



Designation: D6029/D6029M – 20

Standard Practice for (Analytical Procedures) Determining Hydraulic Properties of a Confined Aquifer and a Leaky Confining Bed with Negligible Storage by the Hantush-Jacob Method¹

This standard is issued under the fixed designation D6029/D6029M; the number immediately following the designation indicates the year of original adoption or, in the case of revision, the year of last revision. A number in parentheses indicates the year of last reapproval. A superscript epsilon (ϵ) indicates an editorial change since the last revision or reapproval.

1. Scope*

1.1 This practice covers an analytical procedure for determining the transmissivity and storage coefficient of a confined aquifer and the leakance value of an overlying or underlying confining bed for the case where there is negligible change of water in storage in a confining bed. This practice is used to analyze water-level or head data collected from one or more observation wells or piezometers during the pumping of water from a control well at a constant rate. With appropriate changes in sign, this practice also can be used to analyze the effects of injecting water into a control well at a constant rate.

1.2 This analytical procedure is used in conjunction with Test Method **D4050**.

1.3 *Limitations*—The valid use of the Hantush-Jacob method is limited to the determination of hydraulic properties for aquifers in hydrogeologic settings with reasonable correspondence to the assumptions of the Theis nonequilibrium method (Practice **D4106**) with the exception that in this case the aquifer is overlain, or underlain, everywhere by a confining bed having a uniform hydraulic conductivity and thickness, and in which the gain or loss of water in storage is assumed to be negligible, and that bed, in turn, is bounded on the distal side by a zone in which the head remains constant. The hydraulic conductivity of the other bed confining the aquifer is so small that it is assumed to be impermeable (see **Fig. 1**).

1.4 *Units*—The values stated in either SI units or inch-pound units are to be regarded separately as standard. The values stated in each system may not be exact equivalents; therefore, each system shall be used independently of the other. Combining values from the two systems may result in noncon-

formance with the standard. Reporting of results in units other than SI shall not be regarded as nonconformance with this standard.

1.5 All observed and calculated values shall conform to the guidelines for significant digits and round established in Practice **D6026**, unless superseded by this standard.

1.5.1 The procedures used to specify how data are collected/recorded or calculated, in this standard are regarded as the industry standard. In addition, they are representative of the significant digits that generally should be retained. The procedures used do not consider material variation, purpose for obtaining the data, special purpose studies, or any considerations for the user's objectives; and it is common practice to increase or reduce significant digits of reported data to be commensurate with these considerations. It is beyond the scope of this standard to consider significant digits used in analysis method for engineering design.

1.6 This practice offers a set of instructions for performing one or more specific operations. This document cannot replace education or experience and should be used in conjunction with professional judgment. Not all aspects of the practice may be applicable in all circumstances. This ASTM standard is not intended to represent or replace the standard of care by which the adequacy of a given professional service must be judged, nor should this document be applied without the consideration of a project's many unique aspects. The word "Standard" in the title of this document means only that the document has been approved through the ASTM consensus process.

1.7 *This standard does not purport to address all of the safety concerns, if any, associated with its use. It is the responsibility of the user of this standard to establish appropriate safety, health, and environmental practices and determine the applicability of regulatory limitations prior to use.*

1.8 *This international standard was developed in accordance with internationally recognized principles on standardization established in the Decision on Principles for the*

¹ This practice is under the jurisdiction of ASTM Committee **D18** on Soil and Rock and is the direct responsibility of Subcommittee **D18.21** on Groundwater and Vadose Zone Investigations.

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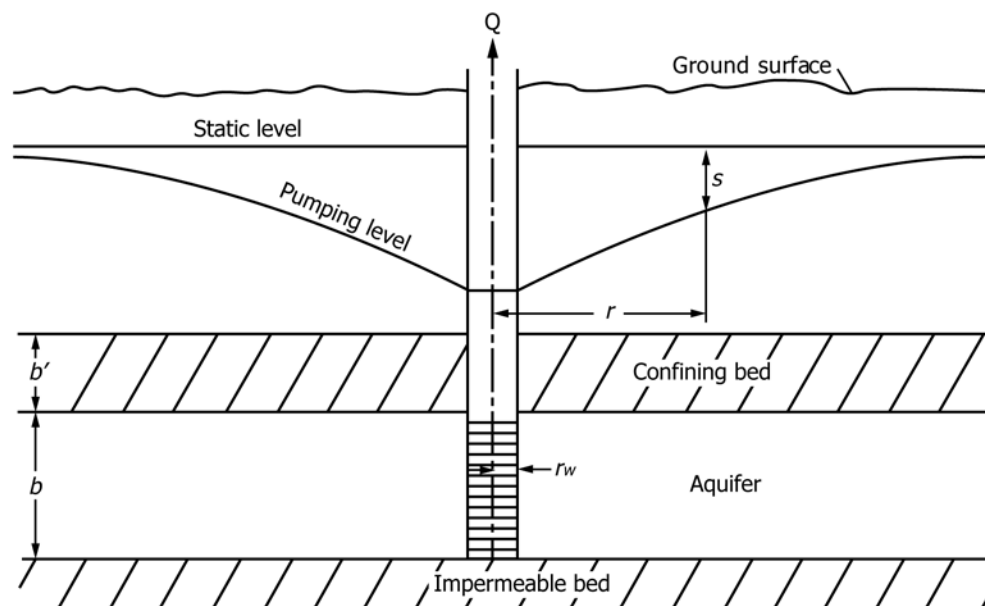


FIG. 1 Cross Section Through a Discharging Well in a Leaky Aquifer (from Reed (1)³). The Confining and Impermeable Bed Locations Can Be Interchanged

Development of International Standards, Guides and Recommendations issued by the World Trade Organization Technical Barriers to Trade (TBT) Committee.

2. Referenced Documents

2.1 ASTM Standards:²

- D653 Terminology Relating to Soil, Rock, and Contained Fluids
- D3740 Practice for Minimum Requirements for Agencies Engaged in Testing and/or Inspection of Soil and Rock as Used in Engineering Design and Construction
- D4050 Test Method for (Field Procedure) for Withdrawal and Injection Well Testing for Determining Hydraulic Properties of Aquifer Systems
- D4106 Practice for (Analytical Procedure) for Determining Transmissivity and Storage Coefficient of Nonleaky Confined Aquifers by the Theis Nonequilibrium Method
- D6026 Practice for Using Significant Digits in Geotechnical Data
- D6028/D6028M Practice for (Analytical Procedure) Determining Hydraulic Properties of a Confined Aquifer Taking into Consideration Storage of Water in Leaky Confining Beds by Modified Hantush Method

3. Terminology

3.1 Definitions:

3.1.1 For definitions of common technical terms used in this standard, refer to Terminology D653.

3.2 Symbols and Dimensions:

3.2.1 K —hydraulic conductivity of the aquifer [LT^{-1}].

3.2.1.1 Discussion—The use of the symbol K for the term hydraulic conductivity is the predominant usage in groundwater literature by hydrogeologists, whereas the symbol k is commonly used for this term in soil and rock mechanics and soil science.

3.2.2 K' —vertical hydraulic conductivity of the confining bed through which leakage can occur [LT^{-1}].

3.2.3 $L(u, v)$ —leakage function of u, v [nd]; equal to $W(u, r/B)$.

3.2.4 Q —discharge [L^3T^{-1}].

3.2.5 $S = bS_s$ —storage coefficient [nd].

3.2.6 S_s —specific storage of the aquifer [L^{-1}].

3.2.7 S'_s —specific storage of the confining bed [L^{-1}].

3.2.8 T —transmissivity [L^2T^{-1}].

3.2.9 $u = \frac{r^2 S}{4Tt}$ [nd].

3.2.10 $W(u, r/B)$ —well function for leaky aquifer systems with negligible storage changes in confining beds [nd].

3.2.11 b —thickness of aquifer [L]. b' —thickness of the confining bed through which leakage can occur [L].

3.2.12 r —radial distance from control well [L].

3.2.13 r_c —radius of the control well casing, or hole if uncased [L].

3.2.14 s —drawdown [L].

3.2.15 $v = \frac{r}{2B} = \frac{r}{2} \sqrt{\frac{K'}{Tb}}$, v —defined by Eq 7 [nd].

3.2.16 $\sqrt{\frac{Tb'}{K'}}$ [L].

3.2.17 t —time since pumping or injection began [T].

3.2.18 $K_0(x)$ —zero-order modified Bessel function of the second kind [nd].

² For referenced ASTM standards, visit the ASTM website, www.astm.org, or contact ASTM Customer Service at service@astm.org. For Annual Book of ASTM Standards volume information, refer to the standard's Document Summary page on the ASTM website.



$$3.2.19 \beta = \frac{r}{4b} \sqrt{\frac{K'S'_s}{KS_s}}$$

4. Summary of Practice

4.1 This practice involves pumping a control well that is fully screened through the confined aquifer and measuring the water-level response in one or more observation wells or piezometers. The well is pumped at a constant rate. The water-level response in the aquifer is a function of the transmissivity and storage coefficient of the aquifer and the leakance coefficient of a confining bed. The other confining bed is assumed to be impermeable. Alternatively, the practice can be performed by injecting water at a constant rate into the control well. Analysis of buildup of water level in response to injection is similar to analysis of drawdown of the water level in response to withdrawal in a confined aquifer. The water-level response data may be analyzed in two ways. The time variation of the water-level response in any one well can be analyzed using one set of type curves, or the water-level responses measured at the same time but in observation wells at different distances from the control well can be analyzed using another set of type curves.

NOTE 1—The quality of the result produced by this standard is dependent on the competence of the personnel performing it, and the suitability of the equipment and facilities used. Agencies that meet the criteria of Practice D3740 are generally considered capable of competent and objective testing/sampling/inspection/etc. Users of this standard are cautioned that compliance with Practice D3740 does not in itself assure reliable results. Reliable results depend on many factors; Practice D3740 provides a means of evaluating some of those factors.

4.2 *Solution*—Hantush and Jacob (2)³ give two mathematically equivalent expressions for the solution which can be written as follows:

$$s = \frac{Q}{4\pi T} \int_u^\infty \frac{1}{z} \exp\left(-z - \frac{r^2}{4B^2z}\right) dz \quad (1)$$

where z is the variable of integration and

$$s = \frac{Q}{4\pi T} \left[2K_0\left(\frac{r}{B}\right) - \int_{\frac{r^2}{4B^2u}}^\infty \frac{1}{z} \exp\left(-z - \frac{r^2}{4B^2z}\right) dz \right] \quad (2)$$

where:

$$u = \frac{r^2 S}{4Tt} \quad (3)$$

$$B^2 = \frac{Tb'}{K'} \quad (4)$$

4.2.1 Because a closed-form expression of the integrals that appear in Eq 1 or Eq 2 are not known, Hantush and Jacob developed equivalent expressions that involve infinite series that can be numerically evaluated. The infinite series for Eq 1 converges more rapidly for early times and the infinite series for Eq 2 converges more rapidly for late times.

4.2.2 Hantush (3) expressed Eq 1 and Eq 2 as follows:

$$s = \frac{Q}{4\pi T} W\left(u, \frac{r}{B}\right) \quad (5)$$

³ The boldface numbers in parentheses refer to a list of references at the end of this practice.

where $W\left(u, \frac{r}{B}\right)$ was called the well function for leaky systems. Hantush tabulated values of this function for a practical range of the parameters u and $\frac{r}{B}$.

4.2.3 Cooper (4) opted to express the Hantush-Jacob solution in the following form:

$$s = \frac{Q}{4\pi T} L(u, v) \quad (6)$$

where Cooper's v = Hantush's $\frac{r}{2B}$

or

$$v = \frac{r}{2B} = \frac{r}{2\sqrt{\frac{Tb'}{K'}}} \quad (7)$$

4.2.4 Cooper prepared two families of type curves. One set of Cooper's curves allow the head changes as a function of time at a fixed distance to be analyzed for the aquifer parameters, and the other set of curves allow the head changes at different distances at some fixed time to be analyzed.

5. Significance and Use

5.1 Assumptions:

5.1.1 The control well discharges at a constant rate, Q .

5.1.2 The control well is of infinitesimal diameter and fully penetrates the aquifer.

5.1.3 The aquifer is homogeneous, isotropic, and areally extensive.

NOTE 2—Slug and pumping tests implicitly assume a porous medium. Fractured rock and carbonate settings may not provide meaningful data and information.

5.1.4 The aquifer remains saturated (that is, water level does not decline below the top of the aquifer).

5.1.5 The aquifer is overlain, or underlain, everywhere by a confining bed having a uniform hydraulic conductivity and thickness. It is assumed that there is no change of water storage in this confining bed and that the hydraulic gradient across this bed changes instantaneously with a change in head in the aquifer. This confining bed is bounded on the distal side by a uniform head source where the head does not change with time.

5.1.6 The other confining bed is impermeable.

5.1.7 Leakage into the aquifer is vertical and proportional to the drawdown, and flow in the aquifer is strictly horizontal.

5.1.8 Flow in the aquifer is two-dimensional and radial in the horizontal plane.

5.2 The geometry of the well and aquifer system is shown in Fig. 1.

5.3 Implications of Assumptions:

5.3.1 Paragraph 5.1.1 indicates that the discharge from the control well is at a constant rate. Section 8.1 of Test Method D4050 discusses the variation from a strictly constant rate that is acceptable. A continuous trend in the change of the discharge rate could result in misinterpretation of the water-level change data unless taken into consideration.

5.3.2 The leaky confining bed problem considered by the Hantush-Jacob solution requires that the control well has an infinitesimal diameter and has no storage. Abdul Khader and Ramadurgaiah (5) developed graphs of a solution for the drawdowns in a large-diameter control well discharging at a constant rate from an aquifer confined by a leaky confining bed. Fig. 2 (Fig. 3 of Abdul Khader and Ramadurgaiah (5)) gives a graph showing variation of dimensionless drawdown with dimensionless time in the control well assuming the aquifer storage coefficient, $S = 10^{-3}$, and the leakage parameter, $\frac{r_w}{B}$. Note that at early dimensionless times the curve for a large-diameter well in a non-leaky aquifer (BCE) and in a leaky aquifer (BCD) are coincident. At later dimensionless times, the curve for a large diameter well in a leaky aquifer coalesces with the curve for an infinitesimal diameter well (ACD) in a leaky aquifer. They coalesce about one logarithmic cycle of dimensionless time before the drawdown becomes sensibly constant. For a value of r_w/B smaller than 10^{-3} , the constant drawdown (D) would occur at a greater value of dimensionless drawdown and there would be a longer period during which well-bore storage effects are negligible (the period where ACD and BCD are coincident) before a steady drawdown is reached.

For values of $\frac{r_w}{B}$ greater than 10^{-3} , the constant drawdown (D) would occur at a smaller value of drawdown and there would be a shorter period of dimensionless time during which well-storage effects are negligible (the period where ACD and BCD are coincident) before a steady drawdown is reached.

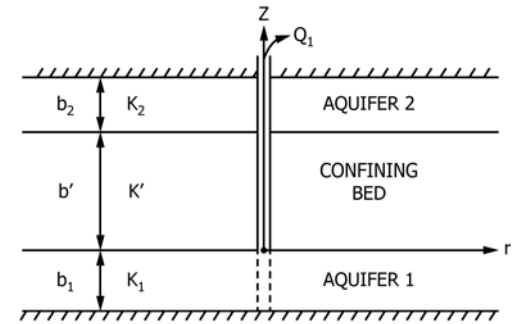


FIG. 3 Schematic Diagram of Two-Aquifer System

Abdul Khader and Ramadurgaiah (5) present graphs of dimensionless time versus dimensionless drawdown in a discharging control well for values of $S = 10^{-1}, 10^{-2}, 10^{-3}, 10^{-4}$, and 10^{-5} and $r_w/B = 10^{-2}, 10^{-3}, 10^{-4}, 10^{-5}, 10^{-6}$, and 0. These graphs can be used in an analysis prior to the aquifer test making use of estimates of the hydraulic properties to estimate the time period during which well-bore storage effects in the control well probably will mask other effects and the drawdowns would not fit the Hantush-Jacob solution.

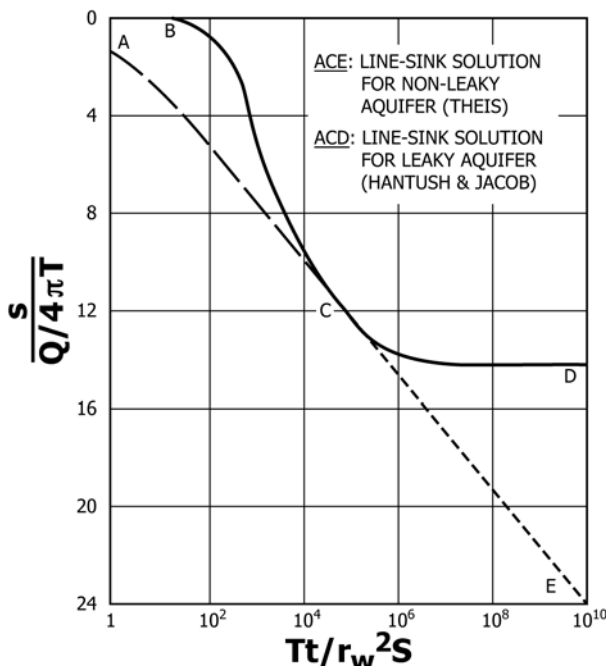
5.3.2.1 The time needed for the effects of control-well bore storage to diminish enough that drawdowns in observation wells should fit the Hantush-Jacob solution is less clear. But the time adopted for when drawdowns in the discharging control well are no longer dominated by well-bore storage affects probably should be the minimum estimate of the time to adopt for observation well data.

5.3.3 The assumption that the aquifer is bounded, above or below, by a leaky layer on one side and a nonleaky layer on the other side is not likely to be entirely satisfied in the field. Neuman and Witherspoon (6, p. 1285) have pointed out that because the Hantush-Jacob formulation uses water-level change data only from the aquifer being pumped (or recharged) it can not be used to distinguish whether the leaking beds are above or below (or from both sides) of the aquifer. Hantush (7) presents a refinement that allows the parameters determined by the aquifer field test analysis to be interpreted as composite parameters that reflect the combined effects of overlying and underlying confined beds. Neuman and Witherspoon (6) describe a method to estimate the hydraulic properties of a confining layer by using the head changes in that layer.

5.3.4 The Hantush-Jacob theoretical development requires that the leakage into the aquifer is proportional to the drawdown, and that the drawdown does not vary in the vertical in the aquifer. These requirements are sometimes described by stating that the flow in the confining beds is essentially vertical and in the aquifer is essentially horizontal. Hantush's (8) analysis of an aquifer bounded only by one leaky confining bed suggested that this approximation is acceptably accurate wherever

$$\frac{K}{K'} > 100 \frac{b}{b'} \quad (8)$$

5.3.5 The Hantush-Jacob method requires that there is no change in water storage in the leaky confining bed. Weeks (9) states that if the "leaky" confining bed is thin and highly



Curve BCE: Solution for well of large diameter in a non-leaky aquifer (Papadopoulos and Cooper (6)).

Curve BCD: Solution for well of large diameter in a leaky aquifer (Abdul Khader and Ramadurgaiah).

FIG. 2 Time—Drawdown Variation in the Control Well for $S = 10^{-3}$ (from Abdul Khader and Ramadurgaiah (5))