



Designation: D 5473 – 93 (Reapproved 2000)

Standard Test Method for (Analytical Procedure for) Analyzing the Effects of Partial Penetration of Control Well and Determining the Horizontal and Vertical Hydraulic Conductivity in a Nonleaky Confined Aquifer¹

This standard is issued under the fixed designation D 5473; the number immediately following the designation indicates the year of original adoption or, in the case of revision, the year of last revision. A number in parentheses indicates the year of last approval. A superscript epsilon (ε) indicates an editorial change since the last revision or reapproval.

1. Scope

1.1 This test method covers an analytical solution for determining the horizontal and vertical hydraulic conductivity of an aquifer by analysis of the response of water levels in the aquifer to the discharge from a well that partially penetrates the aquifer.

1.2 *Limitations*—The limitations of the technique for determination of the horizontal and vertical hydraulic conductivity of aquifers are primarily related to the correspondence between the field situation and the simplifying assumption of this test method.

1.3 The values stated in either inch-pound or SI units are to be regarded separately as the standard. The values given in parentheses are for information only.

1.4 *This standard does not purport to address all of the safety problems, if any, associated with its use. It is the responsibility of the user of this standard to establish appropriate safety and health practices and determine the applicability of regulatory limitations prior to use.*

2. Referenced Documents

2.1 ASTM Standards:

D 653 Terminology Relating to Soil, Rock, and Contained Fluids²

D 4050 Test Method for (Field Procedure for) Withdrawal and Injection Well Tests for Determining Hydraulic Properties of Aquifer Systems²

D 4105 Test Method for (Analytical Procedure for) Determining Transmissivity and Storativity of Nonleaky Confined Aquifers by the Modified Theis Nonequilibrium Method²

D 4750 Test Method for Determining Subsurface Liquid Levels in a Borehole or Monitoring Well (Observation Well)²

3. Terminology

3.1 Definitions:

3.1.1 *aquifer, confined*—an aquifer bounded above and below by confining beds and in which the static head is above the top of the aquifer.

3.1.2 *confining bed*—a hydrogeologic unit of less permeable material bounding one or more aquifers.

3.1.3 *control well*—well by which the head and flow in the aquifer is changed, for example, by pumping, injection, or imposing a constant change of head.

3.1.4 *drawdown*—vertical distance the static head is lowered due to the removal of water.

3.1.5 *hydraulic conductivity*—(field aquifer tests), the volume of water at the existing kinematic viscosity that will move in a unit time under a unit hydraulic gradient through a unit area measured at right angles to the direction of flow.

3.1.6 *observation well*—a well open to all or part of an aquifer.

3.1.7 *piezometer*—a device so constructed and sealed as to measure hydraulic head at a point in the subsurface.

3.1.8 *specific storage*—the volume of water released from or taken into storage per unit volume of the porous medium per unit change in head.

3.1.9 *storage coefficient*—the volume of water an aquifer releases from or takes into storage per unit surface area of the aquifer per unit change in head.

3.1.10 *transmissivity*—the volume of water at the existing kinematic viscosity that will move in a unit time under a unit hydraulic gradient through a unit width of the aquifer.

3.1.11 *unconfined aquifer*—an aquifer that has a water table.

3.1.12 For definitions of other terms used in this test method, see Terminology D 653.

3.2 Symbols: Symbols and Dimensions:

3.2.1 *a* [nd]— $(K_z/K_r)^{1/2}$.

3.2.2 *b* [L]—thickness of aquifer.

3.2.3 *d* [L]—distance from top of aquifer to top of screened interval of control well.

3.2.4 *d'* [L]—distance from top of aquifer to top of screened interval of observation well.

¹ This test method is under the jurisdiction of ASTM Committee D18 on Soil and Rock and is the direct responsibility of Subcommittee D18.21 on Ground Water and Vadose Zone Investigations.

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² Annual Book of ASTM Standards, Vol 04.08.

3.2.5 f_s [nd]—dimensionless drawdown factor.

3.2.6 K [LT^{-1}]—hydraulic conductivity.

3.2.7 K_r [LT^{-1}]—hydraulic conductivity in the plane of the aquifer, radially from the control well.

3.2.8 K_z [LT^{-1}]—hydraulic conductivity normal to the plane of the aquifer.

3.2.9 K_0 —modified Bessel function of the second kind and zero order.

3.2.10 l [L]—distance from top of aquifer to bottom of screened interval of control well.

3.2.11 l' [L]—distance from top of aquifer to bottom of screened interval of observation well.

3.2.12 Q [L^3T^{-1}]—discharge.

3.2.13 r [L]—radial distance from control well.

3.2.14 r_c —distance from pumped well at which an observed drawdown deviation, δs , would occur in the equivalent isotropic aquifer.

3.2.15 S [nd]—storage coefficient.

3.2.16 s [L]—drawdown.

3.2.17 S_s [L^{-1}]—specific storage.

3.2.18 T [L^2T^{-1}]—transmissivity.

3.2.19 u [nd]— $(r^2S)/(4Tt)$.

3.2.20 $W(u)$ [nd]—an exponential integral known in hydrology as the well function of u .

3.2.21 $W(u, f_s)$ —partial-penetration control well function.

3.2.22 δs [L]—drawdown deviation due to partial penetration from that given by equations for purely radial flow.

3.2.23 z [L]—distance from top of aquifer to bottom of piezometer.

4. Summary of Test Method

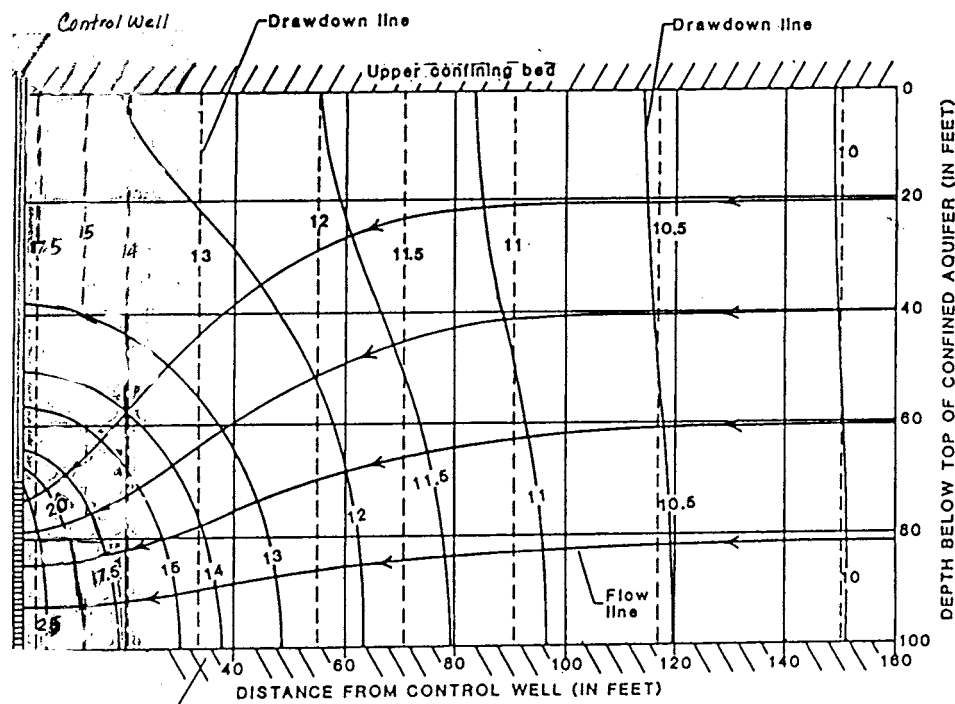
4.1 This test method uses the deviations in drawdown near a partially penetrating control well from those that would occur near a control well fully penetrating the aquifer. These deviations occur when a well partially penetrating the aquifer is pumped because water levels are drawn down more near the level of the screen, and less at levels somewhat above or below the screened interval, than they would be if the pumped well fully penetrated the aquifer. These effects are shown in Fig. 1 by comparing drawdown and flow lines for fully penetrating and partially penetrating control wells in an isotropic aquifer. Drawdown deviations due to partial penetration are amplified when the vertical permeability is less than the horizontal permeability, as often occurs in stratified sediments (1).³ Hantush (2) has shown that at a distance, r , from the control well the drawdown deviation due to pumping a partially penetrating well at a constant rate is the same as that at a distance $r (K_z/K_r)^{1/2}$ if the aquifers were transformed into an equivalent isotropic aquifer.

4.2 *Solutions*—Solutions are given by Hantush (2) for the drawdown near a partially penetrating control well being pumped at a constant rate and tapping a homogeneous, isotropic artesian aquifer:

$$s = \frac{Q}{4\pi T} [W(u) + f_s] \quad (1)$$

where:

³ The boldface numbers in parentheses refer to a list of references at the end of the text.



NOTE 1—Solid lines are for a well screened in the bottom three tenths of the aquifer; dashed lines are for a well screened the full thickness.

FIG. 1 Vertical Section Showing Drawdown Lines and Approximate Flow Paths Near a Pumped Well in an Ideal Artesian Aquifer

$$W(u) = \int_u^{\infty} \frac{e^{-y}}{y} dy \quad (2)$$

and f_s is the dimensionless drawdown correction factor. The function $[W(u) + f_s]$ in Eq 1 can be referred to as the partial penetration well function.

4.2.1 The dimensionless drawdown correction factor for a piezometer is given by:

$$f_s = f\left(u, \frac{ar}{b}, \frac{l}{b}, \frac{d}{b}, \frac{z}{b}\right) \quad (3)$$

$$= \frac{2b}{\pi(l-d)} \sum_{n=1}^{\infty} \frac{1}{n} \left(\sin \frac{n\pi l}{b} - \sin \frac{n\pi d}{b} \right) \cos \frac{n\pi z}{b} W\left(u, \frac{n\pi ar}{b}\right)$$

and the solution for the dimensionless drawdown correction factor for an observation well is given by:

$$f_s = f\left(u, \frac{ar}{b}, \frac{l}{b}, \frac{d}{b}, \frac{l'}{b}, \frac{d'}{b}\right) \quad (4)$$

$$= \frac{2b^2}{\pi^2(l-d)(l'-d')} \sum_{n=1}^{\infty} \frac{1}{n^2} \left(\sin \frac{n\pi l}{b} - \sin \frac{n\pi d}{b} \right) \left(\sin \frac{n\pi l'}{b} - \sin \frac{n\pi d'}{b} \right) W\left(u, \frac{n\pi ar}{b}\right)$$

where:

$$W(m, x) = \int_u^{\infty} \frac{\exp\left(-y - \frac{x^2}{4y}\right)}{y} dy \quad (5)$$

The hydrogeologic conditions and symbols used in connection with piezometer and well geometries are shown in Fig. 2.

4.2.2 For large values of time, that is, for $t > b^2 S / (2a^2 T)$ or $t > b S / (2K_z)$, the effects of partial penetration are constant in time, and $W(u, (n\pi ar)/b)$ can be approximated by $2K_0((n\pi ar)/b)$ (2). K_0 is the modified Bessel function of the second kind of order zero.

4.2.3 Eq 1 can be written

$$s = \frac{Q}{4\pi T} W(u) + \frac{Q}{4\pi T} f_s \quad (6)$$

The first term in Eq 6 is the drawdown in an isotropic homogeneous confined aquifer under radial flow, as given by Theis (3). The second term is deviation from the Theis

drawdown caused by partial penetration of the control well. This term is designated as the drawdown deviation by Weeks (1) and is given by:

$$\delta s = \frac{Q}{4\pi T} f_s \quad (7)$$

4.2.4 The effects of partial penetration need to be considered for $ar/b < 1.5$. There is a response curve for each value of ar/b , d/b , l/b , and either z/b for piezometers, or l'/b and d'/b for observation wells. A table of dimensionless drawdown factors for piezometers from Weeks (1) is given in Table 1 covering 56 different partial-penetration situations. A graph of one of the many families of curves showing the dimensionless drawdown factor f_s versus ar/b for a control well screened, or open, from $z = 0.6b$ to $z = 0.9b$ for various values of piezometer penetration, z/b , is shown in Fig. 3. Because of the even greater number of possible drawdown factors for observation wells, drawdown correction factors for wells are not tabulated.

5. Significance and Use

5.1 Assumptions:

5.1.1 Control well discharges at a constant rate, Q .

5.1.2 Control well is of infinitesimal diameter and partially penetrates the aquifer.

5.1.3 The nonleaky artesian aquifer is homogeneous, and aurally extensive. The aquifer may also be anisotropic and, if so, the directions of maximum and minimum hydraulic conductivity are horizontal and vertical, respectively. The methods may be used to analyze tests on unconfined aquifers under conditions described in a following section.

5.1.4 Discharge from the well is derived exclusively from storage in the aquifer.

5.1.5 The geometry of the assumed aquifer and well conditions are shown in Fig. 2.

5.2 Implications of Assumptions—The vertical flow components in the aquifer are induced by a control well that partially penetrates the aquifer, that is, a well that is not open to the aquifer through its full thickness. The effects of vertical flow components are measured in piezometers near the control well,

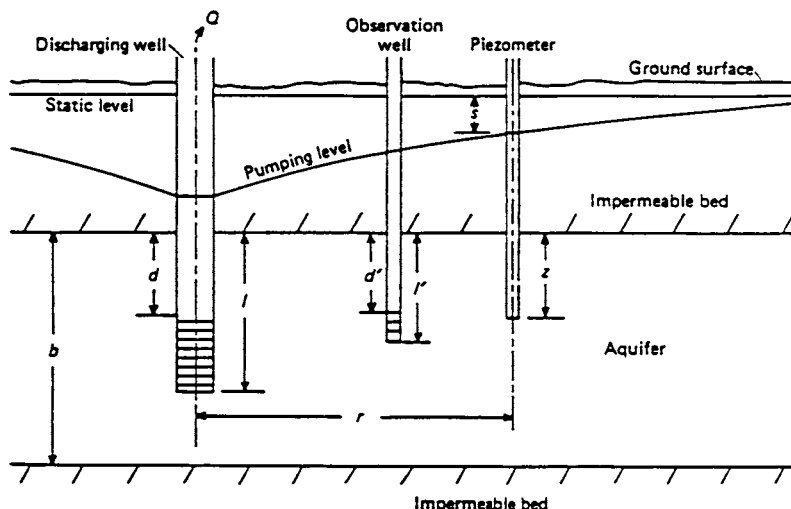


FIG. 2 Cross Section Through a Discharging Well That is Screened in a Part of a Nonleaky Aquifer

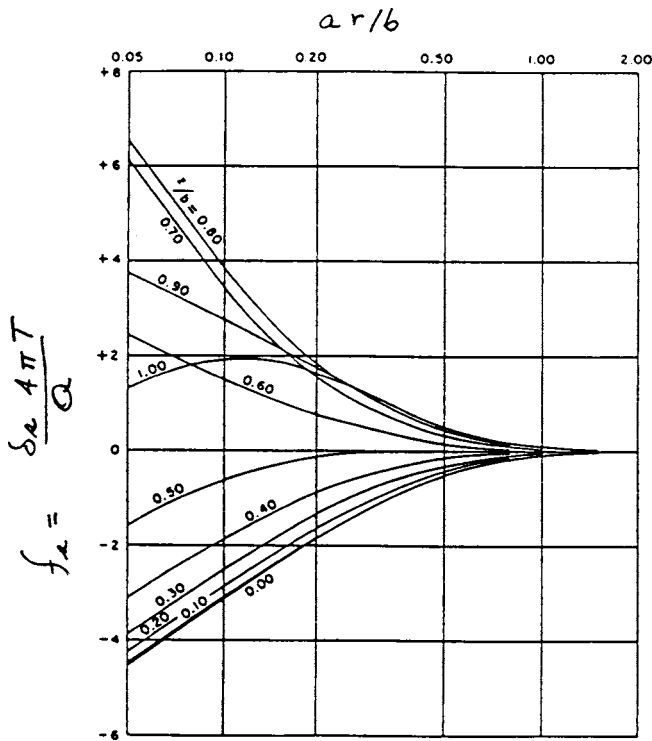


FIG. 3 Graph of Dimensionless Drawdown Factor, f_s , versus ar/b for a Pumped Well Screened from $z = 0.66$ to $z = 0.96$ for Values of Piezometer Penetration, z/b

that is, within a distance, r , in which vertical flow components are significant, that is:

$$r < 1.5b \sqrt{Kr/K_z} \quad (8)$$

5.3 Application of Method to Unconfined Aquifers:

5.3.1 Although the assumptions are applicable to artesian or confined conditions, Weeks (1) has pointed out that the solution may be applied to unconfined aquifers if drawdown is small compared with the saturated thickness of the aquifer or if the drawdown is corrected for reduction in thickness of the aquifer, and the effects of delayed gravity response are small. The effects of gravity response become negligible after a time as given, for piezometers near the water table, by the equation:

$$t = \frac{bS_y}{K_z} \quad (9)$$

for values of $ar/b < 0.4$ and by the equation:

$$t = \frac{bS_y}{K_z} \left(0.5 + 1.25 \frac{r}{b} \sqrt{\frac{K_z}{K_r}} \right) \quad (10)$$

for greater values of ar/b .

5.3.2 Drawdown in an unconfined aquifer is also affected by curvature of the water table or free surface near the control well, and by the decrease in saturated thickness, that causes the transmissivity to decline toward the control well. This test method should be applicable to analysis of tests on water-table aquifers for which the control well is cased to a depth below the pumping level and the drawdown in the control well is less than $0.2b$. Moreover, little error would be introduced by effects of water-table curvature, even for a greater drawdown in the control well, if the term $(s^2/2b)$ for a given piezometer is small compared to the δs term.

5.3.3 The transmissivity decreases as a result of decreasing thickness of the unconfined aquifer near the control well. Jacob (4) has shown that the effect of decreasing transmissivity on the drawdown may be corrected by the equation:

$$s' = s - (s^2/2b) \quad (11)$$

where s is the observed drawdown and s' is the drawdown in an equivalent confined aquifer.

6. Apparatus

6.1 Apparatus for withdrawal tests is given in Test Method D 4050. The apparatus described as follows are those components of the apparatus that require special attributes for this specific test method.

6.2 *Construction of Control Well*—Screen the control well through only part of the vertical extent of the aquifer to be tested. The screened interval of the control well must be known as a function of aquifer thickness.

6.3 *Construction and Placement of Piezometers and Observation Wells*—The requirements for observation wells and piezometers are related to the method of analysis to be used. Two methods of analysis are prescribed in Section 8; the observation well and piezometer requirements for each method are given as follows. The piezometers and observation wells may be on the same or various radial lines from the control well.

6.3.1 The type curve fitting methods require one or more piezometers near the control well within the radial distance affected by vertical flow components. This distance is given by $r < 1.5b/(K_z/K_r)^{1/2}$. The depth of the piezometer opening must be known as a function of the aquifer thickness. Construction of piezometers or wells for a specific test shall be identical with respect to distance from the top of the aquifer to the bottom of the piezometers or the screened interval of the wells.

6.3.2 Method 1 of the drawdown deviation methods requires one or more piezometers or wells near the control well within the radial distance affected by vertical flow components. The depth of these piezometers and the screened interval of wells must be known as a function of aquifer thickness. Construction of piezometers or wells for a specific test within the distance affected by vertical flow components shall be identical with respect to distance from the top of the aquifer to the bottom of the piezometers or the screened interval of the wells. In addition, the method requires two or more observation wells or piezometers at a distance from the control well beyond the effect of vertical flow components.

6.3.3 Method 2 of the drawdown deviation methods requires two or more piezometers within the radial distance affected by vertical flow components. Construction of piezometers or wells for a specific test within the distance affected by vertical flow components shall be identical with respect to distance from the top of the aquifer to the bottom of the piezometers or the screened interval of the wells.

NOTE 1—The drawdown deviation methods were originated by Weeks (1) who published tables of the drawdown correction factors for piezometers. Partially penetrating observation wells may be used in place of or in addition to the piezometers. Weeks (1) has found that data from observation wells screened for less than 20 % of the aquifer thickness, using the center of the screen as the piezometer depth, can be used in place of

piezometers if the position of the screen in the observation well is above or below that of the screen in the pumped well. However, if the observation well is screened at the same level or overlaps that in the pumped well, Eq 1, or the values in Table 1 derived from Eq 1, should be used only when the screen length of the observation well is less than about 5 % of the aquifer thickness. Data obtained from observation wells open or screened in a larger part of the aquifer thickness could be analyzed by values of the drawdown correction factor derived from Eq 4. Drawdown correction factors can be derived from values of $[W(u) + f_s]$, computed from the Fortran code of Reed (5) or the basic code of Dawson and Istok (6).

7. Procedure

7.1 *Pretest Preparations*—Pretest preparations are given in more detail in Test Method D 4050.

7.1.1 *Testing Response of Piezometers and Observation Wells*—The piezometers and observation wells are tested by pumping or injecting water to assure hydraulic connection between the well and the aquifer.

7.1.2 Measure water levels to determine the trend of water levels before the commencement of the test.

7.1.3 *Step Test*—Pump the control well at steady, progressively greater rates to estimate the transmissivity and select a steady rate of pumping for the aquifer test.

7.2 *Aquifer Testing*—The field procedure summarized below for pumping the control well and measurement of water levels is given in detail in Test Method D 4050.

7.2.1 *Pump Control Well*—Pump the control well at a constant rate. Measure well discharge periodically.

7.2.2 *Measure Water Level in Piezometers and Observation Wells*—Measure water levels frequently during the early phase of pumping; increase the interval between measurements logarithmically as pumping continues.

7.3 *Analysis of the Test Data*—The field test data are analyzed by methods described in Section 8.

8. Calculation and Interpretation of Results

8.1 *Type Curve Methods*—Two type curve methods are presented. The first method is employed by plotting drawdown versus time for each observation well and matching the data plot with prepared-type curves of $[W(u) + f_s]$ versus $1/u$. The second method is employed by plotting drawdown versus r^2/t for one or more wells on the same graph and matching with prepared families of type curves of $[W(u) + f_s]$ versus $1/u$.

8.1.1 *Type Curve Method 1*—This test method is applicable where one or more piezometers or wells are within the distance from the control well affected by vertical flow components.

8.1.2 Select a range of values of $a = (K_z/K_r)^{1/2}$ and prepare a set of type curves for each observation well. For each type curve having values of a and ar/b , plot $[W(u) + f_s]$ versus $1/u$ on logarithmic paper (see Fig. 4).

NOTE 2—The type curves can be plotted from values of $[W(u) + f_s]$ calculated from the Fortran program in Table 2.1 of Reed (5) or the Basic program, TYPE6, of Dawson and Istok (6).

8.1.3 For each observation well, prepare plots of data by plotting s versus t using the same logarithmic scales used to plot the type curves (Fig. 4).

8.1.4 Overlay the data plot on the family of type curves developed for that observation well. Shift the plots relative to each other, keeping the axes parallel, until a position of best fit is found between the data plot and one of the type curves.

8.1.5 Select a common match point on the data plot and the type curve. Record the value of a for the type curve and values of $[W(u) + f_s]$, s , u , and t for the data and type curve match point.

8.1.6 Calculate transmissivity, T , from Eq 1.

8.1.7 Calculate $K_r = T/b$.

8.1.8 From the value of $a = (K_z/K_r)^{1/2}$ for the type curve, calculate $K_z = K_r \cdot a^2$.

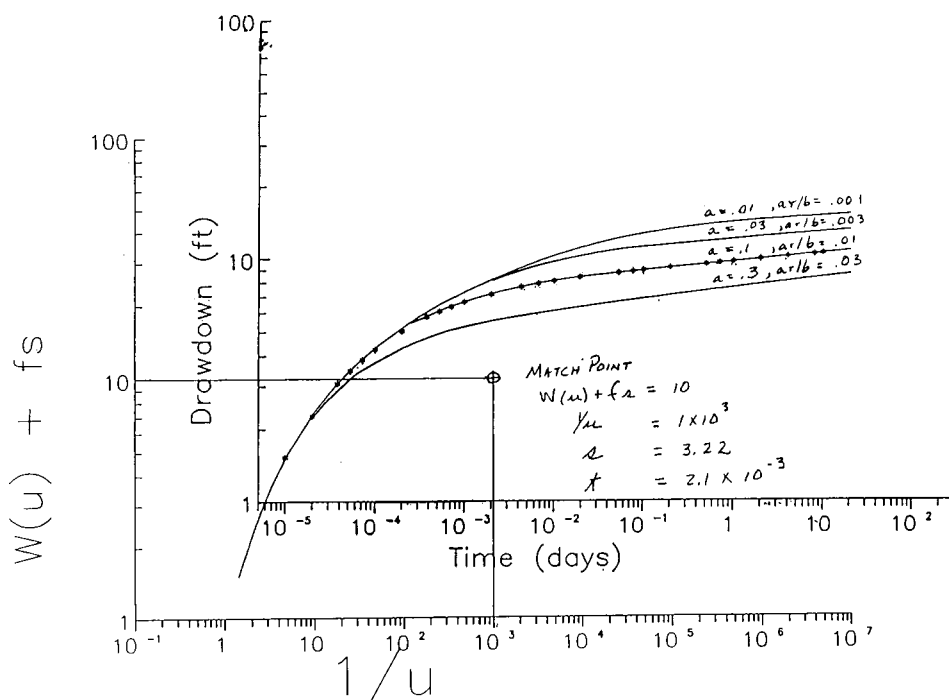


FIG. 4 Data and Type Curves

8.1.9 Substitute values of T , u , t , and r in the equation $u = (r^2 S/4Tt)$ and solve for storage coefficient, S .

NOTE 3—From the match point in Fig. 4, transmissivity is calculated:

$$T = Q/4\pi s (W(u) + f_s) \quad (12)$$

$$T = (19\,250 * 10)/(4 * \pi * 3.22) = 4800 \text{ ft}^2 \text{ day}^{-1} \text{ (rounded)}$$

The hydraulic conductivity radially from the well is calculated:

$$K_r = T/b = 4800/100 = 480 \text{ ft day}^{-1} \quad (13)$$

The hydraulic conductivity normal to the plane of the aquifer is calculated:

$$K_z = 480 * 0.01 = 4.8 \text{ ft day}^{-1} \quad (14)$$

The storage coefficient is calculated:

$$S = 4Tu(t/r^2) \quad (15)$$

$$S = (4 * 4800 * 2.1 * 10^{-3})/(100 * 1000) = 4 * 10^{-5}$$

Note that the curves are similar for both early and late times. In calculating the values for a single well, both early and late water-level measurements are needed to select the proper curve. Without early and late data to select the proper curve, values of transmissivity, and radial and vertical hydraulic conductivity are affected less than the value of storage coefficient.

8.1.10 *Type Curve Method 2*—This test method is applicable where two or more observation wells are within the distance from the control well affected by vertical flow components.

8.1.11 Prepare a set of family-type curves, each family of several curves for selected values of a . For each family of type curves, with equal a , plot $[W(u) + f_s]$ versus $1/u$ on logarithmic paper (see Fig. 5). The type curves can be plotted from values of $[W(u) + f_s]$ calculated from the Fortran program in Table 2.1 of Reed (5) or the basic program, TYPE6, of Dawson and Istok (6).

8.1.12 Prepare a data plot of all observation wells on the same graph. Plot data for each well as s versus r^2/t using the same sized logarithmic scales used to plot the type curves (see Fig. 5).

8.1.13 Overlay the data plots on each family of type curves. Shift the plots relative to each other, keeping the axes parallel, until a position of best fit is found between the data plots and one family of type curves.

8.1.14 Select a common match point on the data plot and the type curve plot. Record values of $[W(u) + f_s]$, s , u , and t for the match point and the value of a for the family of type curves.

8.1.15 Calculate transmissivity, T , from Eq 1.

8.1.16 Calculate the value of $K_r = T/b$.

8.1.17 From the value of $a = (K_z/K_r)^{1/2}$ for the match point, the transmissivity, and the thickness of the aquifer, b , calculate K_z from Eq 12.

8.1.18 Substitute values of T , u , t , and r in the equation $u = (r^2 S/4Tt)$ and solve for storage coefficient, S .

NOTE 4—From the match point in Fig. 5, the transmissivity is calculated:

$$T = Q/4\pi s (W(u) + f_s) \quad (16)$$

$$T = (204\,050 * 1.0)/(4 * \pi * 1.0) = 16\,000 \text{ (rounded)}$$

The hydraulic conductivity radially from the control well is calculated:

$$K_r = T/b = 16\,000/80 = 200 \text{ ft day}^{-1} \quad (17)$$

The hydraulic conductivity normal to the plane of the aquifer is calculated:

$$K_z = 200 * 0.04 = 8 \text{ ft day}^{-1} \quad (18)$$

The storage coefficient is calculated:

$$S = 4Tu(t/r^2) \quad (19)$$

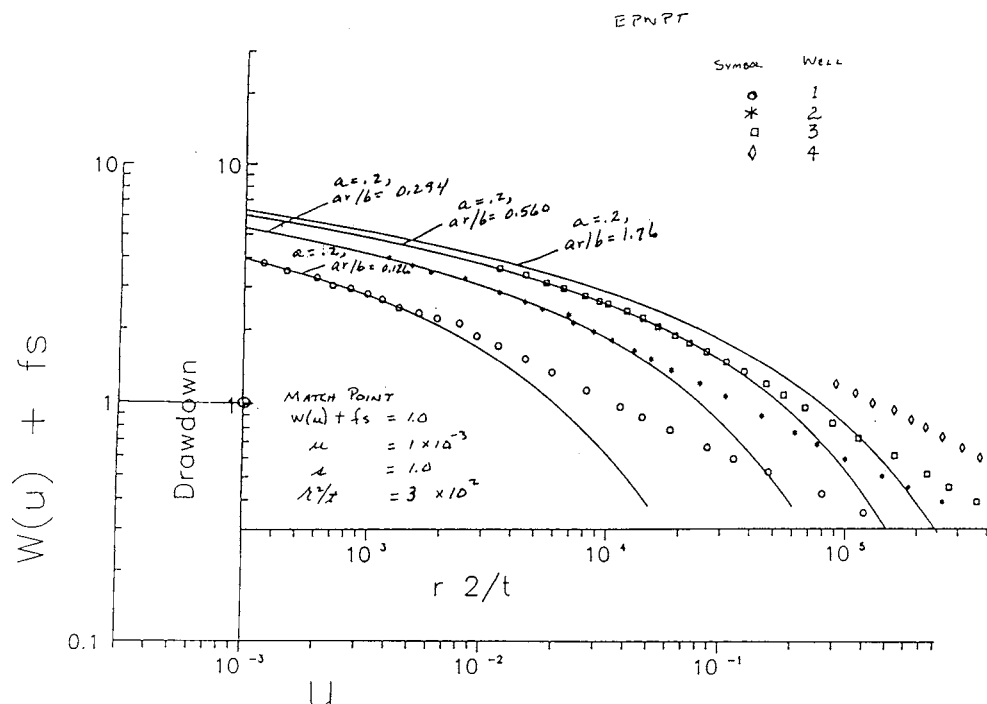


FIG. 5 Data and Type Curves for Multiple Observation Points and Match Point Located