



Standard Practice for Setting an Upper Confidence Bound For a Fraction or Number of Non-Conforming items, or a Rate of Occurrence for Non-conformities, Using Attribute Data, When There is a Zero Response in the Sample¹

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1. Scope

1.1 This practice presents methodology for the setting of an upper confidence bound regarding a unknown fraction or quantity non-conforming, or a rate of occurrence for nonconformities, in cases where the method of attributes is used and there is a zero response in a sample. Three cases are considered.

1.1.1 The sample is selected from a process or a very large population of discrete items, and the number of non-conforming items in the sample is zero.

1.1.2 A sample of items is selected at random from a finite lot of discrete items, and the number of non-conforming items in the sample is zero.

1.1.3 The sample is a portion of a continuum (time, space, volume, area etc.) and the number of non-conformities in the sample is zero.

1.2 Allowance is made for misclassification error in this standard, but only when misclassification rates are well understood or known and can be approximated numerically.

2. Referenced Documents

2.1 ASTM Standards:²

E 141 Practice for Acceptance of Evidence Based on the Results of Probability Sampling

E 456 Terminology Relating to Quality and Statistics

E 1402 Terminology Relating to Sampling

E 1994 Practice for Use of Process Oriented AOQL and LTPD Sampling Plans

2.2 ISO Standards:

ISO 3534-1 Statistics—Vocabulary and Symbols, Part 1:

Probability and General Statistical Terms³
ISO 3534-2 Statistics—Vocabulary and Symbols, Part 2:
Statistical Quality Control³

NOTE 1—Samples discussed in this standard should meet the requirements (or approximately so) of a probability sample as defined in Terminologies E 1402 or E 456.

3. Terminology

3.1 Definitions:

3.1.1 *attributes, method of, n*—measurement of quality by the method of attributes consists of noting the presence (or absence) of some characteristic or attribute in each of the units in the group under consideration, and counting how many of the units do (or do not) possess the quality attribute, or how many such events occur in the unit, group or area. **E 456**

3.1.2 *confidence bound, n*—see *confidence limit*.

3.1.3 *confidence coefficient, n*—the value, C , of the probability associated with a confidence interval or statistical coverage interval. It is often expressed as a percentage.

ISO 3534-1

3.1.4 *confidence level, n*—see *confidence coefficient*.

3.1.5 *confidence limit, n*—each of the limits, T_1 and T_2 , of the two sided confidence interval, or the limit T of the one sided confidence interval.

ISO 3534-1

3.1.6 *one sided confidence interval, n*—when T is a function of the observed values such that, θ being a population parameter to be estimated, the probability $P(T \geq \theta)$ or the probability $P(T \leq \theta)$ is at least equal to C where C is a fixed positive number less than 1. The interval from the smallest value of θ up to T or the interval from T to the largest possible value of θ is a one sided, C , confidence interval for θ .

ISO 3534-1

3.1.7 *confidence limit, n*—each of the limits, T_1 and T_2 , of the two sided confidence interval, or the limit T of the one sided confidence interval.

ISO 3534-1

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² For referenced ASTM Standards, visit the ASTM website, www.astm.org, or contact ASTM Customer Service at service@astm.org. For *Annual Book of ASTM Standards* volume information, refer to the standard's Document Summary page on the ASTM website.

³ Available from American National Standards Institute, 11 W. 42nd Street, 13th Floor, New York, NY 10036.

3.1.8 *non-conformity, n*—the non-fulfillment of a specified requirement. **ISO 3534-2**

3.1.8.1 *Discussion*—The term “defect” is also used in this context.

3.1.9 *non-conforming item, n*—an item containing at least one non-conformity. **ISO 3534-2**

3.1.9.1 *Discussion*—The term “defective item” is also used in this context.

3.1.10 *population, n*—the totality of items or units of material under consideration. **E 456**

3.1.11 *sample, n*—a group of items, observations or test results, or portion of material taken from a large collection of items or quantity of material, which serves to provide information that may be used as a basis for making a decision concerning the larger collection or quantity. **E 456**

3.1.12 *probability sample, n*—a sample of which the sampling units have been selected by a chance process. At each step of selection, a specified probability of selection can be attached to each sampling unit available for selection. **E 1402**

3.1.13 *item, n*—an object or quantity of material on which a set of observations can be made. **E 456**

3.1.13.1 *Discussion*—As used in this standard, “set” denotes a single variable (the defined attribute). The term “sampling unit” is also used to denote an “item” (see Practice E 141).

3.2 Definitions of Terms Specific to This Standard:

3.2.1 *zero response, n*—in the method of attributes, the phrase used to denote that zero non-conforming items or zero non-conformities were found (observed) in the item(s), unit, group or area sampled.

Symbols:

A = the assurance index

C = confidence coefficient as a percent or as a probability value

C_d = the confidence coefficient calculated that a parameter meets a certain requirement, that is, that $p \leq p_0$, that $D \leq D_0$ or that $\lambda \leq \lambda_0$, when there is a zero response in the sample

D = the number of non-conforming items in a finite population containing N items

D_0 = a specified value of D for which a researcher will calculate a confidence coefficient for the statement, $D \leq D_0$, when there is a zero response in the sample

D_u = the upper confidence bound for the parameter D

N = the number of items in a finite population

n = the sample size, that is, the number of items in a sample

n_R = the sample size required

p = a process fraction non-conforming

p_0 = a specified value of p for which a researcher will calculate a confidence coefficient, for the statement $p \leq p_0$, when there is a zero response in the sample

p_u = the upper confidence bound for the parameter p

λ = the mean number of non-conformities (or events) over some area of interest for a Poisson process

λ_0 = a specific value of λ for which a researcher will calculate a confidence coefficient for the statement, $\lambda \leq \lambda_0$, when there is a zero response in the sample

λ_u = the upper confidence bound for the parameter λ

θ_1 = the probability of classifying a conforming item as non-conforming; or of finding a nonconformity where none exists

θ_2 = the probability of classifying a non-conforming item as conforming; or of failing to find a non-conformity where one should have been found

4. Significance and Use

4.1 In Case 1, the sample is selected from a process or a very large population of interest. The population is essentially unlimited, and each item either has or has not the defined attribute. The population (process) has an unknown fraction of items p (long run average process non-conforming) having the attribute. The sample is a group of n discrete items selected at random from the process or population under consideration, and the attribute is not exhibited in the sample. The objective is to determine an upper confidence bound, p_u , for the unknown fraction p whereby one can claim that $p \leq p_u$ with some confidence coefficient (probability) C . The binomial distribution is the sampling distribution in this case.

4.2 In Case 2, a sample of n items is selected at random from a finite lot of N items. Like Case 1, each item either has or has not the defined attribute, and the population has an unknown number, D , of items having the attribute. The sample does not exhibit the attribute. The objective is to determine an upper confidence bound, D_u , for the unknown number D , whereby one can claim that $D \leq D_u$ with some confidence coefficient (probability) C . The hypergeometric distribution is the sampling distribution in this case.

4.3 In Case 3, there is a process, but the output is a continuum, such as area (for example, a roll of paper or other material, a field of crop), volume (for example, a volume of liquid or gas), or time (for example, hours, days, quarterly, etc.) The sample size is defined as that portion of the “continuum” sampled, and the defined attribute may occur any number of times over the sampled portion. There is an unknown average rate of occurrence, λ , for the defined attribute over the sampled interval of the continuum that is of interest. The sample does not exhibit the attribute. For a roll of paper this might be blemishes per 100 ft²; for a volume of liquid, microbes per cubic litre; for a field of crop, spores per acre; for a time interval, calls per hour, customers per day or accidents per quarter. The rate, λ , is proportional to the size of the interval of interest. Thus, if $\lambda = 12$ blemishes per 100 ft² of paper, this is equivalent to 1.2 blemishes per 10 ft² or 30 blemishes per 250 ft². It is important to keep in mind the size of the interval in the analysis and interpretation. The objective is to determine an upper confidence bound, λ_u , for the unknown occurrence rate λ , whereby one can claim that $\lambda \leq \lambda_u$ with some confidence coefficient (probability) C . The Poisson distribution is the sampling distribution in this case.

4.4 A variation on Case 3 is the situation where the sampled “interval” is really a group of discrete items, and the defined attribute may occur any number of times within an item. This might be the case where the continuum is a process producing

discrete items such as metal parts, and the attribute is defined as a scratch. Any number of scratches could occur on any single item. In such a case the occurrence rate, λ , might be defined as scratches per 1000 parts or some similar metric.

4.5 In each case a sample of items or a portion of a continuum is examined for the presence of a defined attribute, and the attribute is not observed (that is, a zero response). The objective is to determine an upper confidence bound for either an unknown proportion, p (Case 1), an unknown quantity, D (Case 2), or an unknown rate of occurrence, λ (Case 3). In this standard, confidence means the probability that the unknown parameter is not more than the upper bound. More generally, these methods determine a relationship among sample size, confidence and the upper confidence bound. They can be used to determine the sample size required to demonstrate a specific p , D or λ with some degree of confidence. They can also be used to determine the degree of confidence achieved in demonstrating a specified p , D or λ .

4.6 In this standard allowance is made for misclassification error but only when misclassification rates are well understood or known, and can be approximated numerically.

4.7 It is possible to impose the language of classical acceptance sampling theory on this method. Terms such as Lot Tolerance Percent Defective, Acceptable Quality Level, Consumer Quality Level are not used in this standard. For more information on these terms, see Practice E 1994.

5. Procedure

5.1 When a sample is inspected and a zero response is exhibited with respect to a defined attribute, we refer to this event as “all_zeros.” Formulas for calculating the probability of “all_zeros” in a sample are based on the binomial, the hypergeometric and the Poisson probability distributions. When there is the possibility of misclassification error, adjustments to these distributions are used. This practice will clarify when each distribution is appropriate and how misclassification error is incorporated. Three basic cases are considered as described in Section 4. Formulas and examples for each case are given below. Mathematical notes are given in Appendix X1.

5.2 In some applications, the measurement method is known to be fallible to some extent resulting in a significant misclassification error. If experiments with repeated measurements have established the rates of misclassification, and they are known to be constant, they should be included in the calculating formulas. Two misclassification error probabilities are defined for this practice:

5.2.1 Let θ_1 be the probability of reporting a non-conforming item when the item is really conforming.

5.2.2 Let θ_2 be the probability of reporting a conforming item when the item is really non-conforming.

5.2.3 Almost all applications of this standard require that θ_1 be known to be 0 (see 6.1.2).

5.3 Formulas for upper confidence bounds in three cases:

5.3.1 *Case 1*—The item is a completely discrete object and the attribute is either present or not within the item. Only one response is recorded per item (either go or no-go). The sample items originate from a process and hence the future population of interest is potentially unlimited in extent so long as the

process remains in statistical control. The item having the attribute is often referred to as a defective item or a non-conforming item or unit. The sample consists of n randomly selected items from the population of interest. The n items are inspected for the defined attribute. The sampling distribution is the binomial with parameters p equal to the process (population) fraction non-conforming and n the sample size. When zero non-conforming items are observed in the sample (the event “all_zeros”), and there are no misclassification errors, the upper confidence bound, p_u , at confidence level C ($0 < C < 1$), for the population proportion non-conforming is:

$$p_u = 1 - \sqrt[n]{1 - C} \quad (1)$$

5.3.1.1 For the case with misclassification errors, when zero non-conforming items are observed in the sample (all_zeros), the upper confidence bound, p_u , at confidence level C is:

$$p_u = \frac{1 - \theta_1 - \sqrt[n]{1 - C}}{(1 - \theta_1 - \theta_2)} \quad (2)$$

5.3.1.2 Eq 2 reduces to Eq 1 when $\theta_1 = \theta_2 = 0$. To find the minimum sample size required (n_R) to state a confidence bound of p_u at confidence C if zero non-conforming items are to be observed in the sample, solve Eq 2 for n . This is:

$$n_R = \frac{\ln(1 - C)}{\ln((1 - p_u)(1 - \theta_1) + p_u\theta_2)} \quad (3)$$

5.3.1.3 To find the confidence demonstrated (C_d) in the claim that an unknown fraction non-conforming p is no more than a specified value, say p_0 , when zero non-conformances are observed in a sample of n items solve Eq 2 for C . This is:

$$C_d = 1 - ((1 - p_0)(1 - \theta_1) + p_0\theta_2) \quad (4)$$

5.3.2 *Case 2*—The item is a completely discrete object and the attribute is either present or not within the item. Only one response is recorded per item (either go or no-go). The sample items originate from a finite lot or population of N items. The sample consists of n randomly selected items from among the N , without replacement. The population proportion defective is $p = D/N$ where the unknown D is the integer number of non-conforming (defective) items among the N . The sampling distribution is the hypergeometric with parameters N , D and n . When zero non-conforming items are observed in the sample (all_zeros), and there are no misclassification errors, the upper confidence bound, at confidence level C , for the unknown number of non-conforming items, D , in the population is found by solving Eq 5 iteratively for D_u .

$$C = 1 - \prod_{i=1}^n \left(1 - \frac{D_u}{N - i + 1} \right) \quad (5)$$

5.3.2.1 For the case with misclassification errors, when zero non-conforming items are observed in the sample (all_zeros), the upper confidence bound, D_u , at confidence level C is found by solving Eq 6 iteratively for D_u .

$$C = 1 - \quad (6)$$

$$\frac{\binom{N - D_u}{n} (1 - \theta_1)^n + \sum_{x=1}^{\min(D_u, n)} \binom{N - D_u}{n - x} (1 - \theta_1)^{n - x} \binom{D_u}{x} \theta_2^x}{\binom{N}{n}}$$

5.3.2.2 Eq 5 and 6 must be solved numerically for D_u . For fixed values of C , N , n , θ_1 and θ_2 , we evaluate the right hand