



Designation: D5881 – 95(Reapproved 2005)

Standard Test Method for (Analytical Procedure) Determining Transmissivity of Confined Nonleaky Aquifers by Critically Damped Well Response to Instantaneous Change in Head (Slug)¹

This standard is issued under the fixed designation D5881; the number immediately following the designation indicates the year of original adoption or, in the case of revision, the year of last revision. A number in parentheses indicates the year of last reapproval. A superscript epsilon (ϵ) indicates an editorial change since the last revision or reapproval.

1. Scope

1.1 This test method covers determination of transmissivity from the measurement of water-level response to a sudden change of water level in a well-aquifer system characterized as being critically damped or in the transition range from underdamped to overdamped. Underdamped response is characterized by oscillatory changes in water level; overdamped response is characterized by return of the water level to the initial static level in an approximately exponential manner. Overdamped response is covered in Guide [D4043](#); underdamped response is covered in [D5785](#), [D4043](#).

1.2 The analytical procedure in this test method is used in conjunction with Guide [D4043](#) and the field procedure in Test Method [D4044](#) for collection of test data.

1.3 *Limitations*—Slug tests are considered to provide an estimate of the transmissivity of an aquifer near the well screen. The method is applicable for systems in which the damping parameter, ζ , is within the range from 0.2 through 5.0. The assumptions of the method prescribe a fully penetrating well (a well open through the full thickness of the aquifer) in a confined, nonleaky aquifer.

1.4 The values stated in SI units are to be regarded as standard.

1.5 *This standard does not purport to address all of the safety concerns, if any, associated with its use. It is the responsibility of the user of this standard to establish appropriate safety and health practices and determine the applicability of regulatory limitations prior to use.*

¹ This test method is under the jurisdiction of ASTM Committee [D18](#) on Soil and Rock and is the direct responsibility of Subcommittee [D18.21](#) on Groundwater and Vadose Zone Investigations.

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2. Referenced Documents

2.1 *ASTM Standards*:²

[D653 Terminology Relating to Soil, Rock, and Contained Fluids](#)

[D4043 Guide for Selection of Aquifer Test Method in Determining Hydraulic Properties by Well Techniques](#)

[D4044 Test Method for \(Field Procedure\) for Instantaneous Change in Head \(Slug\) Tests for Determining Hydraulic Properties of Aquifers](#)

[D4750 Test Method for Determining Subsurface Liquid Levels in a Borehole or Monitoring Well \(Observation Well\)](#) (Withdrawn 2010)³

[D5785 Test Method for \(Analytical Procedure\) for Determining Transmissivity of Confined Nonleaky Aquifers by Underdamped Well Response to Instantaneous Change in Head \(Slug Test\)](#)

3. Terminology

3.1 *Definitions*:

3.1.1 *aquifer, confined*—an aquifer bounded above and below by confining beds and in which the static head is above the top of the aquifer.

3.1.2 *confining bed*—a hydrogeologic unit of less permeable material bounding one or more aquifers.

3.1.3 *control well*—a well by which the head and flow in the aquifer is changed by pumping, injecting, or imposing a constant change of head.

² For referenced ASTM standards, visit the ASTM website, www.astm.org, or contact ASTM Customer Service at service@astm.org. For *Annual Book of ASTM Standards* volume information, refer to the standard's Document Summary page on the ASTM website.

³ The last approved version of this historical standard is referenced on www.astm.org.

3.1.4 *critically damped well response*—characterized by the water level responding in a transitional range between underdamped and overdamped following a sudden change in water level.

3.1.5 *head, static*—the height above a standard datum the surface of a column of water can be supported by the static pressure at a given point.

3.1.6 *observation well*—a well open to all or part of an aquifer.

3.1.7 *overdamped well response*—characterized by the water level returning to the static level in an approximately exponential manner following a sudden change in water level. (See for comparison *underdamped well response*.)

3.1.8 *slug*—a volume of water or solid object used to induce a sudden change of head in a well.

3.1.9 *storage coefficient*—the volume of water an aquifer releases from or takes into storage per unit surface area of the aquifer per unit change in head. For a confined aquifer, the storage coefficient is equal to the product of the specific storage and aquifer thickness.

3.1.10 *transmissivity*—the volume of water at the existing kinematic viscosity that will move in a unit time under a unit hydraulic gradient through a unit width of the aquifer.

3.1.11 *underdamped well response*—response characterized by the water level oscillating about the static water level following a sudden change in water level (See for comparison *overdamped-well response*).

3.1.12 For definitions of other terms used in this test method, see Terminology **D653**.

3.2 Symbols and Dimensions:

3.2.1 T —transmissivity [$L^2 T^{-1}$].

3.2.2 S —storage coefficient [nd].

3.2.3 L —static water column length above top of aquifer [L].

3.2.4 L_e —effective length of water column in a well, equal to $L_c + (r_c^2/r_s^2) (b/2)$ [L].

3.2.5 L_c —length of water column within casing [L].

3.2.6 L_s —length of water column within well screen [L].

3.2.7 g —acceleration of gravity [LT^{-2}].

3.2.8 h —hydraulic head in the aquifer [L].

3.2.9 h_o —initial hydraulic head in the aquifer [L].

3.2.10 h_s —hydraulic head in the well screen [L].

3.2.11 r_c —radius of well casing [L].

3.2.12 r_s —radius of well screen [L].

3.2.13 t —time [T].

3.2.14 t' —dimensionless time [nd].

3.2.15 t —dimensionless time [nd].

3.2.16 w —water level displacement from the initial static level [L].

3.2.17 w_o —initial water level displacement [L].

3.2.18 α —dimensionless storage parameter [nd].

3.2.19 β —dimensionless inertial parameter [nd].

3.2.20 γ —damping constant [T^{-1}].

3.2.21 τ —wavelength [T].

3.2.22 ω —angular frequency [T^{-1}].

3.2.23 ζ —dimensionless damping factor [nd].

4. Summary of Test Method

4.1 This test method describes the analytical procedure for analyzing data collected during an instantaneous head (slug) test for well and aquifer response at and near critical damping. Procedures in conducting a slug test are given in Test Method **D4044**. The analytical procedure consists of analyzing the response of water level in the well following the change in water level induced in the well.

4.2 *Theory*—The equations that govern the response of well to an instantaneous change in head are treated at length by Kipp (1).⁴ The flow in the aquifer is governed by the following equation for cylindrical flow:

$$\frac{S}{T} \frac{dh}{dt} = \frac{1}{r} \frac{d}{dr} \left(r \frac{dh}{dr} \right) \quad (1)$$

where:

h = hydraulic head,

T = aquifer transmissivity, and

S = storage coefficient.

4.2.1 The initial condition is at $t = 0$ and $h = h_o$, and the outer boundary condition is as $r \rightarrow \infty$ and $h = h_o$.

4.2.1.1 An equation is given by Kipp (1) for the skin factor, that is, the effect of aquifer damage during drilling of the well. However, this factor is not treated by Kipp (1) and is not considered in this procedure.

4.2.2 The flow rate balance on the well bore relates the displacement of the water level in the well riser to the flow into the well:

$$\pi r_c^2 \frac{dw}{dt} = 2\pi r_s T \frac{dh}{dr} \Big|_{r=r_s} \quad (2)$$

where:

r_c = radius of the well casing, and

w = displacement of the water level in the well from its initial position.

4.2.3 The fourth equation describing the system relating h_s and w , comes from a momentum balance equation of Bird et al (2) as referenced in Kipp (1):

$$\frac{d}{dt} \int_{-b}^0 \pi r_s^2 p v dz = (-p v_2^2 + p_1 - p_2 - \rho g b) \pi r_s^2 \quad (3)$$

where:

v = velocity in the well screen interval,

b = aquifer thickness,

p = pressure,

ρ = fluid density,

g = gravitational acceleration, and

r_s = well screen radius.

⁴ The boldface numbers in parentheses refer to a list of references at the end of the text.

The numerical subscripts refer to the planes described above and shown in Fig. 1. Atmospheric pressure is taken as zero.

5. Solution

5.1 Kipp (1) derives the following differential equation to represent for the response of the displacement of water level in the well:

$$\frac{d^2 w}{dt^2} + \left(\frac{g}{L_e} \right) w = \frac{g}{(h_s - h_o)/L_e} \quad (4)$$

where:

L_e = effective water column length, defined as:

$$L_e = L + (r_c^2/r_s^2)(b/2) \quad (5)$$

where:

b = aquifer thickness with initial conditions:

$$\text{at } t=0, w=w_o \quad (6)$$

$$dw/dt = w_o^* \quad (7)$$

$$h_s = L = h_o \quad (8)$$

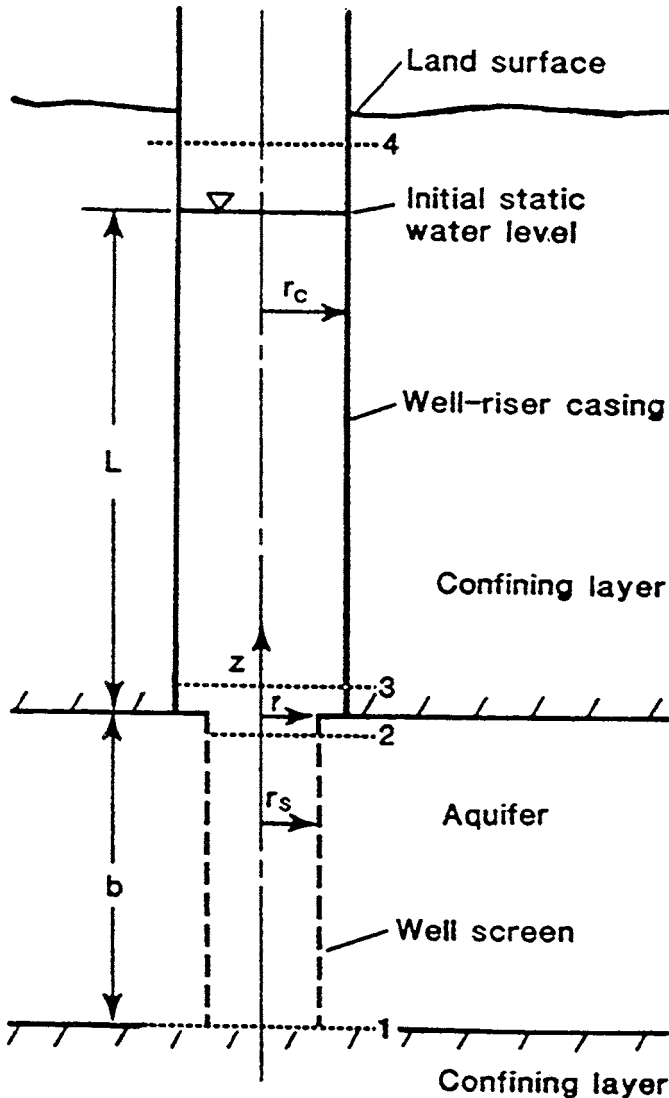


FIG. 1 Well and Aquifer Geometry from Kipp (1)

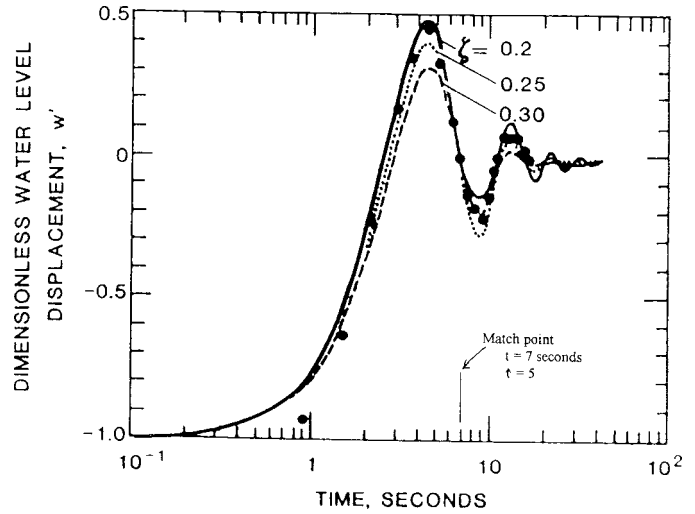


FIG. 2 Slug-Test Data Overlaid on Type Curves for Three Different Damping Factors, Modified from Kipp (1)

5.2 Kipp (1) introduces dimensionless variables and parameters in converting these equations to dimensionless form, solves the equations by Laplace transforms, and inverts the solution by a Laplace-transform-inversion algorithm.

5.2.1 The following dimensionless parameters are among those given by Kipp (1):

dimensionless water-level displacement:

$$w' = -w/w_o \quad (9)$$

dimensionless time:

$$t' = (tT)/(r_s^2 S) \quad (10)$$

and:

$$\hat{t} = t'/\beta^{1/2} \quad (11)$$

dimensionless storage:

$$\alpha = (r_c^2)/(2r_s^2 S) \quad (12)$$

dimensionless inertial parameter:

$$\beta = (Le/g)(T/(r_s^2 S))^2 \quad (13)$$

dimensionless skin factor:

$$\sigma = f/r_s \quad (14)$$

dimensionless frequency parameter:

$$\omega = \frac{[-d^2(\sigma + 1/4 \ln \beta) + 4\beta]^{1/2}}{2\beta} \quad (15)$$

dimensionless decay parameter:

$$\gamma = \frac{\alpha(\sigma + 1/4 \ln \beta)}{2\beta} \quad (16)$$

and dimensionless damping factor:

$$\zeta = \frac{\alpha(\sigma + 1/4 \ln \beta)}{2\beta^{1/2}} \quad (17)$$

5.3 For ζ less than one, the system is underdamped; for ζ greater than one, the system is overdamped. For ζ equal to one, the system is critically damped, yet the inertial effects are quite important (1). For ζ greater than about one, the system