{1}{2}$ is a ChSP-1 plan with $i = 1$. An acceptance number of $\frac{1}{3}$ is a ChSP-1 plan with $i = 2$, and an acceptance number of $\frac{1}{5}$ is a ChSP-1 plan with $i = 4$, that is, for the fraction $1/f$ in ANSI/ISO/ASQ 2859-1, use the ChSP-1 plan with $i = f - 1$.