



Standard Test Method for (Analytical Procedure) Determining Transmissivity of Confined Nonleaky Aquifers by Critically Damped Well Response to Instantaneous Change in Head (Slug)¹

This standard is issued under the fixed designation D5881; the number immediately following the designation indicates the year of original adoption or, in the case of revision, the year of last revision. A number in parentheses indicates the year of last reapproval. A superscript epsilon (ϵ) indicates an editorial change since the last revision or reapproval.

1. Scope*

1.1 This test method covers determination of transmissivity from the measurement of water-level response to a sudden change of water level in a well-aquifer system characterized as being critically damped or in the transition range from underdamped to overdamped. Underdamped response is characterized by oscillatory changes in water level; overdamped response is characterized by return of the water level to the initial static level in an approximately exponential manner. Overdamped response is covered in Guide [D4043](#); underdamped response is covered in [D5785](#), [D4043](#).

1.2 The analytical procedure in this test method is used in conjunction with Guide [D4043](#) and the field procedure in Test Method [D4044](#) for collection of test data.

1.3 *Limitations*—Slug tests are considered to provide an estimate of the transmissivity of an aquifer near the well screen. The method is applicable for systems in which the damping parameter, ζ , is within the range from 0.2 through 5.0. The assumptions of the method prescribe a fully penetrating well (a well open through the full thickness of the aquifer) in a confined, nonleaky aquifer.

1.4 All observed and calculated values shall conform to the guidelines for significant digits and rounding established in Practice [D6026](#).

1.5 *Units*—The values stated in SI units are to be regarded as standard. No other units of measurement are included in this standard.

1.5.1 The procedures used to specify how data are collected/recorded or calculated, in this standard are regarded as the industry standard. In addition, they are representative of the significant digits that generally should be retained. The procedures used do not consider material variation, purpose for obtaining the data, special purpose studies, or any consider-

ations for the user's objectives and it is common practice to increase to reduce significant digits of reported data to be commensurate with these considerations. It is beyond the scope of this standard to consider significant digits used in analytical methods for design.

1.6 *This standard does not purport to address all of the safety concerns, if any, associated with its use. It is the responsibility of the user of this standard to establish appropriate safety and health practices and determine the applicability of regulatory limitations prior to use.*

2. Referenced Documents

2.1 ASTM Standards:²

- [D653 Terminology Relating to Soil, Rock, and Contained Fluids](#)
- [D3740 Practice for Minimum Requirements for Agencies Engaged in Testing and/or Inspection of Soil and Rock as Used in Engineering Design and Construction](#)
- [D4043 Guide for Selection of Aquifer Test Method in Determining Hydraulic Properties by Well Techniques](#)
- [D4044 Test Method for \(Field Procedure\) for Instantaneous Change in Head \(Slug\) Tests for Determining Hydraulic Properties of Aquifers](#)
- [D5785 Test Method for \(Analytical Procedure\) for Determining Transmissivity of Confined Nonleaky Aquifers by Underdamped Well Response to Instantaneous Change in Head \(Slug Test\)](#)
- [D6026 Practice for Using Significant Digits in Geotechnical Data](#)

3. Terminology

3.1 *Definitions*—For definitions of common technical terms in this standard, refer to Terminology [D653](#).

3.2 Definitions of Terms Specific to This Standard:

3.2.1 *aquifer, confined*—an aquifer bounded above and below by confining beds and in which the static head is above the top of the aquifer.

¹ This test method is under the jurisdiction of ASTM Committee [D18](#) on Soil and Rock and is the direct responsibility of Subcommittee [D18.21](#) on Groundwater and Vadose Zone Investigations.

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² For referenced ASTM standards, visit the ASTM website, www.astm.org, or contact ASTM Customer Service at service@astm.org. For *Annual Book of ASTM Standards* volume information, refer to the standard's Document Summary page on the ASTM website.

3.2.2 *critically damped well response*—characterized by the water level responding in a transitional range between underdamped and overdamped following a sudden change in water level.

3.2.3 *observation well*—a well open to all or part of an aquifer.

3.3 Symbols and Dimensions:

3.3.1 T —transmissivity [$L^2 T^{-1}$].

3.3.2 S —storage coefficient [nd].

3.3.3 L —static water column length above top of aquifer [L].

3.3.4 L_e —effective length of water column in a well, equal to $L_c + (r_c^2/r_s^2)(b/2)$ [L].

3.3.5 L_c —length of water column within casing [L].

3.3.6 L_s —length of water column within well screen [L].

3.3.7 g —acceleration of gravity [LT^{-2}].

3.3.8 h —hydraulic head in the aquifer [L].

3.3.9 h_o —initial hydraulic head in the aquifer [L].

3.3.10 h_s —hydraulic head in the well screen [L].

3.3.11 r_c —radius of well casing [L].

3.3.12 r_s —radius of well screen [L].

3.3.13 t —time [T].

3.3.14 t' —dimensionless time [nd].

3.3.15 $\hat{t}$ —dimensionless time [nd].

3.3.16 w —water level displacement from the initial static level [L].

3.3.17 w_o —initial water level displacement [L].

3.3.18 α —dimensionless storage parameter [nd].

3.3.19 β —dimensionless inertial parameter [nd].

3.3.20 γ —damping constant [T^{-1}].

3.3.21 τ —wavelength [T].

3.3.22 ω —angular frequency [T^{-1}].

3.3.23 ζ —dimensionless damping factor [nd].

4. Summary of Test Method

4.1 This test method describes the analytical procedure for analyzing data collected during an instantaneous head (slug) test for well and aquifer response at and near critical damping. Procedures in conducting a slug test are given in Test Method **D4044**. The analytical procedure consists of analyzing the response of water level in the well following the change in water level induced in the well.

4.2 *Theory*—The equations that govern the response of well to an instantaneous change in head are treated at length by Kipp **(1)**.³ The flow in the aquifer is governed by the following equation for cylindrical flow:

$$\frac{S}{T} \frac{dh}{dt} = \frac{1}{r} \frac{d}{dr} \left(r \frac{dh}{dr} \right) \quad (1)$$

³ The boldface numbers in parentheses refer to a list of references at the end of this standard.

where:

h = hydraulic head,

T = aquifer transmissivity, and

S = storage coefficient.

4.2.1 The initial condition is at $t = 0$ and $h = h_o$, and the outer boundary condition is as $r \rightarrow \infty$ and $h = h_o$.

4.2.1.1 An equation is given by Kipp **(1)** for the skin factor, that is, the effect of aquifer damage during drilling of the well. However, this factor is not treated by Kipp **(1)** and is not considered in this procedure.

4.2.2 The flow rate balance on the well bore relates the displacement of the water level in the well riser to the flow into the well:

$$\pi r_c^2 \frac{dw}{dt} = 2\pi r_s T \frac{dh}{dr} \bigg|_{r=r_s} \quad (2)$$

where:

r_c = radius of the well casing, and

w = displacement of the water level in the well from its initial position.

4.2.3 The fourth equation describing the system relating h_s and w , comes from a momentum balance equation of Bird et al **(2)** as referenced in Kipp **(1)**:

$$\frac{d}{dt} \int_{-b}^0 \pi r_s^2 p v dz = (-p v_2^2 + p_1 - p_2 - \rho g b) \pi r_s^2 \quad (3)$$

where:

v = velocity in the well screen interval,

b = aquifer thickness,

p = pressure,

ρ = fluid density,

g = gravitational acceleration, and

r_s = well screen radius.

The numerical subscripts refer to the planes described above and shown in **Fig. 1**. Atmospheric pressure is taken as zero.

5. Solution

5.1 Kipp **(1)** derives the following differential equation to represent for the response of the displacement of water level in the well:

$$\frac{d^2 w}{dt^2} + \left(\frac{g}{L_e} \right) w = \frac{g}{(h_s - h_o)/L_e} \quad (4)$$

where:

L_e = effective water column length, defined as:

$$L_e = L + (r_c^2/r_s^2)(b/2) \quad (5)$$

where:

b = aquifer thickness with initial conditions:

$$\text{at } t = 0, w = w_o \quad (6)$$

$$dw/dt = w_o^* \quad (7)$$

$$h_s = L = h_o \quad (8)$$

5.2 Kipp **(1)** introduces dimensionless variables and parameters in converting these equations to dimensionless form, solves the equations by Laplace transforms, and inverts the solution by a Laplace-transform-inverse Laplace transform.

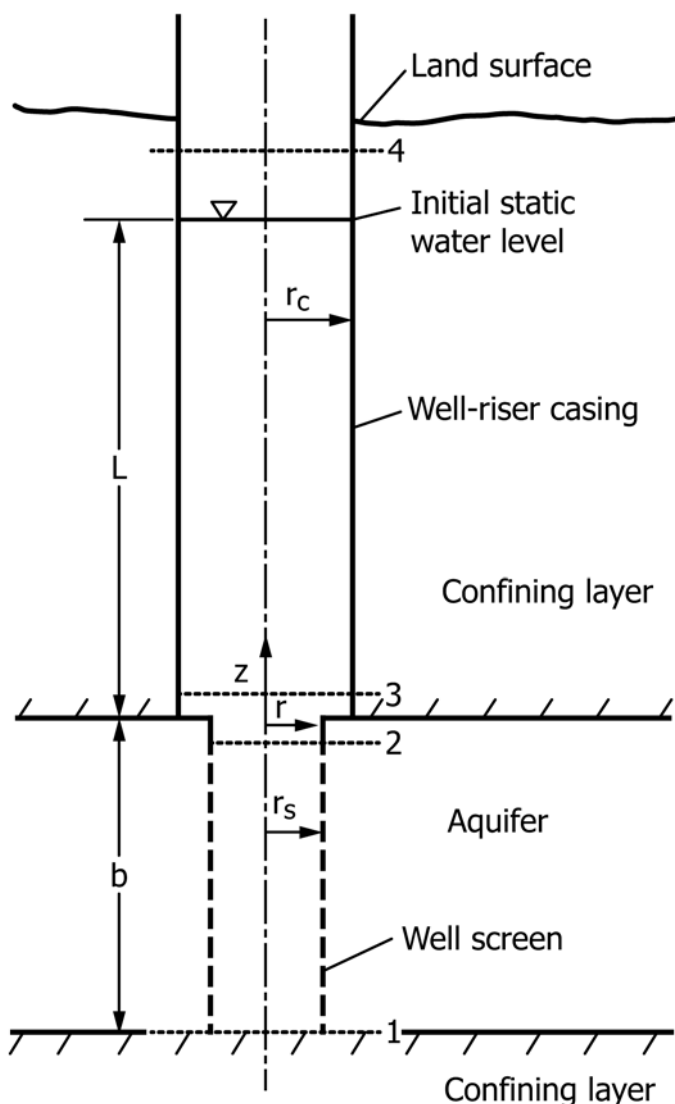


FIG. 1 Well and Aquifer Geometry from Kipp (1)

5.2.1 The following dimensionless parameters are among those given by Kipp (1):

dimensionless water-level displacement:

$$w' = -w/w_o \quad (9)$$

dimensionless time:

$$t' = (tT)/(r_s^2 S) \quad (10)$$

and:

$$\hat{t} = t'/\beta^{1/2} \quad (11)$$

dimensionless storage:

$$\alpha = (r_c^2) (2r_s^2 S) \quad (12)$$

dimensionless inertial parameter:

$$\beta = (Le/g)(T/(r_s^2 S))^2 \quad (13)$$

dimensionless skin factor:

$$\sigma = f/r_s \quad (14)$$

dimensionless frequency parameter:

$$\omega = \frac{[-d^2(\sigma + 1/4 \ln \beta) + 4\beta]^{1/2}}{2\beta} \quad (15)$$

dimensionless decay parameter:

$$\gamma = \frac{\alpha(\sigma + 1/4 \ln \beta)}{2\beta} \quad (16)$$

and dimensionless damping factor:

$$\zeta = \frac{\alpha(\sigma + 1/4 \ln \beta)}{2\beta^{1/2}} \quad (17)$$

5.3 For ζ less than one, the system is underdamped; for ζ greater than one, the system is overdamped. For ζ equal to one, the system is critically damped, yet the inertial effects are quite important (1). For ζ greater than about five, the system responds as if the inertial effects can be neglected and the solution of Cooper et al. (3) (given in Guide D4043) is applicable. For ζ about 0.2 or less, the approximate solution of vander Kamp (4) is valid (given in Test Method D5785). The solution of Kipp (1), the subject of this test method, is applicable for the transition zone between systems that are underdamped and overdamped. Solutions are given here for ζ ranging from 0.2 to 5.0.

6. Significance and Use

6.1 The assumptions of the physical system are given as follows:

6.1.1 The aquifer is of uniform thickness, with impermeable upper and lower confining boundaries.

6.1.2 The aquifer is of constant homogeneous porosity and matrix compressibility and constant homogeneous and isotropic hydraulic conductivity.

6.1.3 The origin of the cylindrical coordinate system is taken to be on the well-bore axis at the top of the aquifer.

6.1.4 The aquifer is fully screened.

6.1.5 The well is 100 % efficient, that is, the skin factor, f , and dimensionless skin factor, σ , are zero.

6.2 The assumptions made in defining the momentum balance are as follows:

6.2.1 The average water velocity in the well is approximately constant over the well-bore section.

6.2.2 Frictional head losses from flow in the well are negligible.

6.2.3 Flow through the well screen is uniformly distributed over the entire aquifer thickness.

6.2.4 Change in momentum from the water velocity changing from radial flow through the screen to vertical flow in the well are negligible.

NOTE 1—The quality of the result produced by this standard is dependent on the competence of the personnel performing it, and the suitability of the equipment and facilities used. Agencies that meet the criteria of Practice D3740 are generally considered capable of competent and objective testing/sampling/inspection/etc. Users of this standard are cautioned that compliance with Practice D3740 does not in itself assure reliable results. Reliable results depend on many factors; Practice D3740 provides a means of evaluating some of those factors.

7. Procedure

7.1 The overall procedure consists of conducting the slug test field procedure (see Test Method D4044) and analysis of the field data using this test method.