



Designation: C 1340 – 99

Standard Practice for Estimation of Heat Gain or Loss Through Ceilings Under Attics Containing Radiant Barriers by Use of a Computer Program¹

This standard is issued under the fixed designation C 1340; the number immediately following the designation indicates the year of original adoption or, in the case of revision, the year of last revision. A number in parentheses indicates the year of last reapproval. A superscript epsilon (ε) indicates an editorial change since the last revision or reapproval.

1. Scope

1.1 This practice covers the estimation of heat gain or loss through ceilings under attics containing radiant barriers by use of a computer program. The computer program included as an adjunct to this practice provides a calculational procedure for estimating the heat loss or gain through the ceiling under an attic containing a truss or rafter mounted radiant barrier. The program also is applicable to the estimation of heat loss or gain through ceilings under an attic without a radiant barrier. This procedure utilizes hour-by-hour weather data to estimate the hour-by-hour ceiling heat flows. The interior of the house below the ceiling is assumed to be maintained at a constant temperature. At present, the procedure is applicable to sloped-roof attics with rectangular floor plans having an unshaded gabled roof, a horizontal ceiling, and no HVAC ducts in the attic. It is not applicable to structures with flat roofs, vaulted ceilings, or cathedral ceilings. The calculational accuracy also is limited by the quality of physical property data for the construction materials, principally the insulation and the radiant barrier, and by the quality of the weather data.

1.2 Under some circumstances, interactions between radiant barriers and HVAC ducts in attics can have a significant effect on the thermal performance of a building. When analysis of these interactions is completed they will be added to the computer program.

2. Referenced Documents

2.1 ASTM Standards:

C 168 Terminology Relating to Thermal Insulating Materials²

2.2 ANSI Standards:

X3.5 Flow Chart Symbols and Their Usage in Information Processing³

X3.9 Standard for Fortran Programming Language³

¹ This practice is under the jurisdiction of ASTM Committee C16 on Thermal Insulation and is the direct responsibility of Subcommittee C16.21 on Reflective Insulation.

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² Annual Book of ASTM Standards, Vol 04.06.

³ Available from American National Standards Institute, 11 W. 42nd St., 13th Floor, New York, NY 10036.

2.3 ASTM Adjuncts:

Computer Program for Estimation of Heat Gain or Loss through Ceilings Under Attics Containing Radiant Barriers⁴

3. Terminology

3.1 *Definitions*—For definitions of terms used in this practice, refer to Terminology C 168.

3.2 *Symbols*—Symbols will be introduced and defined in the detailed description of the development.

4. Summary of Practice

4.1 The procedures used in this practice are based on the thermal response factor method for calculating dynamic heat conduction through multilayer slabs (1,2),⁵ along with a model for convective and radiative heat exchanges inside and outside the attic.

4.2 The operation of the computer program involves the following steps:

4.2.1 *Response Factors*—A separate computer program must be used to calculate the thermal response factors of the solid materials surrounding the attic. Input to this program would consist of the thermal conductivity, specific heat, density, and thickness of each layer, or the thermal resistance of the layer if it has negligible density, and the fraction of the cross-sectional area occupied by the framing. Output of such a program would be a set of response factors for use as input to the main program. The adjunct to this practice contains data files with response factors for several typical attic constructions.

4.2.2 *Data Input to the Main Program*—This input includes the response factors, total hemispherical emittances of the inside and outside surfaces of the attic envelope, solar absorptances of the outside surfaces of the attic envelope, length and width of the attic, slopes of the two roof sections, distance between attic floor and roof at edge of attic, orientation of house, vent areas and type of vents, water vapor permeances of attic surfaces, area of exposed wood inside attic, mass of wood

⁴ Available from ASTM Headquarters. Request Adjunct: ADJC1340.

⁵ The boldface numbers in parentheses refer to the list of references at the end of this standard.

in attic, initial moisture content of wood in attic, rate of exfiltration of air from house into attic space, latitude and longitude, time zone indicator, solar reflectance of the ground, indoor temperature, and indoor humidity.

4.2.3 Analysis—Using hourly weather data consisting of outdoor temperature and humidity ratio, atmospheric pressure, total horizontal and direct solar radiation, wind speed and direction, cloud amount, cloud type, and atmospheric clearness number, the computer program calculates the inside and outside temperatures of the attic envelope and the temperature of the air inside the attic. Using these temperatures, the program calculates the heat flux through the ceiling.

4.2.4 Output—The hourly heat flux through the ceiling is written to a file which can be used for further processing, such as seasonal or annual heat gains or losses.

5. Significance and Use

5.1 Manufacturers of radiant barriers express the performance of their products in terms of the total hemispherical emittance. The purpose of a radiant barrier is to decrease the radiation heat transfer across the attic air space, and hence, to decrease the heat loss or gain through the ceiling below the attic. The amount of decrease in heat flow will depend upon a number of factors, such as weather conditions, amount of mass or reflective insulation in the attic, solar absorptance of the roof, geometry of the attic and roof, and amount and type of attic ventilation. Because of the infinite combinations of these factors, it is not practical to publish data for each possible case.

5.2 The calculation of heat loss or gain of a system containing radiant barriers is mathematically complex, and because of the iterative nature of the method, it is best handled by computers.

5.3 Computers are now widely available to most producers and consumers of radiant barriers to permit the use of this practice.

5.4 The user of this practice may wish to modify the data input to represent accurately the structure. The computer program also may be modified to meet individual needs. Also, additional calculations may be desired, for example, to sum the hourly heat flows in some fashion to obtain estimates of seasonal or annual energy usages. This might be done using the hourly data as inputs to a whole-house model, and by choosing house balance points to use as cutoff points in the summations.

6. Method of Calculation

6.1 Approach:

6.1.1 This calculation of heat loss or gain requires that the following be known:

6.1.1.1 The thermal conductivity, specific heat, and density of the construction materials (that is, insulation, plywood, roofing materials, sheathing, gypsum board);

6.1.1.2 The total hemispherical emittance of all materials facing the attic air space;

6.1.1.3 The solar absorptance of the exterior surfaces of the attic (that is, the roof and gables);

6.1.1.4 The geometry of the attic;

6.1.1.5 The moisture permeance and storage properties of the materials facing the attic space; and

6.1.1.6 The weather conditions.

6.1.2 The solution is a computer procedure that estimates temperatures of both sides of the components of the attic envelope and the temperature of the air in the attic space, uses these estimates of temperatures to refine estimates of convection and radiation heat transfer coefficients, reestimates the temperatures using the new heat transfer coefficients, continues iterating on the temperatures and heat transfer coefficients until convergence is reached, and uses the last estimates of temperatures to calculate the heat gain or loss through the ceiling. This procedure is repeated for each hour of the simulation period (typically a full year).

6.2 Development of Equations—The model that is the basis for this practice is based on the model developed by B. Peavy (3), which was later extended by Wilkes (4-6). The sketch of an attic given in Fig. 1 shows the various heat transfer mechanisms that occur within an attic. Although the sketch shows ventilation occurring at soffit and ridge vents, the location of the vents may be at other locations, such as at the gables. The model treats all of these phenomena through a system of heat balance equations at the interior and exterior surfaces of the ceiling, roof sections, and gables, as well as a heat balance on the air mass within the attic. To handle the case of raised trusses, short vertical walls at the eaves also are included. Each of the surfaces is assumed to be isothermal; thus, for an attic consisting of a ceiling, two roof sections, two gables, two vertical eave sections, and one air space, a total of 15 heat balance equations is used.

6.3 Equations—Conduction:

6.3.1 The model developed here utilizes the thermal response factor method to analyze conduction through building envelope sections. The thermal response factor method was developed by Mitalas and Arseneault (1) and was extended by Kusuda (2). The method is based on an exact analytical solution of the heat conduction equation for one-dimensional heat flow through a multilayer slab having temperature-independent thermal properties. The only approximation is that the surface temperatures are taken to vary linearly with time between time steps. For analysis of buildings, the time step is

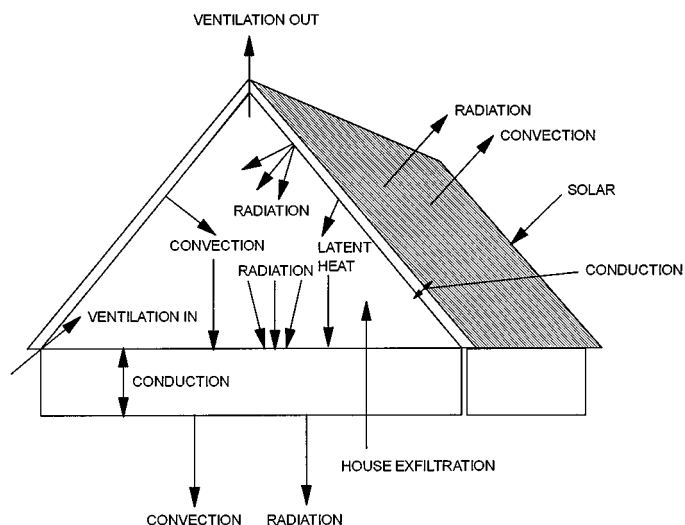


FIG. 1 Schematic of Residential Attic Showing Heat Transfer Phenomena

normally taken to be 1 h. The response factor equations relate the heat fluxes at the surfaces of the slab to the present and previous temperatures at the two surfaces. The equations are:

$$QI = \sum_{j=0}^{\infty} Z'(j) (TIS(j) - TR) - \sum_{j=0}^{\infty} Y'(j) (TOS(j) - TR) \quad (1)$$

$$QO = \sum_{j=0}^{\infty} Y'(j) (TIS(j) - TR) - \sum_{j=0}^{\infty} X'(j) (TOS(j) - TR) \quad (2)$$

where:

QI	= heat flux at inside surface at present time (note that the positive heat flow direction is from the inside to the outside), Btu/h·ft ² ,
QO	= heat flux at outside surface at present time, Btu/h·ft ² ,
$TIS(j)$	= temperature at inside surface j hours previous to present time, °F,
$TOS(j)$	= temperature at outside surface j hours previous to present time, °F,
$X'(j), Y'(j), Z'(j)$	= response factors, Btu/h·ft ² ·°F, and
TR	= reference temperature, °F.

6.3.2 The response factors are determined from a sequence of calculations that involve the thermal diffusivity, thermal conductivity, specific heat, density, and thickness of each of the layers in the multilayer slab. An efficient computer program for calculating the response factors has been developed by George Walton of the National Institute of Standards and Technology (NIST) (7).

6.3.2.1 The efficiency of the response factor calculations can be increased by making use of the fact that after a sufficient number of terms, the ratio of two consecutive response factors becomes constant. This is expressed by:

$$\frac{X'(j+1)}{X'(j)} = \frac{Y'(j+1)}{Y'(j)} = \frac{Z'(j+1)}{Z'(j)} = CR \text{ for } j \geq N \quad (3)$$

CR = the common ratio, and

N = a sufficiently large number.

6.3.2.2 The common ratio is used to define a new set of functions, called the first order conduction transfer functions or simply the conduction transfer functions, $X(j)$, $Y(j)$, and $Z(j)$, which are given by:

$$X(0) = X'(0) \quad (4)$$

$$Y(0) = Y'(0) \quad (5)$$

$$Z(0) = Z'(0) \quad (6)$$

$$X(j) = X'(j) - CR X'(j-1) \text{ for } j \leq N \quad (7)$$

$$Y(j) = Y'(j) - CR Y'(j-1) \text{ for } j \leq N \quad (8)$$

$$Z(j) = Z'(j) - CR Z'(j-1) \text{ for } j \leq N \quad (9)$$

$$X(j) = 0 \text{ for } j > N \quad (10)$$

$$Y(j) = 0 \text{ for } j > N \quad (11)$$

$$Z(j) = 0 \text{ for } j > N \quad (12)$$

6.3.2.3 With the conduction transfer functions, the heat fluxes and surface temperatures are related by:

$$QI = \sum_{j=0}^N Z(j) (TIS(j) - TR) - \sum_{j=0}^N Y(j) (TOS(j) - TR) + CR QI' \quad (13)$$

$$QO = \sum_{j=0}^N Y(j) (TIS(j) - TR) - \sum_{j=0}^N X(j) (TOS(j) - TR) + CR QO' \quad (14)$$

where:

QI' = heat flux at inside surface at previous time step,
 QO' = heat flux at outside surface at previous time step,
 and

N = number of significant conduction transfer functions.

6.3.2.4 When parallel heat flow paths occur in an envelope component, separate response factors for each path may be needed. If the boundary temperatures of the two paths may be assumed to be equal, however, then the response factors may be added together as:

$$X' = A_1 X'_1 + A_2 X'_2 \quad (15)$$

where:

A_1, A_2 = area fractions for paths 1 and 2, and
 $(X'_1, X'_2), (Y'_1, Y'_2) \text{ and } (Z'_1, Z'_2)$ = the response factors for paths 1 and 2.

Parallel conduction transfer functions may be calculated from these parallel response factors, provided that the common ratio for the path with the largest number of significant terms is used.

6.3.2.5 The original derivation of the response factor technique relied upon the assumption of temperature-independent thermal properties. An approximate method has been developed to account for the temperature dependence of the thermal properties (5). The thermal transmission coefficient of the component is taken to vary linearly with temperature as:

$$U = U_{TR} [1 + b(T - TR)] \quad (16)$$

The conduction transfer function equations then become:

$$QI = \sum_{j=0}^N Z(j) (TIS(j) - TR) - \sum_{j=0}^N Y(j) (TOS(j) - TR) + CR QI' + b/2 \sum_{j=0}^N Z(j) (TIS(j) - TR)^2 - b/2 \sum_{j=0}^N Y(j) (TOS(j) - TR)^2 \quad (17)$$

$$QO = \sum_{j=0}^N Y(j) (TIS(j) - TR) - \sum_{j=0}^N X(j) (TOS(j) - TR) + CR QO' \quad (18)$$

$$+ b/2 \sum_{j=0}^N Y(j) (TIS(j) - TR)^2 - b/2 \sum_{j=0}^N X(j) (TOS(j) - TR)^2$$

These equations are used in the system of heat balance equations.

6.4 Equations—Convection:

6.4.1 Convection heat transfer from the interior and exterior surfaces of the envelope components is calculated using correlations from the literature (8). The coefficients are based on correlations that have been developed for isolated isothermal flat plates. The correlations are in the form of a Nusselt number, Nu , as a function of a Rayleigh number, Ra , Grashof number, Gr , or a Reynolds number, Re , where:

$$Nu = hL/k \quad (19)$$

$$Ra = \frac{g\beta\rho C_p \Delta T L^3}{\nu k} \quad (20)$$

$$Gr = Ra/Pr \quad (21)$$

$$Pr = \nu/\alpha \quad (22)$$

$$Re = VL/\nu \quad (23)$$

and:

- h = convection heat transfer coefficient, Btu/h·ft²·°F,
- L = characteristic length of plate, ft,
- k = thermal conductivity of air, Btu/h·ft·°F,
- g = acceleration of gravity, ft/h²,
- β = volume coefficient of expansion of air, °R⁻¹,
- ρ = density of air, lb/ft³,
- C_p = specific heat of air, Btu/lb·°F,
- ΔT = temperature difference between surface and air, °F,
- ν = kinematic viscosity of air, ft²/h,
- Pr = Prandtl number for air, dimensionless,
- α = thermal diffusivity of air, ft²/h, and
- V = velocity of air stream, ft/h.

6.4.2 The model accounts for the temperature dependence of the properties of air by evaluating them at the film temperature, which is defined as the average of the temperatures of the surface and the air. Relationships for the temperature dependent properties were obtained from NBS Circular 564 (9).

6.4.2.1 The model utilizes correlations that have been developed for various orientations of the plate with respect to gravity and for the direction of heat flow (up versus down). Correlations for both laminar and turbulent flow are used, with the choice depending upon the magnitude of the Rayleigh number for natural convection and of the Reynolds number for forced convection. Separate coefficients are calculated for natural and forced flow, and a mixed coefficient is calculated by taking the third root of the sum of the cubes of the two separate coefficients (10).

6.4.2.2 The correlations used in the model are given in Table 1. They account for the effects of surface-to-air temperature difference, heat flow direction, film temperature, surface size, and surface orientation. The correlations are contained in a subroutine called HCON. Values of surface temperature, air temperature, plate tilt angle, plate characteristic length, a flag

TABLE 1 Correlations for Convection Coefficients

I. Natural Convection:
A. Horizontal surface, heat flow up $Nu = 0.54 Ra^{1/4}$ for $Ra < 8 \times 10^6$ $Nu = 0.15 Ra^{1/3}$ for $Ra > 8 \times 10^6$
B. Horizontal surface, heat flow down $Nu = 0.58 Ra^{0.2}$
C. Vertical surface $Nu = 0.59 Ra^{1/4}$ for $Ra < 1 \times 10^9$ $Nu = 0.10 Ra^{1/3}$ for $Ra > 1 \times 10^9$
D. Nearly horizontal surface (tilt angle less than 2°), heat flow down $Nu = 0.58 Ra^{0.2}$
E. Tilted surfaces (greater than 2° tilt), heat flow down $Nu = 0.56 (Ra \sin(\Phi))^{1/4}$ Φ = tilt angle
F. Tilted surface, heat flow up $Nu = 0.56 (Ra \sin(\Phi))^{1/4}$ for $Ra/Pr < Gr_c$ $Nu = 0.14 (Ra^{1/3} - (Gr_c Pr)^{1/3})$ $+ 0.56 (Gr_c Pr \sin(\Phi))^{1/4}$ for $Ra/Pr > Gr_c$ $Gr_c = 1 \times 10^6$ for $\Phi < 15^\circ$ $Gr_c = 10^{(4\Phi/(1.1870 + 0.0870 \times \Phi))}$ for $15^\circ < \Phi < 75^\circ$ $Gr_c = 5 \times 10^9$ for $\Phi > 75^\circ$
II. Forced Convection:
$Nu = 0.664 Pr^{1/3} Re^{1/2}$ for $Re < 5 \times 10^5$ $Nu = Pr^{1/3}(0.037 Re^{0.8} - 850)$ for $Re > 5 \times 10^5$

to denote whether the plate faces up or down, and air speed past the plate are passed to the subroutine. The subroutine returns the mixed convection coefficient. For exterior surfaces, the air speed is taken to be the wind speed obtained from meteorological data. For interior surfaces, a crude estimate of air speed is obtained by dividing the attic ventilation volume flow rate by an average cross-sectional area for the attic. For the interior surfaces, a crude estimate for air speed should suffice, since natural convection should dominate over forced convection.

6.5 Equations—Radiation:

6.5.1 Radiation interchanges within the attic space are handled using the enclosure method described by Sparrow and Cess (11). With this method, each of the surfaces is assumed to be plane, opaque, gray, and isothermal, to be diffusely emitting and reflecting, and to have a uniform radiant flux over the surface. The assumption of diffuse emission and reflection for radiant barrier surfaces may be questioned. Very flat radiant barrier surfaces would be expected to exhibit specular properties. Real radiant barrier surfaces usually are not flat, and hence the diffuse assumption is probably appropriate.

6.5.1.1 The net radiant heat flux away from surface i is given by:

$$Q_{ri} = \sum_{j=1}^N G_{ij} \sigma (T_i^4 - T_j^4) \quad (24)$$

with the following relations:

$$G_{ij} = \frac{\epsilon_i}{1 - \epsilon_i} \psi_{ij} \quad (25)$$

$$\psi_{ij} = \text{inverse of the matrix } \chi_{ij}$$

$$\chi_{ij} = \frac{\delta_{ij} - (1 - \epsilon_i) F_{ij}}{\epsilon_i}$$

$$\delta_{ij} = 0 \text{ for } i \neq j; \delta_{ij} = 1 \text{ for } i = j$$

where:

F_{ij} = radiation view factor from surface i to surface j ,

ϵ_i = emittance of surface i ,



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σ = Stefan-Boltzmann constant, 1.714×10^{-9} Btu/h·ft²·R⁴, and
 N = number of surfaces in enclosure.

6.5.1.2 The heat flux equation may be cast into a form that is linear in the temperatures by factoring as:

$$Q_{ri} = \sum_{j=1}^N G_{ij} \sigma (T_i^2 + T_j^2)(T_i + T_j)(T_i - T_j) \quad (26)$$

$$= \sum_{j=1}^N HR_{ij} (T_i - T_j)$$

6.5.1.3 The quantities appearing in these equations are calculated in several subprograms: subroutine VIEW2, and functions FMN, FP, and HRAD. Function FMN calculates the view factor between two rectangular surfaces that meet at an angle and share a common edge, while function FP calculates the view factor between two parallel surfaces that have the same dimensions. Expressions for the two view factors are given by Sparrow and Cess (11). The functions themselves are based on those given by Peavy, with some necessary modifications (3). The functions have been verified by comparison with the calculations of Feingold (12). Subroutine VIEW2 calculates the view factors among the surfaces that face the attic space, using view factor algebra and calls to functions FMN and FP. The G_{ij} matrix is also calculated within VIEW2. The HR_{ij} matrix is calculated by function HRAD using G_{ij} , T_i , and T_j as inputs.

6.5.1.4 The algorithm for radiation interchanges allows each of the surfaces to have different emittances. The algorithm accounts for all interreflections within the enclosure and properly accounts for the T^4 (Stefan-Boltzmann) law for radiation exchange.

6.6 Equations—Ventilation::

6.6.1 Heat transfer to the ventilation air is treated by an extension of the method used by Peavy (3). With this method, the temperature of the air is assumed to increase (or decrease) as it moves along a flow path and picks up heat by convection from the surrounding surfaces. This is expressed by the following first order ordinary differential equation:

$$\dot{m} C_p \frac{dT}{dx} = \sum_{j=1}^M A(j) HC(j) (TIS(j) - T) \quad (27)$$

$\dot{m}$ = mass flow rate of ventilation air, lb/h,
 C_p = specific heat of air, Btu/lb·°F,
 T = air temperature at position, °F,
 x = normalized position (0–1) along flow path,
 $A(j)$ = area of surface j in contact with ventilation air, ft²,
 $HC(j)$ = convection heat transfer coefficient at surface j , Btu/h·ft²·°F,
 $TIS(j)$ = temperature of surface j , °F, and,
 M = number of surfaces in contact with ventilation air.

6.6.2 In the absence of a detailed picture of the ventilation air flow pattern within the attic, the simplifying assumption has been made that equal proportions of area of each surface are contacted as the air flows along a differential length of its flow path. Integrating this equation yields the following expression for the temperature as a function of flow path position:

$$T = T_o \exp(-ax) + b[1 - \exp(-ax)] \quad (28)$$

where:

T_o = temperature at $x = 0$,

$a = \frac{\sum_{j=1}^M A(j) HC(j) / \dot{m} C_p}{\sum_{j=1}^M A(j) HC(j) TIS(j)}$, and

$b = \frac{\sum_{j=1}^M A(j) HC(j) TIS(j)}{\sum_{j=1}^M A(j) HC(j)}$.

6.6.3 An average air temperature is defined by:

$$TA = \int_0^1 T dx = b[1 + (\exp(-a) - 1)/a] - T_o/a[\exp(-a) - 1] \quad (29)$$

and the temperature of the air exiting from the attic at $x = 1$ is:

$$TE = b[1 - \exp(-a)] + T_o \exp(-a) \quad (30)$$

6.6.4 The flow rate of ventilation air is determined from a combination of stack and wind pressure effects. For the stack effect calculation, the air inside the attic space is assumed to be well-mixed, and relations from Chapter 22 of the 1985 ASHRAE Handbook of Fundamentals are used (13). The volume flow rate due to the stack effect is given by:

$$\dot{V}_S = C_D A [2 gh (TA - T_o)/TA]^{1/2} \text{ for } TA > T_o \quad (31)$$

$$\dot{V}_S = C_D A [2 gh (T_o - TA)/T_o]^{1/2} \text{ for } T_o > TA \quad (32)$$

where:

$\dot{V}_S$ = volume flow rate due to stack effect, ft³/h,

C_D = discharge coefficient taken to be 0.65,

A = the lesser of the net free areas of the vent inlet and outlet, ft²,

h = height of the neutral pressure level from the lower opening, ft,

T_o = absolute temperature of inlet air, °R, and

TA = average absolute temperature of air in attic, °R.

6.6.5 The height of the neutral pressure level is given by:

$$h = \frac{H}{1 + \left(\frac{AI}{AO}\right)^2 \frac{TA}{T_o}} \text{ for } TA > T_o \quad (33)$$

$$h = \frac{H}{1 + \left(\frac{AI}{AO}\right)^2 \frac{T_o}{TA}} \text{ for } T_o > TA \quad (34)$$

where:

H = difference in elevation between inlet and outlet vents, ft,

AI = net free area of inlet vents, ft², and

AO = net free area of outlet vents, ft².

6.6.6 The volume flow rate due to wind pressure is given by:

$$\dot{V}_W = C_F A WS \quad (35)$$

where:

C_F = discharge coefficient,

A = the lesser of the net free areas of the vent inlet and outlet, ft², and

WS = wind speed, ft/h.

