



Standard Guide for Reliability Demonstration Testing¹

This standard is issued under the fixed designation E3291; the number immediately following the designation indicates the year of original adoption or, in the case of revision, the year of last revision. A number in parentheses indicates the year of last reapproval. A superscript epsilon (ϵ) indicates an editorial change since the last revision or reapproval.

1. Scope

1.1 This standard covers fundamental concepts, applications and mathematical relationships associated with the planning of reliability demonstration tests as applied to components and materials testing.

1.2 The system of units for this guide is not specified. Quantities and examples are presented only as illustrations of a method or a calculation. Any examples used are not binding on any particular product or industry.

1.3 *This standard does not purport to address all of the safety concerns, if any, associated with its use. It is the responsibility of the user of this standard to establish appropriate safety, health, and environmental practices and determine the applicability of regulatory limitations prior to use.*

1.4 *This international standard was developed in accordance with internationally recognized principles on standardization established in the Decision on Principles for the Development of International Standards, Guides and Recommendations issued by the World Trade Organization Technical Barriers to Trade (TBT) Committee.*

2. Referenced Documents

2.1 *ASTM Standards:*²

E456 Terminology Relating to Quality and Statistics

E2555 Practice for Factors and Procedures for Applying the MIL-STD-105 Plans in Life and Reliability Inspection

E2696 Practice for Life and Reliability Testing Based on the Exponential Distribution

E3159 Guide for General Reliability

2.2 *ISO Standards:*³

ISO 3534-1 Statistics – Vocabulary and symbols, Part 1: Probability and general statistical terms

ISO Guide 73 Risk management vocabulary

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² For referenced ASTM standards, visit the ASTM website, www.astm.org, or contact ASTM Customer Service at service@astm.org. For *Annual Book of ASTM Standards* volume information, refer to the standard's Document Summary page on the ASTM website.

³ Available from American National Standards Institute (ANSI), 25 W. 43rd St., 4th Floor, New York, NY 10036, <http://www.ansi.org>.

3. Terminology

3.1 Definitions:

3.1.1 Unless otherwise noted, terms relating to quality and statistics are as defined in Terminology E456. Other general statistical terms and terms related to risk are defined in ISO 3534-1 and ISO Guide 73.

3.1.2 B_p life, n —for continuous variables, the life at which there is a probability, p , (expressed as a percentage) of failure at or less than this value. **E3159**

3.1.3 failure mode, n —the way in which a device, process or system has failed. **E3159**

3.1.4 hazard rate, n —differential fraction of items failing at time t among those surviving up to time t , symbolized by $h(t)$. **E2555**

3.1.5 mean time between failures (MTBF), n —the average time to failure for a repairable item. **E3159**

3.1.6 mean time to failure (MTTF), θ , n —in life testing, the average length of life of items in a lot. **E2696**

3.1.7 reliability, n —the probability that a component, device, product, process or system will function or fulfill a function after a specified duration of time or usage under specified conditions. **E3159**

3.2 Symbols:

3.2.1 The following symbols are used extensively in the discussion.

3.2.2 C —confidence coefficient (decimal value between 0 and 1).

3.2.3 n —sample size, (positive integer at least 1).

3.2.4 p —failure probability (decimal between 0 and 1, or percentage between 0 and 100).

3.2.5 R —reliability. $R = 1 - p$ or $(100 - p)\%$ for p expressed as a percentage.

3.2.6 r —number of failures allowed ($0 \leq r < n$).

3.2.7 β —Weibull shape parameter, also referred to as the “Weibull slope.”

3.2.8 θ —for the exponential model, the mean (MTTF) of the distribution.

3.2.9 η —for the Weibull model, the characteristic life or scale parameter.

3.2.10 λ —for the exponential model, the failure rate; also equal to $1 / \theta$.

3.2.11 f —scatter factor equal to the ratio $B_{50}/B_{0.1}$

4. Significance and Use

4.1 Reliability demonstration testing is a methodology for qualifying or validating a product's performance capability. Demonstration methods are useful for components, devices, assemblies, materials, processes, and systems. Many industries require demonstration testing either for new product development and product introduction, in validating a change to an existing product or as part of an audit. Test plans generally try to answer the questions, "How long will a product last?" or "What is its reliability?", under stated conditions at some specific time. When time is being used as a life variable, it must be cast in some kind of "time" units. Typical time units are hours (or minutes), cycles of usage, calendar time or some variation of these. In certain cases, "time" can be accelerated in order to reduce a plan's completion time. In the automotive industry mileage may be used as the time variable. Certain means of accelerating tests involve the use of increased power, voltage, mechanical load, humidity, vibration, or temperature (often in the form of thermal cycling).

4.2 Two fundamental objectives in reliability test planning are: (a) demonstrating that a product meets a specific life requirement, and (b) demonstrating what a product can do – its life capability. In the first case, a requirement is specified; in the second case a series of test results are used to state a result at the present time – its current capability. Both cases share similar inputs and outputs.

4.3 Often a life distribution model is specified such as the Weibull, the exponential, the lognormal or the normal distribution. In addition, for the specific distribution assumed, a parameter is typically assumed (or a range of values for a parameter). For example, in the Weibull case, the shape parameter, β , is assumed; in the lognormal case the scale parameter, σ , is assumed and in the normal case the standard deviation, σ , is assumed. In other cases, a non-parametric analysis can be used. Non-parametric cases typically require a larger sample size than parametric cases. This standard will discuss conditions under which distributions and associated parameters can be assumed.

4.4 Generally, a life requirement is cast as a mission time and associated reliability, for example, to demonstrate a reliability of 99% at time $t=1000$ hours of usage. In another case the requirement might be cast as a B_p life requirement, such as the B_5 life. For example, if $B_5 = 10\,000$ cycles are specified, this means to demonstrate a reliability of 95 % at $t = 10\,000$ cycles. Other life requirements might be a mean life, a median life (B_{50}) or a failure rate not to be exceeded at a specified time t . In other cases, the requirement might mean withstanding a load for some duration. Demonstration necessarily means to demonstrate with some statistical confidence. Thus, a confidence value is a standard input in any plan. Commonly used confidence values are 99 %, 95 %, 90 %, and 63.2 %.

4.4.1 When a requirement and a confidence value have been stated, a derived plan will determine a sample size, n , a test

time, t , and a maximum number of failures, r , allowed by the plan. A test concludes and is successful if the n units tested result in not more than r failures by time t . In another scenario, the sample size, number of failures allowed and confidence value are first stated and the plan returns the test time requirement.

4.5 The "RC" nomenclature for specifying a test requirement is often used, where R stands for reliability and C for confidence. For example, to state a requirement of 2000 hours at R99C90 means that the requirement is to demonstrate 99 % reliability at 2000 hours with 90 % confidence. Alternatively, this also means to demonstrate a B_1 life of 2000 hours with 90 % confidence.

4.6 This guide considers, the Weibull, lognormal and normal parametric cases as well as the basic non-parametric case for attribute reliability. The common exponential case is a Weibull distribution with assumed shape parameter $\beta = 1$, but is considered as a separate case, distinct from the Weibull.

5. General Introduction

5.1 Before reliability can be assessed, and a test plan specified, it is useful to know something about the nature of the failure modes that might occur. The type of failure mode is important in selecting a life distribution and any assumed parameter where variable (continuous distribution) data is used.

5.1.1 In general, there are three classes of failure modes that operate in electro-mechanical systems: (a) random, (b) wear out, and (c) "infant mortality". These are related to the failure rates operative in field applications.

5.2 In a random type failure mode, the failure mechanism is not related to the age of the object tested or fielded, and the failure rate is constant throughout this portion of life (constant failure rate or CFR). A new object and any unfailed object having seen usage each have the same propensity to fail in the future. Causes for this type of failure mode are often related to some external stimulus, operability robustness or to a complex of rare factors that might come together, rarely, in just the right mix to cause a failure.

5.2.1 Random stress spikes under field conditions, such as those driven by electrical, mechanical, chemical or thermal shock may cause random failures at any point in a product life cycle. Operating a product outside of specification limits at random times may render a product prone to failure at any time in its life. Other outside stimuli such as those caused by foreign object damage or biological interference are possible and may lead to random type failures.

5.3 Wear out mechanisms are related to the age of the object and have increasing failure rates with time/usage. This is called an increasing failure rate (IFR). The more usage the object has, the greater the likelihood of failures for the surviving population. Wear-out or performance degradation is generally a gradual process as usage increases. Ultimately, this results in loss of robustness and eventual failure. In electro-mechanical applications, causes of this type of failure may be driven by gradual chemical, thermal, mechanical, electrical or radiation stresses.

5.4 An “infant mortality” type failure mode relates to certain severe conditions possessed by some objects in a population rendering these select units prone to early failure, while the remaining population may last many times longer. The older a unit is, the less likely such failures would appear because the affected unit would most likely have failed early in the life cycle if it had the severe condition. Essentially, there is a failure mode and some items in the population have the failure conditions more severely than others, resulting in early failure. The failure rate in these cases is decreasing with time. This is called a decreasing failure rate (DFR). There are numerous causes for this case ranging from assembly issues to material problems.

5.5 The three general classes of failure are depicted in the so-called *bathtub curve* shown in Fig. 1. Fig. 1 is a highly idealized portrait of a product’s life cycle. It suggests that there are three phases to life: (a) the burn-in period is the trouble shooting or corrective action phase where the failure rate is decreasing, (b) the random period is the useful life phase where the failure rate is constant, and (c) the wear out period with an increasing failure rate leads to end of life or retirement. Not every product will experience all three phases and the degree of each phase depicted may be of very dissimilar proportion.

5.5.1 *Infant Mortality*—Failures of this type are generally the result of some components that do not meet specifications or workmanship standards. These types of failures are typically not design-related issues, but quality-related issues. As a product is put into service, early failures are observed as a result of these or similar conditions. Corrections are gradually made, and the failure rate decreases for a given time until it may reach a steady state or constant rate. The infant mortality period, then, is characterized by a decreasing failure rate. The Weibull distribution with shape parameter $0 < \beta < 1$ is commonly used to characterize infant mortality conditions.

5.5.2 *Constant Failure Rate*—Once the failures due to components and workmanship are for the most part eliminated, the constant failure rate period is entered, and called the random failure rate period. “Random” means that failure events are proportional to time in use, but this is not dependent on age at the start of any time interval. The constant failure rate period is the most common time frame for making reliability predictions, where the exponential distribution is used. The exponential – which is equivalent to a Weibull with $\beta = 1$ – can

be used to describe this phase. The number of such random failures occurring within this period is Poisson distributed with some mean number of events. A failure rate parameter, λ , in the units of failure per unit time is used to characterize this portion of life.

5.5.3 *Wear Out Period*—As components begin to fatigue or wear out, failures occur at increasing rates for a specified interval. An increasing or sometimes sharp rise in failure rate can be the result of fatigue and other physical actions. As time goes on, failures occur more and more frequently to a point where it may no longer be practical to continue operating the product. Several distributions may be appropriate to model the wear-out period. The Weibull and lognormal distributions are often used. For the Weibull distribution, an increasing failure rate occurs where the shape parameter $\beta > 1$.

5.6 For purposes of validation of a new product introduction or for a change to an existing product it is common practice to use the exponential distribution which covers the useful (random) portion of product life. This assumption is conservative in that it takes more test time and/or a larger sample size to validate a specific requirement for the random life case than for the wear out case. In general, the infant mortality case is not typical in validating procedures (although in theory, this case would be valid). The infant mortality case is more often used in working with a fielded population that shows signs of early failures. It may also be used to develop a burn-in or screening period that a product is exposed to prior to its field application, in an attempt to find early type failures before customer use.

5.7 In some cases, the failure rate may exhibit increases and decreases as a function of age. This is typically distribution-specific and would be the case in certain parameter cases when the lognormal distribution is used.

6. The Nonparametric Case

6.1 In the nonparametric case (NP) there is no life distribution being used to model variable data. Testing is of the pass-fail type and governed by the binomial distribution. In this attribute reliability case, n is the sample size, and p is the failure probability at the specified test condition. The reliability is cast as $R = 1 - p$. The parameters for this test case are C the specified confidence, R the reliability, n , the sample size, and r the number of failures in n allowed by the plan.

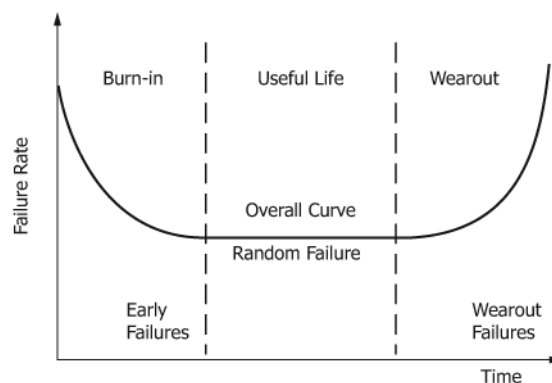


FIG. 1 The “Bathtub” Curve

6.1.1 A random variable X is said to have a binomial distribution if x takes on integer values between 0 and n , and is a count of the number of occurrences of a defined condition, where each of the n sample units either has or does not have the condition. Each sample unit has a probability p of having the condition and the n sample values behave independently of one another. We say that X is binomially distributed with parameters n and p .

6.1.2 A “failure event” is defined as the failure of a component, a material specimen or other entity to meet a well-defined requirement. A requirement may be a variable quantity such as a number of cycles or a minimum load condition in a fatigue test, a variable dimensional requirement, or a basic attribute tested as go or no-go type of characteristic. Each test either meets or fails to meet the requirement. For pure attribute pass or fail testing, a “zero failure” test plan is a common theme. For the zero-failure test plan the following equation, based on the binomial distribution, relates sample size, n , confidence, C , and reliability, R (1, 2).⁴

$$R \geq \sqrt[n]{1 - C} \quad (1)$$

6.1.3 In Eq 1, C is the chosen confidence coefficient ($0 < C < 1$), n is the sample size ($n > 0$), and R is the demonstrated reliability ($0 < R < 1$) when n tests result in zero failures. Eq 1 may be solved for either C or n giving.

$$C \geq 1 - R^n \quad (2)$$

$$n = \frac{\ln(1 - C)}{\ln(R)} \quad (3)$$

6.1.4 In Eq 2, the confidence in meeting the reliability requirement R using sample size n is at least C . In Eq 3, the sample size needed to claim reliability R with confidence C is n . In each case $r = 0$ failures are observed in a sample of size n .

6.1.5 Example 1—Suppose $n = 22$ units have been run to a specific requirement and 0 failures were observed. Then at $C=90\%$ confidence the minimum reliability demonstrated is $\sqrt[22]{1-0.9} = 0.90$ or 90 %, using Eq 1. This is often referred to as an “R90C90” value (reliability at least 90 % with 90 % confidence). Other “RC” combinations are in use. Suppose further that $R = 90$ % is not good enough and the practitioner requires 95 % reliability, what sample size is necessary? Use Eq 3 with $R = 0.95$ and $C = 0.9$ giving $n = \ln(1 - 0.9) / \ln(0.95) = 44.89$ or $n = 45$ units required without failure. What is the confidence in meeting a reliability of at least 95 % using $n = 22$ tests where 0 failure have occurred? Use Eq 2 with $n = 22$ and $R = 0.95$ giving $C \geq 1 - 0.95^{22} = 0.676$ or about 67.6 % confidence.

6.2 Another commonly occurring case is to allow 1 failure in n (not more than 1 failure among n units tested). In that case, the relation among n , C and R , based on a binomial model and simplified from the relation between a binomial and beta distribution is (1):

$$nR^{n-1} - (n - 1)R^n \geq 1 - C \quad (4)$$

6.2.1 Eq 4 may be solved numerically for any variable when the remaining two are known or assumed.

6.2.2 Example 2—Suppose 1 failure in $n = 22$ tests are observed. What is the reliability demonstrated using $C = 90$ % confidence? Use Eq 4 with $n = 22$ and $C = 0.9$, and iterating on R until the inequality in Eq 4 is just met, gives $R = 0.8344$ or about 83.4 % reliability. If the desire is to achieve 99 % reliability using 90 % confidence, what sample size is required if we allow not more than 1 failure in n ? Use Eq 4 with $C = 0.9$ and $R = 0.99$. Then iterating on n until the inequality is just met find that $n = 388$.

6.3 The variables in the general case include a sample size n , confidence C , reliability R and r the number of allowable failures in n . The solution is a generalization of the $r = 1$ case discussed in section 6.2. The beta distribution equivalent to the binomial probability expression $P(X \leq r) \geq 1 - C$ is used. Let p be the failure probability. Then $R = 1 - p$. Given a confidence level C , a sample size n , and a number of allowed failures r , the relationship is (3):

$$C \geq \int_0^{1-R} \beta(r + 1, n - r) dy \quad (5)$$

6.3.1 Eq 5 is a cumulative beta distribution with parameters $r + 1$ and $n - r$ evaluated at the point $1 - R$. Refer to the *cdf* integral as $G(1 - R)$. Then $C = G(1 - R)$ and the parameter R is calculated using the inverse beta *cdf* evaluated at C . This is $G^{-1}(C) \geq 1 - R$. Then the demonstrated reliability R is calculated as:

$$R \geq 1 - G^{-1}(C) \quad (6)$$

6.3.2 This calculation is carried out numerically using for example a spreadsheet-type program having beta distribution capability. When R , n , and r are specified, Eq 5 is used directly to calculate the confidence demonstrated. If R , C and r are specified or if R , C and n are specified, then Eq 5 is iterated on the variable being solved for until the inequality in Eq 6 is just met. It is important, that r must be strictly less than n in all cases for these types of plans. In addition, for specified R , C and n , there may not exist a suitable r that meets the RC requirement. In that case, the sample size needs to increase.

6.3.3 Example 3—A test plan was conducted using a sample of $n = 250$ units with $r = 3$ failures observed. What reliability is demonstrated at $C = 95$ % confidence? Use Eq 6 with beta distribution parameters $r + 1 = 4$ and $n - r = 247$. This results in $R \geq 0.969$ or 96.9 %. If an R95C95 plan is desired that allows 3 failures in n , what sample size is required? Use Eq 6 with $C = 0.95$, $R = 0.95$ and $r = 3$. Iterate n in the beta parameters until the inequality in Eq 6 is just met. This results in $n = 153$ as the required sample size.

6.3.4 Example 4—If $n = 450$ units are available for test and R95C90 is the requirement, what number of failures is allowed? Use Eq 6 with $R = 0.95$, $C = 0.9$ and $n = 450$, iterating on r until the inequality in Eq 6 is just met. Find that $r = 16$ is the maximum number of failures allowed. Note that this combination of n and r is one of several possible combinations that would satisfy the R95C90 requirement. For example, $n=282$ and $r = 9$ or $n = 209$ and $r = 6$ would also work. A smaller sample size would be possible and more economical. Process capability should be taken into account when

⁴ The boldface numbers in parentheses refer to the list of references at the end of this standard.

feasible, when deciding on any plan. Table 1 is a short summary comparing sample sizes for the $r = 0$, $r = 1$ and $r = 2$ cases and several common “RC” combinations.

7. The Exponential Case

7.1 The exponential distribution case is one the most commonly required testing scenarios found in reliability. In this section, theory and methods are outlined for the exponential life case where the failure mode is assumed to be of the random type. Further related information is given in the Appendices. The exponential distribution is shown in Fig. 2. It is used for predicting the reliability of items in the constant failure period (see Fig. 1). This is typically the starting point in design reliability requirements determination.

7.2 The exponential probability density function (pdf) is:

$$f(t) = \frac{1}{\theta} e^{-t/\theta}, t > 0 \quad (7)$$

7.2.1 The parameter θ is often called the mean time to failure (or MTTF) and is the mean of the distribution of t for single use items or first occurrence failure times. In repairable systems, units are repaired upon failure and reinstalled until failure a second or a third time, the average time between repair cycles is called the mean time between failures (MTBF). In general, this should not be confused with the MTTF because repair does not always render a repaired item in like new condition; however, for the exponential case, it does, at least in theory, and we have that MTTF = MTBF.

7.2.2 Eq 7 may be recast using $\lambda = 1/\theta$ where λ is called the failure rate (failures per unit time).

$$f(t) = \lambda e^{-\lambda t} \quad (8)$$

7.2.3 Integrating Eq 8 gives the cumulative distribution function (cdf), $F(t)$.

$$\int_0^t f(y) dy = F(t) = 1 - e^{-\lambda t} \quad (9)$$

7.2.4 Where, again, λ is substituted for $1/\theta$. $F(t)$ is the failure probability at time t for objects having a constant failure rate λ . The quantity $1 - F(t) = R(t)$ is the reliability function at time t . Another name for reliability is survival probability, used particularly in the life sciences.

$$R(t) = 1 - F(t) = e^{-\lambda t} \quad (10)$$

7.2.5 The hazard function, $h(t)$, of any failure time distribution is the ratio $f(t)/R(t)$. It is the instantaneous failure rate at

time t for the population of surviving units at that time. For the exponential, $h(t) = \lambda$, a constant throughout time.

7.2.6 The exponential distribution has the following key properties:

7.2.7 The mean and standard deviation are the same value.

7.2.8 Approximately 63.2 % of the area under the curve falls below the mean (θ). This further means that the failure probability at the mean life is $F(\theta) = 63.2\%$ leaving the reliability at the mean life of $100 - 63.2 = 36.8\%$. This property is not greatly appreciated by practitioners and can be the source of numerous types of errors and misunderstandings concerning reliability with this model.

7.2.9 The hazard function (failure rate) is constant throughout life.

7.2.10 The quantity λt is called the cumulative hazard. It is the expected number of failures at time t . For given time interval t , the number of failures in the interval $[0, t]$ is Poisson distributed with mean λt provided the random behavior remains homogeneous through time t . For the exponential, this is independent of the starting time of the interval.

7.2.11 *Memoryless Property*—For an exponentially distributed life, the probability that an item of age t , still surviving, lasts an additional time s without failing is identical to a brand-new item lasting through a time s without failure. Essentially, an item’s propensity to fail is not age dependent. A surviving item at any age has the same probability of failing within or surviving a future time duration s as a new item does. Thus, there is no wear out mechanism.

7.2.12 A key property that holds only for the exponential distribution is:

$$R(t + s) = R(t)R(s) \quad (11)$$

7.2.12.1 This can be manipulated further as:

$$R(ns) = \{R(s)\}^n \quad (12)$$

7.2.12.2 For example, if we know that the reliability at 1 cycle is 0.9999 ($s = 1$), then the reliability at 1000 cycles ($n = 1000$) is 0.9999^{1000} or about 0.905.

7.2.13 *Reproductive or Closure Property*—The smallest order statistic in a sample of size n from an exponential distribution with mean θ is also exponentially distributed with mean θ/n . The smallest in n inherits the exponential property with mean proportional to $1/n$.

7.2.14 *B-life Formula*—The B_p life of a failure distribution is that time by which $p\%$ of the population is expected to fail by. For the exponential distribution this is found by inverting the cdf, Eq 7:

$$B_p = -\theta \ln\left(1 - \frac{p}{100}\right) \quad (13)$$

7.2.15 *Simulation*—A random observation from an exponential distribution with mean θ is determined using:

$$t = -\theta \ln(u) \quad (14)$$

where:

u = a random observation from a uniform distribution on $[0, 1]$.

TABLE 1 Sample Sizes for Nonparametric RC^A Test Plan Cases with r Failures Allowed

RC case	$r=0$	$r=1$	$r=2$
R90C90	22	38	52
R90C95	29	46	61
R90C99	44	64	81
R95C90	45	77	105
R95C95	59	93	124
R95C99	90	130	165
R99C90	230	388	531
R99C95	299	473	628
R99C99	459	662	838

^A “R” = reliability, “C” = confidence

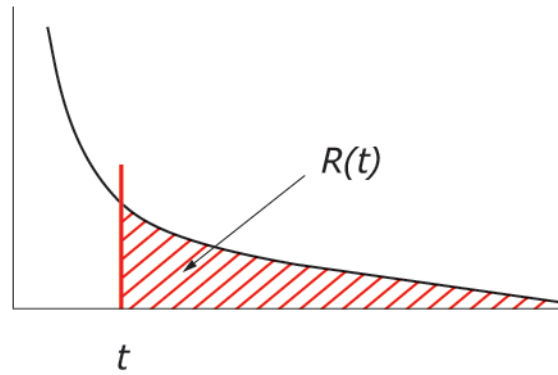


FIG. 2 The Exponential Distribution

7.2.16 Both the time units and the failure rate may be expressed in several ways. Hours or cycles are the typical time units used. Several commonly used metrics are:

- (1) Failures/hr,
- (2) Failures per 100k hours,
- (3) Failures per million hours, and
- (4) %/K hours.

7.2.17 The last expression, %/K hours, represents % failing in 1000 hours of service. For example, if the rate is 1.5 %/K and 10 000 units each operate for 500 hours we would expect 75 failures because 10 000 units at 500 hours each is equivalent to 5 000 units at 1000 hours each. 1.5 % of 5000 is 75 failures.

7.2.18 The standard rate in failures per hour is then $75 / (10\,000 \times 500) = 1.5\text{E-}5$. The relationship between %/K and the failure rate λ in failures per hour is: $\%/K = 10^5\lambda$.

7.2.19 Another variation is “failures in time” abbreviated as FIT and equivalent to parts per million per 1000 hours of service or PPM/K. 1 PPM/K means 1 failure in 1 million units each operating for 1000 hours or 1 failure per billion operating hours. This is equivalent to $\text{FIT} = 10\,000(\%/K)$. In the above example, the FIT number is $10\,000 \times (1.5\%/K) = 15\,000$ FIT. Table 2 shows a conversion among λ , %/K and FIT in the time units of hours.

7.2.20 Eq 9 can be re-parameterized in the following way substituting any B_p life in for θ . This equation is useful in some applications.

$$F(t) = 1 - (1 - p / 100)^{\frac{t}{B_p}} \quad (15)$$

7.2.21 Example 5—A certain type of transistor is known to have a constant failure rate with rate 0.04 %/K. What is the probability that one of these transistors fails before 15 000 hours of use? How long do we have to wait to expect 1% of the population to have failed? What is the FIT measure of the failure rate?

TABLE 2 Equivalent Failure Rates in Different Metrics^A

rate, λ	%/K	FIT
1E-04	10	100000
1E-05	1	10000
1E-06	0.1	1000
1E-07	0.01	100
1E-08	0.001	10
1E-09	0.0001	1

^A $\lambda = 1\text{E-}5\text{X}(\%/K) = 1\text{E-}9\text{X}(\text{FIT})$; $\text{FIT} = 1\text{E}4\text{X}(\%/K)$

7.2.21.1 First, convert to the failure metric to failures per hour, λ . Multiply %/K by 10^{-5} to get $\lambda = 4\text{E-}7$. The probability of failure by 15 000 hours is $F(15\,000)$ or:

$$F(15\,000) = 1 - e^{-(4\text{E-}7)(15000)} = 0.006$$

7.2.21.2 The 1 % failure time is the B_1 life. Use Eq 13 with $\theta = 1 / \lambda = 1 / (4\text{E-}7) = 2\,500\,000$ hours.

$$B_1 = -2,500,000 \ln\left(1 - \frac{1}{100}\right) = 25,126$$

7.2.21.3 The FIT measure of failure is $10^9\lambda = 10^9(4\text{E-}7) = 400$ failures per billion operating hours.

7.2.22 Example 6—A certain type of aerospace system uses five components operating simultaneously in series. The component MTTF is known to be 250,000 cycles. The system fails if any of the 5 components fail. Its mission time is 50 cycles of usage. Calculate the system reliability at the mission time. Calculate the $B_{0.1}$ life of the system.

7.2.22.1 The system will fail when the first failure in $n = 5$ occurs. The first of $n = 5$ distribution is also exponential with $\text{MTTF} = \theta/n = 250\,000 / 5 = 50\,000$ cycles (see 7.2.13). The probability of failure by the mission time of 50 cycles is:

$$F(50) = 1 - e^{-50/50000} = 0.001$$

7.2.22.2 Thus, the $t = 50$ cycle value is the $B_{0.1}$ system life.

7.3 Test Planning—A test plan for the exponential distribution requires several key design parameters.

- (1) Sample size (n) and number of failures allowed (r),
- (2) Objective MTTF (θ) or failure rate (λ), and
- (3) Confidence level (C).

7.3.1 As an alternative objective, a mission time and associated reliability may be specified, and this is equivalent to specifying a B_p life. For example, if a reliability of 0.99 is required for a mission time of 1200 hours, this is equivalent to stating a B_1 life requirement of 1200 hours.

7.3.2 We may also ask what MTTF, B_p life or reliability at a stated mission time T has been demonstrated given a set of failure time data at the present time during a test. In addition, we can solve for the demonstrated confidence in meeting a requirement.

7.3.3 The most commonly occurring questions about test planning are: (1) what sample size (n) should I use? And (2) what test time (t) is appropriate? Associated with both of these is the number of failures allowed (r). This analysis starts with

a relationship involving, n , r , the confidence value C and the beta distribution on the interval $[0,1]$. Let p be beta distributed with parameters $r+1$ and $n-r$. Its cumulative distribution is denoted $G(p)$. Set $G(p) = C$ and solve for p . This uses the inverse beta integral:

$$p_c = G^{-1}(C) \quad (16)$$

7.3.4 Alternatively, we have the beta integral itself:

$$G(p_c) = C \quad (17)$$

7.3.5 Eq 16 and Eq 17 can be solved using many software programs including spreadsheet-type programs. For clarity of notation, a subscript “c” is attached to p indicating that it satisfies Eq 16 and Eq 17. The quantity p_c is the upper 100C % non-parametric confidence bound on a failure probability p when n units are tested and r failures occur where $0 \leq r < n$.

7.3.6 Let $F(t)$ be the cumulative exponential failure probability where t is the test time and θ is an objective MTTF (see 7.2). Note that we could use λ , θ , or a B_p life, converting one from the other. Use the relation $p_c = F(t)$.

$$p_c = 1 - e^{-\frac{t}{\theta}} \quad (18)$$

7.3.7 Eq 18 then involves the key variables of the test plan and may be solved or manipulated for any variable. It is noted that Eq 18 depends on the ratio of test time to the objective MTTF (t / θ). So long as that remains the same, the test plan will be the same. This further means that for any test plan, the test time will be proportional to the objective MTTF desired. For example, if $t / \theta = 0.1$ then we could use a test time of $t = 100$ to demonstrate $\theta = 1000$ or $t = 10$ to demonstrate $\theta = 100$.

7.3.8 Example 7—Sample size required; r , C , t , θ specified. An engineer wants to claim an MTTF of $\theta = 50\,000$ hours with 90 % confidence ($C = 0.90$). He is willing to allow up to $r = 5$ failures during the test and can run each of the samples for 500 hours. What sample size, n , is required?

7.3.8.1 First solve Eq 18 for p_c , using $\theta = 50\,000$ and $t = 500$. This results in $p_c = 0.00995$. Next, solve the beta function $G(p_c) = C$ targeting $C = 0.90$, using $r = 5$ held constant and adjusting n in the formula until $G(p_c) = 0.9$ results. It is useful to use a spreadsheet type of calculator in executing the calculation. The exhibit below shows several steps toward the solution. We want to solve $G(p_c) = 0.9$ for n using $C = 0.9$ and $p_c = 0.00995$.

Solution Exhibit for Example 7

n	$G(p_c) = C$
900	0.88278
910	0.88879
915	0.89169
920	0.89453
925	0.89730
926	0.89785
929	0.89948
930	0.90002

7.3.8.2 Here we see that $n = 930$ is the solution. This method may also be used if r is the unknown. In this case, n remains fixed and r is varied until Eq 16 is true.

7.3.9 Example 8—Number of failures (r) required; n , C , t , θ specified. Suppose in Example 7 that $n = 1200$ units can be tested for the increased time of $t = 600$. We want to determine

the maximum number of failures for the same confidence level, $C = 0.90$, and objective MTTF of 50 000.

7.3.9.1 Again, solve Eq 18 for p_c , using $\theta = 50\,000$ and $t = 600$. This result is $p_c = 0.011928$. Next, solve the beta function $G(p_c) = C$ targeting $C = 0.90$, using $n = 1200$ held constant and adjusting r in the formula until $G(p_c) = 0.9$ results.

Solution Exhibit for Example 8

r	$G(p_c)=C$
6	0.9886
7	0.9741
8	0.9479
9	0.9060
10	0.8458

7.3.9.2 Here we see that $r = 9$ is the solution.

7.3.10 To solve for the test time per unit required first use Eq 16 solving for p_c with an assumed confidence, C , sample size n , and number of failures allowed, r . Then using p_c and the objective MTBF, θ , solve Eq 18 for t . This results in:

$$t = -\theta \ln(1 - p_c) \quad (19)$$

7.3.11 Example 9—Test time required; n , r , C and θ specified. Suppose we want to demonstrate a B_1 life of 2500 cycles using 95 % confidence. We will use a sample size of $n = 120$ units and allow $r = 2$ failures. What test time should be used?

7.3.11.1 First, use Eq 13 with $p = 1$ and $B_1 = 2500$, solving for θ , and finding $\theta = 248\,748$. We simplify this by rounding up to 249 000. Next, use Eq 16 with $n = 120$, $r = 2$ and $C = 0.95$ finding $p_c = 0.051534$. Then use Eq 18, solving for t finding $t = 13\,174.4$ cycles.

7.3.12 Example 10—Confidence demonstrated required; n , r , t and θ specified. If 150 units on test have resulted in 3 failures at $t = 100$ cycles, what confidence can we claim in stating an MTTF requirement of 2500 cycles has been met?

7.3.12.1 Use Eq 17 directly with $p_c = 0.63212$. This is the failure probability in theory at $t=\theta$. Use the beta integral with parameters $r + 1$ and $n - r$ or 4 and 147 respectively. The beta integral evaluates to 0.8433 making the confidence demonstrated 84.3 %. Iterating on sample size, n , shows that $n = 169$ would be the requirement to state at least 90% confidence with $r = 3$ failures.

7.3.13 The Zero-Failure Case—Many organizations require zero failure test plans. In such a plan, a sample of n units are tested, where the test times may be different for different units tested, and zero failures is the requirement. To determine a plan, is to determine the total test time, T , required to demonstrate an MTTF of θ using confidence C . A total time T can be calculated using $nt = T$ where t is the time per unit tested in for each of n units. Sample size and test time can be interchanged as long as $nt = T$ is preserved. Further we can have variable test times as long as $T = \sum t$ is preserved.

7.3.13.1 The equation governing this case is:

$$T = -\theta \ln(1 - C) \quad (20)$$

7.3.13.2 In using Eq 20, T is the total time on test for all units tested. Once we have T we can decide on the number of test units and distribute the test time in any way that is convenient.

7.3.14 Example 11—Determine the total test time, T , required for a plan to demonstrate that the MTTF is at least