This international standard was developed in accordance with internationally recognized principles on standardization established in the Decision on Principles for the Development of International Standards, Guides and Recommendations issued by the World Trade Organization Technical Barriers to Trade (TBT) Committee.

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Standard Practice for Applying Analytical Hierarchy Process (AHP) to Multiattribute Decision Analysis of Investments Related to Projects, Products, and Processes[1]

This standard is issued under the fixed designation E1765; the number immediately following the designation indicates the year of original adoption or, in the case of revision, the year of last revision. A number in parentheses indicates the year of last reapproval. A superscript epsilon (ε) indicates an editorial change since the last revision or reapproval.

INTRODUCTION

The analytical hierarchy process (AHP) is one of a set of multi-attribute decision analysis (MADA) methods that considers nonmonetary attributes (qualitative and quantitative) in addition to common economic evaluation measures (such as life-cycle costing or net benefits) when evaluating project, product, and process alternatives. Investment decisions depend in part on how competing options perform with respect to nonmonetary attributes. This practice complements existing ASTM standards on building economics by incorporating the existing economic/monetary measures of worth described in those standards into a more comprehensive standard method of evaluation that includes nonmonetary (quantitative and nonquantitative) benefits and costs. The AHP is the MADA method described in this practice.[2] It has three significant strengths: an efficient attribute weighting process of pairwise comparisons; hierarchical descriptions of attributes, which keep the number of pairwise comparisons manageable; and available software to facilitate its use.[3]

1. Scope

1.1 This practice presents a procedure for calculating and interpreting AHP scores of a project's/product's/process' total overall desirability when making capital investment decisions.[3] Projects include design, construction, operation, and disposal of commercial and residential buildings and other engineered structures.[4] Products include materials, components, systems, and equipment.[5] Processes include procurement, materials management, work flow, fabrication and assembly, quality control, and services.

1.2 In addition to monetary benefits and costs, the procedure allows for the consideration of characteristics or attributes which decision makers regard as important, but which are not readily expressed in monetary terms. Examples of such attributes that pertain to the selection among project/product/process alternatives are: a construction projects's building alternatives whose nonmonetary attributes are location/accessibility, site security, maintainability, quality of the sound and visual environment, and image to the public and occupants; building products based on their economic and environmental performance; and sustainability-related issues for key construction processes that address environmental needs, while considering project safety, cost, and schedule.

1.3 *This standard does not purport to address all of the safety concerns, if any, associated with its use. It is the responsibility of the user of this standard to establish appropriate safety, health, and environmental practices and determine the applicability of regulatory limitations prior to use.*

[1] This practice is under the jurisdiction of ASTM Committee E06 on Performance of Buildings and is the direct responsibility of Subcommittee E06.81 on Building Economics.

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[2] For an extensive overview of MADA methods and a detailed treatment of how to apply two MADA methods (one of which is AHP) to building-related decisions, see Norris, G A., and Marshall, H.E., *Multiattribute Decision Analysis: Recommended Method for Evaluating Buildings and Building Systems*, National Institute of Standards and Technology, 1995.

[3] This practice presents a stand-alone procedure for performing an AHP analysis. In addition, an ASTM software product for performing AHP analyses has been developed to support and facilitate use of this practice. *Software to Support ASTM E1765: Standard Practice for Applying Analytical Hierarchy Process (AHP) to Multiattribute Decision Analysis of Investments Related to Buildings and Building Systems*, MNL 29, ASTM, 1998.

[4] Projects also include analytical studies that identify alternative means for achieving organizational objectives as well as research and development activities that support the deployment of new products and processes.

[5] Typical construction-related products for each product type are: (*1*) materials—concrete; (*2*) components—structural steel members; (*3*) systems—heating, ventilating, and air-conditioning system; and (*4*) equipment—heat pump.

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1.4 *This international standard was developed in accordance with internationally recognized principles on standardization established in the Decision on Principles for the Development of International Standards, Guides and Recommendations issued by the World Trade Organization Technical Barriers to Trade (TBT) Committee.*

2. Referenced Documents

2.1 *ASTM Standards:*[6]

E631 Terminology of Building Constructions
E833 Terminology of Building Economics
E917 Practice for Measuring Life-Cycle Costs of Buildings and Building Systems
E964 Practice for Measuring Benefit-to-Cost and Savings-to-Investment Ratios for Buildings and Building Systems
E1057 Practice for Measuring Internal Rate of Return and Adjusted Internal Rate of Return for Investments in Buildings and Building Systems
E1074 Practice for Measuring Net Benefits and Net Savings for Investments in Buildings and Building Systems
E1121 Practice for Measuring Payback for Investments in Buildings and Building Systems
E1480 Terminology of Facility Management (Building-Related)
E1557 Classification for Building Elements and Related Sitework—UNIFORMAT II
E1660 Classification for Serviceability of an Office Facility for Support for Office Work
E1661 Classification for Serviceability of an Office Facility for Meetings and Group Effectiveness
E1662 Classification for Serviceability of an Office Facility for Sound and Visual Environment
E1663 Classification for Serviceability of an Office Facility for Typical Office Information Technology
E1664 Classification for Serviceability of an Office Facility for Layout and Building Factors
E1665 Classification for Serviceability of an Office Facility for Facility Protection
E1666 Classification for Serviceability of an Office Facility for Work Outside Normal Hours or Conditions
E1667 Classification for Serviceability of an Office Facility for Image to the Public and Occupants
E1668 Classification for Serviceability of an Office Facility for Amenities to Attract and Retain Staff
E1669 Classification for Serviceability of an Office Facility for Location, Access and Wayfinding
E1670 Classification for Serviceability of an Office Facility for Management of Operations and Maintenance
E1671 Classification for Serviceability of an Office Facility for Cleanliness
E1679 Practice for Setting the Requirements for the Serviceability of a Building or Building-Related Facility, and for Determining What Serviceability is Provided or Proposed
E1692 Classification for Serviceability of an Office Facility for Change and Churn by Occupants
E1693 Classification for Serviceability of an Office Facility for Protection of Occupant Assets
E1694 Classification for Serviceability of an Office Facility for Special Facilities and Technologies
E1700 Classification for Serviceability of an Office Facility for Structure and Building Envelope
E1701 Classification for Serviceability of an Office Facility for Manageability
E2114 Terminology for Sustainability
E2320 Classification for Serviceability of an Office Facility for Thermal Environment and Indoor Air Conditions
E2432 Guide for General Principles of Sustainability Relative to the Built Environment

2.2 *ASTM Adjunct:*[7]

Discount Factor Tables - Adjunct to E917 Practice for Measuring Life-Cycle Costs of Buildings and Building Systems - Includes Excel and PDF Files

2.3 *ASTM Software Product:*

MNL 29 Software to Support ASTM E1765: Standard Practice for Applying Analytical Hierarchy Process (AHP) to Multiattribute Decision Analysis of Investments Related to Buildings and Building Systems

3. Terminology

3.1 *Definitions*—For definitions of general terms related to building construction used in this practice, refer to Terminology E631; for general terms related to building economics, refer to Terminology E833; and for general terms related to whole buildings and facilities, refer to Terminology E1480. For definitions of general terms related to sustainability relative to the performance of buildings, refer to Terminology E2114.

4. Summary of Practice

4.1 This practice helps you identify a MADA application, describe the elements that make up a MADA problem, and recognize the three types of problems that MADA can address: screening alternatives, ranking alternatives, and choosing a final "best" alternative.

4.2 A comprehensive list of selected attributes (monetary and nonmonetary) for evaluating building decisions provides a pick list for customizing an AHP model that best fits your building-related decision. Three types of building decisions to which the list applies are choosing among buildings, choosing among building components, and choosing among building materials. Examples of these typical building-related decisions are provided.

4.3 A case illustration of a building choice decision shows how to structure a problem in a hierarchical fashion, describe the attributes of each alternative in a decision matrix, compute attribute weights, check for consistency in pairwise comparisons, and develop the final desirability scores of each alternative.

[6] For referenced ASTM standards, visit the ASTM website, www.astm.org, or contact ASTM Customer Service at service@astm.org. For *Annual Book of ASTM Standards* volume information, refer to the standard's Document Summary page on the ASTM website.

[7] Available from ASTM International Headquarters. Order Adjunct No. ADJE091717-EA. Original adjunct produced in 1984 [illegible] 2017.

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4.4 A description of the applications and limitations of the AHP method concludes this practice.

5. Significance and Use

5.1 The AHP method allows you to generate a single measure of desirability for project/product/process alternatives with respect to multiple attributes (qualitative and quantitative). By contrast, life-cycle cost (Practice E917), net savings (Practice E1074), savings-to-investment ratio (Practice E964), internal rate-of-return (Practice E1057), and payback (Practice E1121) methods all require you to put a monetary value on benefits and costs in order to include them in a measure of project/product/process worth.

5.2 Use AHP to evaluate a finite and generally small set of discrete and predetermined options or alternatives. Specific AHP applications are ranking and choosing among alternatives. For example, rank alternative building locations with AHP to see how they measure up to one another, or use AHP to choose among building materials to see which is best for your application.

5.3 Use AHP if no single alternative exhibits the most preferred available value or performance for all attributes. This is often the result of an underlying trade-off relationship among attributes. An example is the trade-off between low desired energy costs and large glass window areas (which may raise heating and cooling costs while lowering lighting costs).

5.4 Use AHP to evaluate alternatives whose attributes are not all measurable in the same units. Also use AHP when performance relative to some or all of the attributes is impractical, impossible, or too costly to measure. For example, while life-cycle costs are directly measured in monetary units, the number and size of offices are measured in other units, and the public image of a building may not be practically measurable in any unit. To help you choose among candidate buildings with these diverse attributes, use AHP to evaluate your alternatives.

5.5 The AHP method is well-suited for application to a variety of sustainability-related topics. Guide E2432 states when applying the concept of sustainability, it is necessary to assess and balance three dissimilar yet interrelated general principles—environment, economic, and social—based on the best information available at the time the decision is made. Use AHP for pairwise comparisons among environmental attributes, among economic attributes, and among social attributes, and for establishing relative importance weights for each attribute and for each of the three general principles to which the attributes are attached. Use the AHP-established relative importance weights to select the preferred project/product/process from among the competing alternatives.

5.6 Potential users of AHP include architects, developers, owners, or lessors of buildings, real estate professionals (commercial and residential), facility managers, building material manufacturers, equipment manufacturers, product and process engineers, life cycle assessment experts, and agencies managing building portfolios.

6. Procedure

6.1 To carry out a MADA analysis using AHP, follow this procedure:[8]

6.1.1 Identify the elements of your problem to confirm that a MADA analysis is appropriate (see 6.2);

6.1.2 Determine the goal or objective of the analysis, select the attributes on the basis of which you plan to choose an alternative, arrange the attributes in a hierarchy, identify the attribute sets in the hierarchy, identify the leaf attributes in the hierarchy, and identify alternatives to consider (see 6.3);

6.1.3 Construct a decision matrix summarizing available data on the performance of each alternative with respect to each leaf attribute (see 6.4);

6.1.4 Compare in pairwise fashion each alternative against every other alternative as to how much better one is than the other with respect to each leaf attribute (see 6.5);

6.1.5 Make pairwise comparisons, starting from the bottom of the hierarchy, of the relative importance of each attribute in a given set with respect to the attribute or goal immediately above that set in the hierarchy (see 6.6); and

6.1.6 Compute the final overall desirability score for each alternative (see 6.7).

6.2 Confirm that a MADA analysis is appropriate. Three elements are typically common to MADA problems.

6.2.1 MADA problems involve analysis of a finite and generally small set of discrete and predetermined options or alternatives. They do *not* involve the design of a "best" alternative from among a theoretically infinite set of possible designs where the decision maker considers trade-offs among interacting continuous decision variables. Selecting a replacement HVAC system for an existing building is a MADA problem. In contrast, the integrated design and sizing of a future building and its HVAC system is not a MADA problem.

6.2.2 In MADA problems, no single alternative is dominant, that is, no alternative exhibits the most preferred value or performance for all attributes. If one alternative is dominant, a MADA analysis is not needed. You simply choose that alternative. The lack of a dominant alternative is often the result of an underlying trade-off relationship among attributes. An example is the trade-off between proximity to the central business district for convenient meetings with business clients and the desire for a suburban location that is convenient for commuting to residential neighborhoods and relatively free of street crime.

6.2.3 The attributes in a MADA problem are not all measurable in the same units. Some attributes may be either impractical, impossible, or too costly to measure at all. For example, in an office building, energy costs are measurable in life-cycle cost terms. But the architectural statement of the building may not be practically measurable in any unit. If all relevant attributes characterizing alternative buildings can be expressed in terms of monetary costs or benefits scheduled to occur at specifiable times, then the ranking and selection of a building does not require the application of MADA.

[8] Paragraphs 6.1 – 6.4 are common to many MADA methods. Paragraphs 6.5 – 6.7 pertain specifically to the AHP method.

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6.3 Identify the goal of the analysis, the attributes to be considered, and the alternatives to evaluate. Display the goal and attributes in a hierarchy.

6.3.1 The following case example of a search for public office space illustrates how to organize and display the constituents of a hierarchy.

6.3.1.1 A state agency needs, within the next 18 months, office space for 300 workers. It seeks a location convenient to the state capitol building by shuttle. The agency seeks to minimize the travel time and will not accept travel times greater than 10 min. It also has telecommunications and computer infrastructure requirements that will exclude many buildings. The goal of the analysis is to find the best building for the agency.

6.3.1.2 The specification of a 10 min maximum travel time from the site to the capitol eliminates all buildings outside a certain radius. Having up to 18 months to occupy allows either the construction of a new building or the retrofitting of an existing building, either of which could be rented or leased. Telecommunications and computer infrastructure requirements will limit the search even more. These specifications help the analyst define the "attributes" and building "alternatives" for the MADA analysis.

6.3.1.3 Attributes selected for the hierarchy, displayed in Fig. 1, are occupancy availability (within 18 months); information technology (available telecommunications and computer support infrastructure); economics (life-cycle costs of alternative buildings, owned or leased); and location (how convenient to capitol building). The analyst works with the decision maker to make sure that all significant needs of the decision maker are covered by the hierarchy of attributes.

6.3.2 Fig. 2 covers attribute sets and leaf attributes.

6.3.2.1 A set of attributes refers to a complete group of attributes in the hierarchy which is located under another attribute or under the problem goal. There are four separate sets of attributes in the hierarchy displayed in Fig. 2. Each set is enclosed by dashed lines.

6.3.2.2 A leaf attribute is an attribute which has no attributes below it in the hierarchy. The eleven leaf attributes present in the hierarchy in Fig. 2 are shaded.

6.4 Construct a decision matrix with data on the performance of each alternative with respect to each leaf attribute.

6.4.1 Characterize your MADA problem with a decision matrix similar to Table 1. The decision matrix indicates both the set of alternatives and the set of leaf attributes being considered in a given problem, and it summarizes the "raw" data available to the decision maker at the start of the analysis. A decision matrix has a row corresponding to each alternative being considered and a column corresponding to each leaf attribute being considered. Each element of the matrix contains the available information about that row's alternative with respect to that column's attribute. Put quantitative data in the decision matrix if available; use nonquantitative data otherwise.

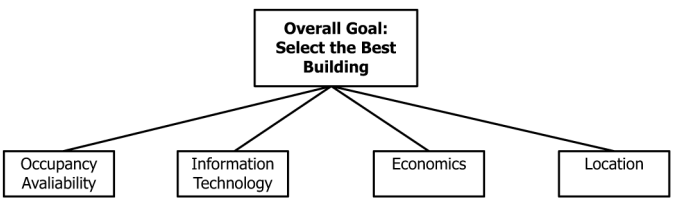

FIG. 1 An Example Hierarchy for the Problem of Selecting a Building

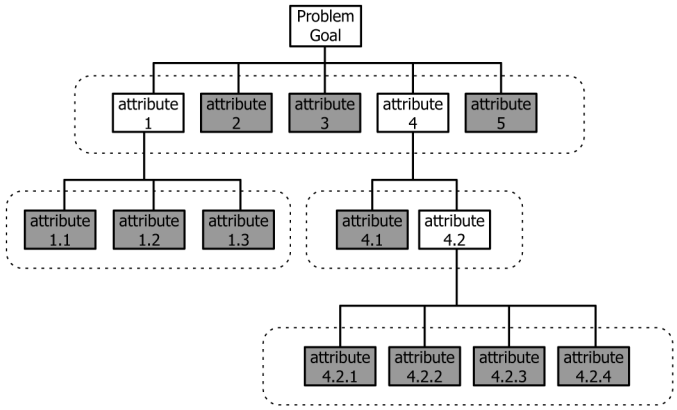

FIG. 2 A Hierarchy Illustrating Attribute Sets and Leaf Attributes

TABLE 1 Heating System Decision Matrix

	Leaf Attributes		
	Life-Cycle Cost, K$	Duration of Warranty, years	Familiarity with the Technology
Alternative 1	10	3	high
Alternative 2	15	1	medium
Alternative 3	20	10	low

6.4.2 Table 1 is a hypothetical and simplified decision matrix for the problem of selecting the "best" heating system for a building. Note that the first column pertains to a monetary attribute: life-cycle costs. The next attribute, warranty period, is measured quantitatively, but not in monetary terms. The last attribute, familiarity with the technology, is characterized only qualitatively.

6.4.3 Include in the decision matrix and analysis only those attributes which the decision maker considers important and which vary significantly among one or more alternatives. For example, heating capacity is clearly an important attribute of any heating system, but if the alternatives in Table 1 include only systems which match the capacity requirements of the building in question, then capacity is not a distinguishing attribute and is not to be included in the decision matrix or in the MADA analysis.

6.4.4 The MADA methods allow one to use the information in a problem's decision matrix together with additional information from the decision maker in determining a final ranking or selection from among the alternatives. For example, the decision matrix alone provides neither information about the relative importance of the different attributes to the decision maker, nor about any minimum acceptable, maximum acceptable, or target values for particular attributes.

6.4.5 For analytical and procedural simplicity, it is common practice when employing MADA to neglect both uncertainties and imprecision inherent in the decision matrix data as well as in the additional information about attributes and alternatives elicited from the decision maker. While there are ways to

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incorporate uncertainty and imprecision in MADA analyses, they are not addressed here.

6.5 Compare in pairwise fashion each alternative against every other alternative as to how much better one is than the other with respect to each leaf attribute. Repeat this process for each leaf attribute in the hierarchy. This and subsequent steps in the procedure describe the AHP method of performing MADA analysis.

6.5.1 The AHP summarizes the results of pairwise judgments in a matrix of pairwise comparisons (MPC), as shown in Fig. 3. For each pair of alternatives, the decision maker specifies a judgment about how much more desirable or how much better in terms of strength of preference one alternative is than the other with respect to the attribute in question. Each pairwise comparison requires the decision maker to provide an answer to the question, "Alternative 1 is how much more desirable than Alternative 2, relative to the attribute of interest?" This procedure is repeated for each leaf attribute in the hierarchy.

6.5.2 Note that the decision maker responds to questions about how much more desirable one alternative is than another. It helps responders if the question is framed this way, since all answers will result in a number greater than or equal to one. As shown in Fig. 3, however, the entries in the MPC always characterize the desirability of the row alternative versus the column alternative. Therefore, in cases where the column alternative is more desirable than the row alternative, the decision maker must answer the question, "How much more desirable is the column alternative than the row alternative?" In such cases, enter the reciprocal of the resulting number into the MPC.

6.5.3 There are three types of approaches for specifying pairwise comparison judgments in AHP: numerical, graphically mediated, and verbally mediated. Each method requires the decision maker to answer a series of questions of the form, "How much more desirable is Alternative 1 than Alternative 2 with respect to the attribute of interest?"

	Alternative 1	Alternative 2	. . .	Alternative *j*	Alternative *k*	. . .	Alternative *n*
Alternative 1	1	Desirability of Alt. 1 versus Alt. 2	. . .	Desirability of Alt. 1 versus Alt. *j*	Desirability of Alt. 1 versus Alt. *k*	. . .	Desirability of Alt. 1 versus Alt. *n*
Alternative 2	Desirability of Alt. 2 versus Alt. 1	1	. . .	Desirability of Alt. 2 versus Alt. *j*	Desirability of Alt. 2 versus Alt. *k*	. . .	Desirability of Alt. 2 versus Alt. *n*
. . .	. . .	. . .	1	. . .	. . .	. . .	. . .
Alternative *j*	Desirability of Alt. *j* versus Alt. 1	Desirability of Alt. *j* versus Alt. 2	. . .	1	Desirability of Alt. *j* versus Alt. *k*	. . .	Desirability of Alt. *j* versus Alt. *n*
Alternative *k*	Desirability of Alt. *k* versus Alt. 1	Desirability of Alt. *k* versus Alt. 2	. . .	Desirability of Alt. *k* versus Alt. *j*	1	. . .	Desirability of Alt. *k* versus Alt. *n*
. . .	. . .	. . .	. . .	. . .	. . .	1	. . .
Alternative *n*	Desirability of Alt. *n* versus Alt. 1	Desirability of Alt. *n* versus Alt. 2	. . .	Desirability of Alt. *n* versus Alt. *j*	Desirability of Alt. *n* versus Alt. *k*	. . .	1

NOTE 1—A separate MPC comparing the alternatives is completed for each leaf attribute in the hierarchy. Within a given MPC, all comparisons of the desirability of Alternative *j* versus Alternative *k* are made with respect to the given leaf attribute of interest.

NOTE 2—Only the $n(n-1)/2$ shaded elements of the matrix (those above the matrix's diagonal) need to be filled in by the decision maker. The n diagonal elements are all equal to 1 by definition because each alternative is "exactly as desirable as itself." The $n(n-1)/2$ elements below the diagonal are equal to the reciprocals of the corresponding elements above the diagonal. This is because, for example, if Alternative 1 is twice as desirable as Alternative 2, then Alternative 2 must be half as desirable as Alternative 1.

FIG. 3 A Matrix of Paired Comparisons (MPC) Among Alternatives

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TABLE 2 Verbal Expressions and Their Numerical Counterparts[A]

NOTE 1—Use numerical values that are intermediate between those listed in the "numerical counterpart" column when preferences are intermediate between those listed in the "verbal expression" column of the table. For these intermediate numerical values, use either integers or non-integers.

Verbal Expression	Numerical Counterpart
Equal importance of attributes/Equal desirability of alternatives	1
Moderate importance of one attribute over another/*Moderate* desirability of one alternative over another	3
Strong importance of one attribute over another/*Strong* desirability of one alternative over another	5
Very Strong importance of one attribute over another/*Very Strong* desirability of one alternative over another	7
Extreme importance of one attribute over another/*Extreme* desirability of one alternative over another	9

[A] This table comes from the *Expert Choice User's Guide*, Decision Support Software, Inc., Pittsburgh, PA, 1993.

6.5.3.1 For the numerical approach, have the decision maker answer each question with a number, as in "Alternative 1 is *3* times as desirable as Alternative 2."[9]

6.5.3.2 For graphically mediated judgments, use an interactive software display to help the decision maker establish the degree of preference.

6.5.3.3 For verbally mediated judgments, have the decision maker answer each question with a verbal expression selected from Table 2 as in "Alternative 1 is moderately more desirable than Alternative 2." Then convert the verbal expressions to their numerical counterparts in Table 2. Be aware, however, that with verbal mediation, the final desirability scores for the alternatives are sensitive to the numerical scale underlying the approach.

6.6 Make pairwise comparisons of the relative importance of each attribute in a given set (starting with sets at the bottom of the hierarchy) with respect to the attribute or goal immediately above that set. (Attribute sets are defined in 6.3.2.1.) Use the same MPC approach that was described in 6.5 for making a series of pairwise comparisons.

6.6.1 Compare in pairwise fashion the relative importance of each attribute with respect to the attribute or goal above its set in the hierarchy. For each pair of attributes, the decision maker specifies a judgment about how much more important one attribute is than the other. Each pairwise comparison requires the decision maker to provide an answer to the question," Attribute 1 is how much more important than Attribute 2, relative to the attribute or goal above it in the hierarchy?"

6.6.2 Note that the decision maker responds to questions about how much more important one attribute is than another. It helps responders if the question is framed this way, since all answers will result in a number greater than or equal to one. Recall from Fig. 3, however, that the entries in an MPC always characterize the importance of each row attribute versus each column attribute. Therefore, in cases where the column attribute is more important than the row attribute, the decision maker shall answer the question, "How much more important is the column attribute than the row attribute?" In such cases, enter the reciprocal of the resulting number into the MPC.

6.6.3 Use numerical, graphically mediated, or verbally mediated judgments.

6.6.3.1 For example, in the numerical approach, have the decision maker answer each question with a number, as in "Attribute 1 is *2* times as important as Attribute 2."

6.6.3.2 For graphical judgments, use an interactive software display to help the decision maker establish the degree of preference.

6.6.3.3 For verbally mediated judgments, have the decision maker respond with a verbal expression selected from Table 2 as in" Attribute 1 is *moderately more important* than Attribute 2." Then convert the verbal expressions to their numerical counterparts in Table 2. Again be aware, however, that with verbal mediation the final desirability scores for the alternatives are sensitive to the underlying numerical scale underlying the approach.

6.6.4 Repeat the procedure for each set of attributes in the hierarchy.

6.7 Compute the final, overall desirability score for each alternative.

6.7.1 Obtain a vector of weights for each MPC using the principal eigenvector method. Find the principal eigenvector e^* which solves Eq 1, where M is the MPC of interest and λ_{max} is the principal eigenvalue of the matrix M.

$$\lambda_{max} e^* = M e^* \tag{1}$$

6.7.2 Normalize the eigenvector so that its elements sum to 1.0. To solve for the normalized principle eigenvector p, divide each of the n elements of the principal eigenvector e^* by the sum of the elements of e^*, as shown in Eq 2. The elements of the normalized principal eigenvector p are the weights derived from the MPC using the principal eigenvector method.

$$p = \left(\frac{1}{\sum_{i=1}^{n} e^*_i} \right) e^* \tag{2}$$

Use the AHP/Expert Choice for ASTM Building Evaluation software product or similar commercially available software to compute the principal eigenvector of each MPC. Simpler hand calculations which develop approximate solutions to Eq 1 do not reliably provide an accurate solution to the principal eigenvector problem.

6.7.3 Use the principal eigenvalue to calculate a heuristic check of consistency among the pairwise comparisons in a given MPC. Do a consistency check for each MPC in the problem both on comparisons among alternatives and among attributes.

6.7.3.1 Perfect consistency among pairwise comparisons is equivalent to perfect cardinal transitivity among the comparisons. That is, if Attribute 1 is twice as important as Attribute 2, and Attribute 2 is three times as important as Attribute 3, then perfect cardinal transitivity requires that Attribute 1 is six (two times three) times as important as Attribute 3.

[9] Integer answers are not required. For example, it is appropriate to say Alternative 1 is 1.2 times as desirable as Alternative 2 if that is your best estimate of relative desirability.

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