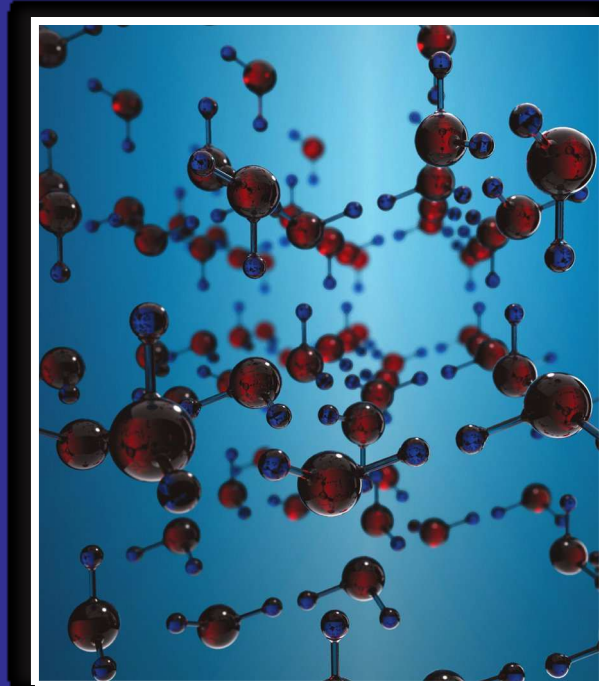


Principles and Practices of Water Supply Operations

BASIC SCIENCE CONCEPTS AND APPLICATIONS



Fourth Edition



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Basic Science Concepts and Applications

Fourth Edition



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Foreword

Basic Science Concepts and Applications is a supplementary reference book that accompanies the other four books in the series titled Principles and Practices of Water Supply Operations. The sections of the reference book—Mathematics, Hydraulics, Chemistry, and Electricity—provide extended discussions of the principles and operational calculations related to selected topics in the four books.

The other books in the series are

Water Sources

Water Treatment

Water Transmission and Distribution

Water Quality

Additionally, there are 13 videos associated with these five books. All are available at the AWWA Bookstore.

Acknowledgments

The fourth edition of *Basic Science Concepts and Applications* was revised by Nick Pizzi, Vice-President of Environmental Engineering and Technology. He also revised the third edition to include some additional graphing techniques as applicable to the Interim Enhanced Surface Water Treatment Rule (IESWTR), and some alternative horsepower calculations. A chlorine contact time ($C \times T$) calculation was added to the Dosage section of the text, and subsequent examples have been renumbered.

This edition maintains the traditions of this scientific text and adds updated materials especially in areas where Regulations have driven the need.

The second edition of *Basic Science Concepts and Applications* was revised to include metric examples, and material was updated where necessary. In addition, a student workbook and instructor guide were created for use with this reference book.

Technical review of the second edition is credited to Bill Hester, Denver Water, Denver, Colo.; Stephen E. Jones, Iowa State University, Ames, Iowa; Kenneth D. Kerri, California State University, Sacramento, Calif.; Gary B. Logsdon, Black & Veatch, Inc., Cincinnati, Ohio; Ken Morrison, M. M. Dillon Limited, London, Ontario; Dennis Mutti, M. M. Dillon Limited, London, Ontario; and Robert K. Weir, Denver Water, Denver, Colo. American Water Works Association staff who reviewed the manuscript included Brian Murphy, Frederick W. Pontius, and Steve Posavec.

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The electricity section was reprinted with minor changes from the first edition of AWWA Manual M2, *Automation and Instrumentation*, which was developed by the AWWA Distribution Division Committee on Automation and Instrumentation.

Portions of the text were adapted from *Mathematics for Water and Wastewater Treatment Plant Operators* (1973), published by Ann Arbor Science Publishers, Inc.

Introduction

Subject matter in this reference book is organized into four main sections—mathematics, hydraulics, chemistry, and electricity—supporting a broad range of topics in the other four books in the series. The chapters in each main section need not be read in the order in which they are presented. Where topics in other sections of this work are related and considered helpful to the reader, a footnote reference is given to the related discussion. A glossary at the end of the book defines important terms.

In general, calculations and answers are carried to two decimal places where the number is less than 999.99 (example: 27.12). Final zeros after the decimal point are not printed (example: 39.7, not 39.70). Numbers greater than 999.99 are given only to the nearest whole number (example: 1,718). Tables and graphs are read with whatever precision possible. These rules are adopted for consistency; they do not reflect the accuracy of the data used in the calculations. In practice, the operator should realize that the accuracy of a final answer can never be better than the accuracy of the data used.

Mathematics

Powers and Scientific Notation

Two common methods of expressing a number—powers notation and scientific notation—will be discussed in this chapter.

POWERS NOTATION

The most basic form of powers notation is merely a shorthand method of writing multiplication. For example, 5×5 can be written as

$$5^2$$

This is referred to as *5 to the second power*, or *5 squared*. The small 2 is the exponent, or power. It tells you how many 5s are to be multiplied together: two. In expanded form,

$$5^2 = (5)(5)$$

The expression *5 to the third power* (usually called *5 cubed*) is written as

$$5^3$$

In expanded form, this notation means

$$5^3 = (5)(5)(5)$$

The following examples further illustrate this concept of powers notation.

Example 1

How is the term 5^3 written in expanded form?

The power (or exponent) of 3 means that the number is multiplied by itself three times:

$$5^3 = (5)(5)(5)$$

Example 2

How is the term ft^2 written in expanded form?

The power or exponent of 2 means that the term is multiplied by itself two times:

$$\text{ft}^2 = (\text{ft})(\text{ft})$$

Example 3

How is the term 10^6 written in expanded form?

The exponent of 6 indicates that 10 is multiplied by itself six times:

$$10^6 = (10)(10)(10)(10)(10)(10)$$

Example 4

How is the term $(\frac{3}{4})^2$ written in expanded form?

When parentheses are used, the exponent refers to the entire term within the parentheses. Therefore, in this problem, $(\frac{3}{4})^2$ means

$$\left(\frac{3}{4}\right)^2 = \left(\frac{3}{4}\right)\left(\frac{3}{4}\right)$$

Sometimes a negative exponent is used with a number or term. A number with a negative exponent can be re-expressed using a positive exponent:

$$3^{-2} = \frac{1}{3^2}$$

Another example is

$$12^{-3} = \frac{1}{12^3}$$

Example 5

How is the term 7^{-2} written in expanded form?

$$7^{-2} = \frac{1}{7^2} = \frac{1}{(7)(7)}$$

Example 6

How is the term 10^{-5} written in expanded form?

$$10^{-5} = \frac{1}{10^5} = \frac{1}{(10)(10)(10)(10)(10)}$$

If a term is given in expanded form, you should be able to determine how it would be written in exponential (or power) form. For example,

$$(ft)(ft) = ft^2$$

or

$$(3)(3)(3)(3) = 3^4$$

Example 7

How would the following expanded term be rewritten in exponential form?

$$(mm)(mm)(mm)$$

Since the term is multiplied by itself three times, it would be written in exponential form as

$$mm^3$$

Example 8

Write the following term in exponential form:

$$\frac{(4)(4)(4)}{(5)(5)(5)(5)}$$

The exponent for the numerator of the fraction is 2 and the exponent for the denominator is 3. Therefore, the term would be written as

$$\frac{4^3}{5^4}$$

Since the exponents are not the same, parentheses cannot be placed around the fraction and a single exponent cannot be used.

Example 9

Write the following term in exponential form:

$$\frac{(ft)(ft)}{(in.)(in.)}$$

The exponent of both the numerator and denominator is 2:

$$\frac{\text{ft}^2}{\text{in.}^2}$$

Since the exponents are the same, parentheses can be used to express this term, if desired:

$$\left(\frac{\text{ft}}{\text{in.}} \right)^2$$

Perhaps the two most common situations in which you may see powers used with a number or term are in denoting area or volume units (in.^2 , ft^2 , in.^3 , ft^3) and in scientific notation.

SCIENTIFIC NOTATION

Scientific notation is a method by which any number can be expressed as a term multiplied by a power of 10. The term itself is greater than or equal to 1 but less than 10. Examples of numbers written in scientific notation are

$$5.4 \times 10^1$$

$$1.2 \times 10^3$$

$$9.789 \times 10^4$$

$$3.62 \times 10^{-2}$$

The numbers can be taken out of scientific notation by performing the indicated multiplication. For example,

$$5.4 \times 10^1 = (5.4)(10) = 54$$

$$1.2 \times 10^3 = (1.2)(10)(10)(10) = 1,200$$

$$9.789 \times 10^4 = (9.789)(10)(10)(10)(10) = 97,890$$

$$3.62 \times 10^{-2} = (3.62) \frac{1}{10^2} = (3.62) \frac{(1)}{(10)(10)} = \frac{3.62}{(10)(10)} = 0.0362$$

An easier way to take a number out of scientific notation is by moving the decimal point the number of places indicated by the exponent.

Rule 1

When a number is taken *out* of scientific notation, a *positive* exponent indicates a decimal point move to the *right*, and a *negative* exponent indicates a decimal point move to the *left*.

Let's look again at the examples above, using the decimal point move rather than the multiplication method. The first example is

$$5.4 \times 10^1$$

The *positive* exponent of 1 indicates that the decimal point in 5.4 should be moved one place to the *right*:

$$5.4 \rightarrow 54$$

The next example is

$$1.2 \times 10^3$$

The *positive* exponent of 3 indicates that the decimal point in 1.2 should be moved three places to the *right*:

$$1.200 \rightarrow 1,200$$

The next example is

$$9.789 \times 10^4$$

The *positive* exponent of 4 indicates that the decimal point should be moved four places to the *right*:

$$9.7890 \rightarrow 97,890$$

The final example is

$$3.62 \times 10^{-2}$$

The *negative* exponent of 2 indicates that the decimal point should be moved two places to the *left*:

$$\underbrace{03.62}_{\text{left}} = 0.0362$$

Example 10

Take the following number out of scientific notation:

$$7.992 \times 10^5$$

The positive exponent of 5 indicates that the decimal point should be moved five places to the right:

$$7.\underbrace{99200}_{\text{right}} = 799,200$$

Example 11

Take the following number out of scientific notation:

$$2.119 \times 10^{-3}$$

The negative exponent of 3 indicates that the decimal point should be moved three places to the left:

$$\underbrace{002.119}_{\text{left}} = 0.002119$$

Although there will be very few instances in which you will need to put a number *into* scientific notation, the method is discussed below.

To put a number into scientific notation, the decimal point is moved the number of places necessary to result in a number between 1 and 9. This number is multiplied by a power of 10, with the exponent equal to the number of places that the decimal point was

moved. (Remember that if no decimal point is shown in the number to be converted, it is assumed to be at the end of the number.)

Rule 2

When a number is put *into* scientific notation, a decimal point move to the *left* indicates a *positive* exponent; a decimal point move to the *right* indicates a *negative* exponent.

Now let's try converting a few numbers into scientific notation, using the same numbers as in the previous examples.

First, let's convert

54

To obtain a number between 1 and 9, the decimal point should be moved one place to the left. This move of one place gives the exponent, and the move to the *left* means that the exponent is *positive*:

$$\underbrace{54}_{\text{1 place}} = 5.4 \times 10^1$$

The next number to be put into scientific notation is

1,200

To obtain a number between 1 and 9, the decimal point should be moved three places to the left. The number of place moves (3) becomes the exponent of the power of 10, and the move to the *left* indicates a *positive* exponent:

$$\underbrace{1,200}_{\text{3 places}} = 1.2 \times 10^3$$

The next example is

97,890

To obtain a number between 1 and 9, the decimal point should be moved four places to the *left*, resulting in a *positive* exponent of 4:

$$\underbrace{97,890}_{\text{4 places}} = 9.789 \times 10^4$$

The final example is

$$0.0362$$

To obtain a number between 1 and 9, the decimal point must be moved two places to the right. This indicates an exponent of 2, and the move to the *right* requires a *negative* exponent:

$$\underbrace{0.0362}_{\text{move 2 places right}} = 3.62 \times 10^{-2}$$

Example 12

Put the following number into scientific notation:

$$4,573,000$$

To obtain a number between 1 and 9, the decimal point should be moved six places to the left, resulting in a positive exponent of 6:

$$\underbrace{4,573,000}_{\text{move 6 places left}} = 4.573 \times 10^6$$

Example 13

Convert the following decimal to scientific notation:

$$0.000375$$

To obtain a number between 1 and 9, the decimal point should be moved four places to the right, resulting in a negative exponent of 4:

$$\underbrace{0.000375}_{\text{move 4 places right}} = 3.75 \times 10^{-4}$$

Dimensional Analysis

Dimensional analysis is a tool that you can use to determine whether you have set up a problem correctly. In checking a math setup using dimensional analysis, you work only with the dimensions or units of measure and not with the numbers themselves.

To use the dimensional analysis method, you must know three things:

- how to express a horizontal fraction (such as gal/ft^3) as a vertical fraction (such as $\frac{\text{gal}}{\text{ft}^3}$)
- how to divide by a fraction
- how to divide out or cancel terms in the numerator and denominator of a fraction

These techniques are reviewed briefly below.

When you are using dimensional analysis to check a problem, it is often desirable to write any horizontal fractions as vertical fractions, thus

$$\text{ft}^3/\text{min} = \frac{\text{ft}^3}{\text{min}}$$

$$\text{s}/\text{min} = \frac{\text{s}}{\text{min}}$$

$$\frac{\text{gal}}{\text{min}} \div \frac{\text{gal}}{\text{ft}^3} = \frac{\text{gal}}{\text{min}} \times \frac{\text{ft}^3}{\text{gal}}$$

When a problem involves division by a fraction, the rule is to invert (or turn over) the terms in the denominator and then multiply. For example,

$$\frac{\text{gal}}{\text{min}} \div \frac{\text{gal}}{\text{ft}^3} = \frac{\text{gal}}{\text{min}} \times \frac{\text{ft}^3}{\text{gal}}$$

or

$$\frac{\text{lb}}{\text{min}} \div \frac{\text{d}}{\text{min}} = \frac{\text{lb}}{\text{min}} \times \frac{\text{min}}{\text{d}}$$

or

$$\frac{\text{mm}^2}{\text{mm}^2} \div \frac{\text{m}^2}{\text{mm}^2} = \frac{\text{mm}^2}{\text{mm}^2} \times \frac{\text{mm}^2}{\text{m}^2}$$

Once the fractions in a problem, if any, have been rewritten in the vertical form, and division by a fraction has been re-expressed as multiplication as shown above, then the terms can be divided out or canceled. For every term canceled in the numerator of a fraction, a similar term must be canceled in the denominator, and vice versa, as shown below

$$\frac{\cancel{\text{gal}}}{\text{min}} = \frac{\text{ft}^3}{\cancel{\text{gal}}} = \frac{\text{ft}^3}{\text{min}}$$

$$\frac{\text{kg}}{\cancel{\text{d}}} = \frac{\cancel{\text{d}}}{\text{min}} = \frac{\text{kg}}{\text{min}}$$

$$\frac{\cancel{\text{mm}}^2}{\cancel{\text{mm}}^2} = \frac{\text{m}^2}{\cancel{\text{mm}}^2} = \text{m}^2$$

$$\frac{\cancel{\text{ft}}^3}{\cancel{\text{s}}} = \frac{\text{gal}}{\cancel{\text{ft}}^3} = \frac{\cancel{\text{s}}}{\cancel{\text{min}}} = \frac{\cancel{\text{min}}}{\text{d}} = \frac{\text{gal}}{\text{d}}$$

You may wish to review the concept of powers* before continuing with the following examples in dimensional analysis.

Suppose you wish to convert 2,500 ft³ volume to gallons, and suppose that you know you will use 7.48 gal/ft³ in the conversion but that you don't know whether to multiply or divide by 7.48. Let's look at both possible ways and see how dimensional analysis can be used to choose the correct way. *Only the dimensions* will be used to determine if the math setup is correct.

* Mathematics 1, Powers and Scientific Notation.

First, try multiplying the dimensions:

$$(\text{ft}^3)(\text{gal}/\text{ft}^3) = (\text{ft}^3)\left(\frac{\text{gal}}{\text{ft}^3}\right)$$

Then multiply the numerators and denominators to get

$$= \frac{(\text{ft}^3)(\text{gal})}{\text{ft}^3}$$

and cancel common terms to get

$$\begin{aligned} &= \frac{(\cancel{\text{ft}^3})(\text{gal})}{\cancel{\text{ft}^3}} \\ &= \text{gal} \end{aligned}$$

So, by dimensional analysis you know that if you *multiply* the two dimensions (ft^3 and gal/ft^3), the answer you get will be in gallons, which is what you want. Therefore, since the math setup is correct, you would then multiply the numbers to obtain the number of gallons.

$$(2,500 \text{ ft}^3)(7.48 \text{ gal}/\text{ft}^3) = 18,700 \text{ gal}$$

What would have happened if you had divided the dimensions instead of multiplying?

$$\begin{aligned} \frac{\text{ft}^3}{\text{gal}/\text{ft}^3} &= \frac{\text{ft}^3}{\frac{\text{gal}}{\text{ft}^3}} \\ &= \text{ft}^3\left(\frac{\text{ft}^3}{\text{gal}}\right) \end{aligned}$$

Then multiply the numerators and denominators of the fraction to get

$$= \frac{\text{ft}^6}{\text{gal}}$$

So had you *divided* the two dimensions (ft^3 and gal/ft^3), the units of the answer would have been ft^6/gal , *not* gal. Clearly you do not want to divide in making this conversion.

Example 1

You wish to obtain an answer in square feet. If you are given the two terms— $30 \text{ ft}^3/\text{s}$ and $2.5 \text{ ft}/\text{s}$ —is the following math setup correct?

$$(30 \text{ ft}^3/\text{s})(2.5 \text{ ft}/\text{s})$$

First, only the dimensions are used to determine if the math setup is correct. By multiplying the two dimensions, you get

$$(\text{ft}^3/\text{s})(\text{ft}/\text{s}) = \left(\frac{\text{ft}^3}{\text{s}} \right) \left(\frac{\text{ft}}{\text{s}} \right)$$

Then multiply the terms in the numerators and denominators of the fraction:

$$\begin{aligned} &= \frac{(\text{ft}^3)(\text{ft})}{(\text{s})(\text{s})} \\ &= \frac{\text{ft}^4}{\text{s}^2} \end{aligned}$$

The math setup is wrong since the dimensions of the answer are not square feet. Therefore, if you multiply the numbers just as you did the dimensions, the answer will be wrong.

Let's try division of the two dimensions instead.

$$\frac{\text{ft}^3/\text{s}}{\text{ft}/\text{s}} = \frac{\frac{\text{ft}^3}{\text{s}}}{\frac{\text{ft}}{\text{s}}}$$

Then invert the denominator and multiply to get

$$= \left(\frac{\text{ft}^3}{\text{s}} \right) \left(\frac{\text{s}}{\text{ft}} \right)$$

$$\begin{aligned}
 &= \frac{(\text{ft})(\text{ft})(\text{ft})(\text{s})}{(\text{s})(\text{ft})} \\
 &= \frac{(\text{ft})(\text{ft})(\cancel{\text{ft}})(\cancel{\text{s}})}{(\cancel{\text{s}})(\cancel{\text{ft}})} \\
 &= \text{ft}^2
 \end{aligned}$$

This math setup is correct since the dimensions of the answer are square feet. Therefore, if you *divide* the numbers as you did the units, the answer will also be correct:

$$\frac{30 \text{ ft}^3}{2.5 \text{ ft/s}} = 12 \text{ ft}^2$$

Example 2

Suppose you have two terms—4 m/s and 5 m²—and you wish to obtain an answer in cubic meters per second (m³/s). Is multiplying the two terms the correct math setup?

$$(\text{m/s})(\text{m}^2) = \frac{\text{m}}{\text{s}} \times \text{m}^2$$

Then multiply the numerators and denominators of the fraction:

$$\begin{aligned}
 &= \frac{(\text{m})(\text{m}^2)}{\text{s}} \\
 &= \frac{\text{m}^3}{\text{s}}
 \end{aligned}$$

The math setup is correct since the dimensions of the answer are cubic meters per second (m³/s). So if you multiply the numbers, just as you did the dimensions, you will get the correct answer:

$$(4 \text{ m/s})(5 \text{ m}^2) = 20 \text{ m}^3/\text{s}$$

Example 3

Suppose you have been given the following problem: “The flow rate in a water line is 2.3 ft³/s. What is the flow rate expressed as gallons per minute?” You then set up the math problem as shown below. Use dimensional analysis to determine if this math setup is correct.

$$(2.8 \text{ ft}^3/\text{s})(7.48 \text{ gal}/\text{ft}^3)(60 \text{ s}/\text{min})$$

Dimensional analysis is used to check the math setup:

$$\begin{aligned}(\text{ft}^3/\text{s})(\text{gal}/\text{ft}^3)(\text{s}/\text{min}) &= \left(\frac{\text{ft}^3}{\text{s}} \right) \left(\frac{\text{gal}}{\text{ft}^3} \right) \left(\frac{\text{s}}{\text{min}} \right) \\&= \left(\frac{\cancel{\text{ft}^3}}{\cancel{\text{s}}} \right) \left(\frac{\text{gal}}{\cancel{\text{ft}^3}} \right) \left(\frac{\cancel{\text{s}}}{\text{min}} \right) \\&= \frac{\text{gal}}{\text{min}}\end{aligned}$$

This analysis indicates that the math setup is correct as shown above.

Example 4

You have been given the following problem: “A channel is 5 ft wide with water flowing to a depth of 25 ft. The velocity in the channel is found to be 1.8 ft/s. What is the flow rate in the channel in cubic feet per second?” You then set up the math problem as shown below. Use dimensional analysis to determine if this math setup is correct.

$$\frac{(5 \text{ ft})(25 \text{ ft})}{1.8 \text{ ft}/\text{s}}$$

Dimensional analysis is used to check the math setup:

$$\begin{aligned}\frac{(\text{ft})(\text{ft})}{\text{ft}/\text{s}} &= \frac{(\text{ft})(\text{ft})}{\frac{\text{ft}}{\text{s}}} \\&= (\text{ft})(\text{ft}) \frac{\text{s}}{\text{ft}} \\&= \frac{(\text{ft})(\text{ft})(\text{s})}{\text{ft}} \\&= \frac{\cancel{\text{ft}}(\text{ft})(\text{s})}{\cancel{\text{ft}}} \\&= \text{ft s}\end{aligned}$$

Since the dimensions of the answer are incorrect, the math setup shown in example 4 is also incorrect. Had the math setup instead been $(5 \text{ ft})(25 \text{ ft})(1.8 \text{ ft/s})$, the dimensions of the answer would have been correct.

Rounding and Estimating

The practice of estimating the size of the answer to a calculation helps you to avoid reporting an answer as 5,000, for example, when it should be 50,000, or 40,000 when it should be 4,000,000. Suppose when you are solving a problem that you forget to multiply or divide by a certain number, or suppose you punch a wrong button on the calculator. If you have an idea of what the approximate answer should be, then you can recognize an incorrect answer and recheck your arithmetic.

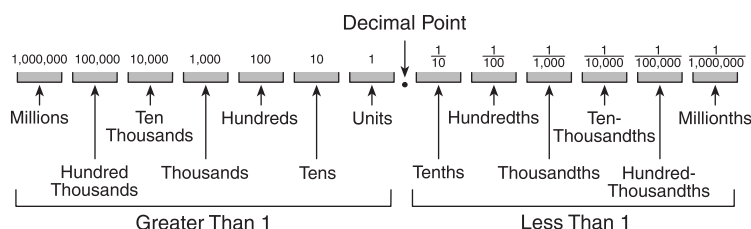
In general, the process of estimating the size of an answer involves two steps:

- rounding the numbers
- completing the calculation using the rounded numbers

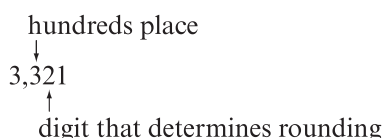
Rounding means replacing the final digits of a number with zeros, thus expressing the number as tens, hundreds, thousands, or tenths, hundredths, thousandths, etc. (for example, expressing 398 as 400; 0.19 as 0.2; or 5,825,393 as 6,000,000). Equations using rounded numbers are often written with an approximate sign ($\approx$) instead of an equals sign ($=$) to show that the answer is an estimate.

ROUNDING

The technique of rounding numbers is based on a particular place value in the decimal system. The various place values are reviewed below to emphasize the importance of understanding this concept before proceeding further.



Suppose you want to round the number 3,321 to the nearest hundred. Rounding this number depends only on the *size of the digit just to the right of the hundreds place*:



Similarly, if you want to round the number 323,772 to the nearest thousand, this rounding depends only on the *size of the digit just to the right* of the thousands place:

thousands place
↓
323,772
↑
digit that determines rounding

Now consider the rules for *rounding whole numbers*. The procedure depends on whether the digit just to the right of the rounding place is less than 5, is 5, or is greater than 5.

Rule 1

If the digit is less than 5: When rounding to any desired place, if the digit to the right of that place is less than 5, replace all digits to the right of the rounding place with zeros.

Rule 2

If the digit is 5 or greater than 5: When rounding to any desired place, if the digit to the right of that place is 5 or greater than 5, increase the digit in the rounding place by 1 and replace all digits to the right of the increase with zeros.

The best way to understand rounding rules is to look at a few examples of rounding.

Example 1

Round 37,926 to the nearest hundred.

The procedure used in this rounding depends on the digit just to the right of the hundreds place:

37,926
↑
hundreds place

Since the digit to the right of the hundreds place is less than 5, the 9 is not changed and all the digits to the right of the 9 are replaced with zeros:

$$37,926 \approx 37,900 \text{ (rounded to the nearest hundred)}$$

Example 2

Round 351,655 to the nearest thousand.

The procedure used depends on the digit just to the right of the thousands place:

351,655
↑
thousands place

Since the digit to the right of the thousands place is greater than 5, the 1 in the thousands place is increased by 1 and all the digits to the right of the 1 are replaced with zeros:

$$351,655 \approx 352,000 \text{ (rounded to the nearest thousand)}$$

Example 3

Round 25,675 to the nearest ten thousand.

The digit just to the right of the ten-thousands place determines how the number is rounded:

25,675
↑
ten-thousands place

Since the digit to the right of the ten-thousands place is 5, the 2 is increased by 1 and all digits to the right of the 2 are replaced with zeros:

$$25,675 \approx 30,000 \text{ (rounded to the nearest ten thousand)}$$

Example 4

Round 14,974 to the nearest hundred.

The digit to the right of the hundreds place determines how the number is rounded:

14,974
↑
hundreds place

Since the digit to the right of the hundreds place is 7, the 9 should be increased by 1 and all digits to the right replaced with zeros. But notice that increasing the 9 by 1 changes it to 10. The zero in the 10 replaces the 9, and the 1 is carried and added to the 4 in the thousands place:

$$14,974 \approx 15,000 \text{ (rounded to the nearest hundred)}$$

The rules for rounding decimal numbers are basically the same as those for rounding whole numbers, with the following modification:

Rule 3

When rounding decimal numbers to a place to the right of the decimal point, instead of replacing the rounded digits with zeros, drop the rounded digits.

The following examples illustrate this procedure.

Example 5

Round 25.17239 to the nearest thousandth.

As in rounding whole numbers, the procedure used depends on the size of the digit just to the right of the rounding place:

25.17239
 ↑
 thousandths place

According to the basic rounding rules, since the digit to the right of the thousandths place is less than 5, the 2 remains unchanged and all digits to the right of the 2 are dropped:

$$25.17239 \approx 25.172 \text{ (rounded to the nearest thousandth)}$$

Example 6

Round 4.271 to the nearest tenth.

4.271
 ↑
 tenths place

The digit to the right of the tenths place is 7. Therefore, the 2 is increased by 1 and all digits to the right are dropped:

$$4.271 \approx 4.3 \text{ (rounded to the nearest tenth)}$$

Example 7

Round 483.16 to units.

483.16
 ↑
 units place

Even though the decimal point falls between the rounding place and the digit just to the right, the procedure still follows the basic rounding rules. The digit to the right of the units place is less than 5; therefore it (and all the digits after it) should be replaced with zeros. But, rounded digits on the right side of the decimal point are dropped. Therefore, the decimal number is rounded to a whole number.

$$483.16 \approx 483 \text{ (rounded to the nearest unit)}$$

ESTIMATING

When estimating the answer to a problem, you will find it helpful to round each number in the calculation so that only one digit remains, with the rest of the digits in the number either changed to zeros or dropped in accordance with the rounding rules. In this way the estimation can often be done in your head, or at least with a minimum of computing with pencil or calculator.

However, you should be aware that the more rounding you do in an estimation, the greater the difference may be between the digits of your estimated answer and those of the actual answer. Keep this principle in mind:

Rounding and estimating indicate the approximate size (place value) of a calculated answer but do not necessarily indicate the numerical value of the answer.

For instance, if the estimate to a problem is 40,000, you can expect the calculated answer to be in the tens of thousands (not hundreds or millions); but the value of the calculated answer could fall between 10,000 and 90,000.

Also consider what the place value of your answer could be if the digit in the estimated answer is 1 or 9. With an estimate of 900, the actual answer will probably be in the hundreds (somewhere between 100 and 900)—but it could also be a little more than 900 (such as 1,020). On the other hand, if the estimate is 1,000, then the calculated answer could be in the thousands, or a little less than 1,000 (in the high hundreds).

Now let's look at the mechanics of estimating. Suppose you have rounded two numbers so that your estimate of the answer is 40×50 . In making this estimate, first multiply the two digits (4 and 5):

$$\begin{array}{cc} (40) & (50) \\ \downarrow & \downarrow \\ 4 \times 5 & = 20 \end{array}$$

Then count all the zeros in the calculation and put that number of zeros after the 20:

$$\begin{array}{ccc} (40) & (50) & = 2000 \\ \downarrow & \downarrow & \downarrow \downarrow \\ 2 \text{ zeros} & & 2 \text{ zeros} \end{array}$$

The estimated answer is 2000, so the actual answer should be in the thousands.

Let's look at a few more examples of this procedure.

Example 8

A calculation has been rounded so that it can be estimated. What is the estimated answer for the calculation?

$$(600)(3,000)$$

First the 6 and the 3 are multiplied, then the total number of zeros are added:

$$(600)(3,000) = \begin{array}{c} \boxed{18} \\ | \\ 6 \times 3 \end{array} \quad \begin{array}{c} \boxed{00000} \\ | \\ 5 \text{ zeros} \end{array}$$

Therefore, the estimated answer for the calculation is 1,800,000; the actual answer should be in the millions.

Example 9

A calculation has been rounded to the numbers shown below. What is the estimated answer for the calculation?

$$(400)(70,000)$$

First the 4 and 7 are multiplied, then the total number of zeros are added:

$$(400)(70,000) = \begin{array}{c} \boxed{28} \\ | \\ 4 \times 7 \end{array} \quad \begin{array}{c} \boxed{000000} \\ | \\ 6 \text{ zeros} \end{array}$$

The estimated answer is therefore 28,000,000; the actual answer should be in the ten millions.

Example 10

What is the estimated answer for the following rounded calculation?

$$(20)(400)(30)$$

As in the two preceding examples, the 2, 4, and 3 are multiplied and then the total number of zeros are added:

$$(20)(400)(30) = \begin{array}{c} \boxed{24} \\ | \\ 2 \times 4 \times 3 \end{array} \quad \begin{array}{c} \boxed{0000} \\ | \\ 4 \text{ zeros} \end{array}$$

The estimated answer is therefore 240,000; the actual answer should be in the hundred thousands.

When three or more digits must be multiplied, rounding is sometimes required during the multiplication process. In the last example, the three digits could be multiplied easily, so no rounding was needed. Suppose, however, that you wanted to estimate the answer to

$$(800)(7,000)(800)$$

In this case the multiplication of the digits is not quite so obvious. Therefore, rounding during multiplication is permissible:

$$\begin{array}{ccc} (80)(7,000)(800) & \gg & \begin{array}{cc} 480 & 000000 \\ \downarrow & \downarrow \\ \boxed{60 \times 8} & \boxed{6 \text{ zeros}} \end{array} \\ \begin{array}{c} \downarrow \downarrow \\ \boxed{8 \times 7} \\ \downarrow \\ 56 \\ \downarrow \\ 60 \times 8 \end{array} & & \end{array}$$

The following two examples illustrate the technique of rounding during multiplication of digits.

Example 11

The estimate of a calculation is shown below. What is the estimated answer?

$$(50)(50)(3,000)$$

As in the other examples, the 5, 5, and 3 are multiplied; then the zeros are added:

$$\begin{array}{ccc} (50)(50)(3,000) & \gg & \begin{array}{cc} 90 & 00000 \\ \downarrow & \downarrow \\ \boxed{30 \times 3} & \boxed{5 \text{ zeros}} \end{array} \\ \begin{array}{c} \downarrow \downarrow \\ \boxed{5 \times 5} \\ \downarrow \\ 25 \\ \downarrow \\ 30 \times 3 \end{array} & & \end{array}$$

The estimated answer is therefore 9,000,000; the actual answer should be in the millions or ten millions.

Example 12

What is the estimated answer to the following calculation?

$$(9)(700)(60)(70)$$

First the 9, 7, 6, and 7 are multiplied; then the zeros are added. This problem requires more rounding during the multiplication step than was required in the previous examples:

$$\begin{array}{rcl}
 (9) & (700) & (60)(70) \\
 \downarrow & \downarrow & \downarrow \downarrow \\
 \boxed{9 \times 7} & & \boxed{60 \times 6} \\
 \downarrow & & \downarrow \\
 63 & & 360 \\
 \downarrow & & \downarrow \\
 \boxed{60 \times 6} & & 400 \times 7 \\
 \downarrow & & \downarrow \\
 360 & & 2800
 \end{array}
 \quad \gg \quad
 \begin{array}{r}
 2,800 \\
 \boxed{400 \times 7}
 \end{array}
 \quad
 \begin{array}{r}
 0000 \\
 \boxed{4 \text{ zeros}}
 \end{array}$$

The estimated answer is 28,000,000; so the actual answer should be in the ten millions.

Sometimes the calculation being estimated involves division. When this occurs, the estimate can often be simplified by dividing out (or crossing off) final zeros; that is, zeros at the end of a whole number. The next few examples illustrate this technique.

Example 13

What is the estimated answer to the following calculation?

$$\frac{50,000}{400}$$

In this problem the estimate may first be simplified. When a final zero is crossed off in the denominator (bottom) of the fraction, a final zero must be crossed off in the numerator (top) of the fraction. The simplified estimate is therefore

$$\frac{50,0\cancel{00}}{4\cancel{00}} = \frac{500}{4}$$

Then the estimate can be completed:

$$\frac{500}{4} = 125$$

Next, let's consider problems where the *estimated answer is a decimal less than 1*. For estimating purposes, it is necessary to obtain only the first nonzero digit to the right of the decimal point. If an estimated answer is 0.01, for example, the actual answer could be 0.014, or 0.02, or even 0.009. The estimate of 0.01 merely shows that the answer will probably be in the *low* hundredths (although it could be in the high thousandths). The point is that the estimate should give you a “ballpark” answer, but *you must take particular care in determining where to place the decimal point* in the calculated answer.

Example 14

Complete the following estimate:

$$\begin{array}{r} 900 \\ 7,000 \end{array}$$

First the estimate is simplified by crossing out final zeros, and then the problem is completed by division. As previously stated, for estimated decimal answers less than 1, it is necessary to obtain only the first nonzero digit to the right of the decimal point.

$$\begin{array}{r} 9\cancel{00} \\ 7,00\cancel{0} \end{array} = \frac{9}{70}$$

$$\begin{array}{r} .1 \\ 70 \overline{) 9.0} \\ \underline{7.0} \end{array}$$

Therefore, the estimated answer is 0.1; the actual answer should be in the tenths (0.1 to 0.9).

Example 15

The estimate of a calculation is shown below. What is the estimated answer?

$$\begin{array}{r} 2,000 \\ 7,000,000 \end{array}$$

This estimate is first simplified by crossing out final zeros:

$$\begin{array}{r} 2,\cancel{000} \\ 7,000,\cancel{000} \end{array} = \frac{2}{7,000}$$

Then the problem is completed by division. As in the last example, it is necessary to obtain only the first nonzero digit (after the required number of zeros) to the right of the decimal point.

$$\begin{array}{r} .0002 \\ 7,000 \overline{) 2.0000} \\ \underline{1.4000} \end{array}$$

Therefore, the answer to this estimate is 0.0002; the actual answer should be in the ten thousandths.

In many practical applications involving rounding and estimating, the problem requires both multiplication and division. When this is the case, first simplify as much as possible by crossing out zeros. Next, multiply the numbers in the numerator, then those in the denominator. Finally, complete the problem by division. The next two examples illustrate this technique.

Example 16

Calculate the estimated answer to

$$\frac{(20)(400)}{(50)(80)}$$

First, the problem is simplified by crossing out final zeros where possible:

$$\frac{(20)(\cancel{400})}{(50)(\cancel{80})} = \frac{(20)(4)}{(5)(8)}$$

The numerator and denominator are each multiplied:

$$\frac{(20)(4)}{(5)(8)} = \frac{80}{40}$$

Additional zeros may be crossed out and the problem completed by division:

$$\begin{aligned} \frac{(\cancel{80})}{(\cancel{40})} &= \frac{8}{4} \\ &= 2 \end{aligned}$$

The estimated answer is 2, so the actual answer should be in the units.

Example 17

Calculate the answer to the following estimated problem:

$$\frac{(4,000)(40)}{(60)(3)}$$

First, the problem is simplified by crossing out zeros:

$$\frac{(\cancel{4,000})(40)}{(\cancel{60})(3)} = \frac{(400)(40)}{(6)(3)}$$

The numerator and denominator are each multiplied:

$$\frac{(400)(40)}{(6)(3)} = \frac{16,000}{18}$$

Before the problem is completed by division, the numbers in the fraction can be rounded and additional zeros crossed out:

$$\begin{aligned} \frac{16,000}{18} &\approx \frac{20,000}{20} \\ &\approx \frac{2,000}{2} \\ &\approx 1,000 \end{aligned}$$

The estimated answer is therefore 1,000, and the actual answer should be in the thousands or high hundreds.

In examples 16 and 17, the numbers were already in rounded form. In practical applications, however, you must round the numbers in the problem before continuing with the estimate. And as noted at the beginning of this section, the more rounding you do, the greater the difference may be between the digits of your estimated answer and the actual answer. The estimated answer will show you what the approximate place value of the calculated answer should be, but the numerical values of the two answers may vary considerably.

Let's look at a few examples that require you to round the numbers before estimating. In each case the estimate is calculated according to the rules given in this section. Then the actual answer, based on the numbers before rounding, is given to show the comparison between the estimated and actual answers.

Example 18

Estimate the answer to the following problem:

$$\frac{(29)(244)}{(12)(32)}$$

First, each term in the calculation is rounded:

$$\frac{(29)(244)}{(12)(32)} \approx \frac{(30)(200)}{(10)(30)}$$

The estimate can now be completed as in the previous examples. The problem is first simplified by crossing out zeros; then the numerator and denominator are each multiplied, and finally the estimated answer is obtained by division:

$$\begin{aligned}\frac{(3\cancel{0})(2\cancel{0}\cancel{0})}{(1\cancel{0})(3\cancel{0})} &= \frac{60}{3} \\ &= 20\end{aligned}$$

The estimated answer is 20; the actual answer is 18.43.

Example 19

Estimate the answer to the following problem:

$$\frac{(430)(61,702)}{(65)(65)}$$

Each term in the calculation is first rounded; then the estimation is completed as in previous examples.

$$\begin{aligned}\frac{(430)(61,702)}{(65)(65)} &\approx \frac{(4\cancel{0}\cancel{0})(60,000)}{(7\cancel{0})(7\cancel{0})} \\ &\approx \frac{240,000}{49}\end{aligned}$$

Then round the terms before division, cancel zeros, and complete the estimate:

$$\begin{aligned} &\approx \frac{200,000}{50} \\ &\approx 4,000 \end{aligned}$$

The estimated answer is 4,000; the actual answer is 6,280.

Example 20

Estimate the answer to the following problem:

$$\frac{(291)(34)(419)}{(21)(169)}$$

First, each term is rounded and the problem is simplified by crossing out final zeros. Then the numerator and denominator are each multiplied; finally, the answer is obtained by division.

$$\begin{aligned} \frac{(291)(34)(419)}{(21)(169)} &\approx \frac{(300)(30)(400)}{(20)(200)} \\ &\approx \frac{(3)(3)(400)}{(2)(2)} \\ &\approx \frac{3,600}{4} \\ &\approx 900 \end{aligned}$$

The estimated answer is 900; the actual answer is 1,168. This problem shows how the place value of the estimated and actual answers can vary.

Up to this point, we have been considering estimates using primarily rounded *whole* numbers. In some practical problems, however, you will be working with decimals less than 1 or with combinations of whole numbers and decimals less than 1. The rounding rules for decimals have already been illustrated. But in performing the necessary multiplication and division steps in an estimate, you may have to do some of the arithmetic in longhand in order to determine the position of the decimal point (and therefore the place value) of the answer.

Suppose, for example, you must round and estimate the following calculation:

$$\frac{(2.375)(4.75)(247)}{(61)(92)(0.785)}$$

First, round each term to one digit (plus the necessary number of zeros); then cancel the final zeros where possible:

$$\frac{(2.375)(4.75)(247)}{(61)(92)(0.785)} \approx \frac{(2)(5)(200)}{(60)(90)(0.8)}$$

Next, multiply the terms in the numerator, then those in the denominator. Round and cancel final zeros again.

$$\begin{aligned}\frac{(2)(5)(2)}{(6)(9)(0.8)} &\approx \frac{20}{(54)(0.8)} \\ &\approx \frac{20}{(50)(0.8)} \\ &\approx \frac{2}{4}\end{aligned}$$

Finally, divide to obtain the estimated answer:

$$\begin{array}{r} .5 \\ 4 \overline{) 2.0} \end{array}$$

The estimated answer is 0.5; the actual answer is 0.63.

Now consider another example:

$$\frac{(271)(32)(0.06725)}{(725)(0.0024)}$$

Round each term to one digit plus the required number of zeros:

$$\frac{(300)(30)(0.07)}{(700)(0.002)}$$

Now cancel the final zeros of whole numbers, as possible. Note that *the only zeros that can be canceled are the final zeros of whole numbers*.

$$\frac{(300)(30)(0.07)}{(700)(0.002)} = \frac{(3)(30)(0.07)}{(7)(0.002)}$$

Next, multiply the numerator, then the denominator. You will probably want to do the arithmetic in longhand to place the decimal points properly.

$$\begin{aligned}
 &= \frac{(90)(0.07)}{(7)(0.002)} \\
 &= \frac{6.3}{0.014}
 \end{aligned}$$

Round each term again, then divide to complete the estimate:

$$\begin{aligned}
 \frac{6.3}{0.014} &\approx \frac{6}{0.01} \\
 &= \frac{600.}{0.01} \approx 600.
 \end{aligned}$$

The estimated answer is 600; the actual answer is 335.17.

As you can see from these two examples, estimates involving decimal numbers less than 1 can be time consuming, and if you misplace the decimal point, your answer could be wrong by one or more place values. You need to be particularly careful in estimating this type of problem because the possibility of error greatly increases.

An easier way to estimate problems that have decimal numbers less than 1 is to put all the rounded numbers of the estimate into scientific notation. The rules for converting numbers to and from scientific notation are given in another section* and are not discussed here. However, there are some basic rules for multiplying and dividing numbers in scientific notation that you must know.

Rule 1

When you *multiply* the numbers in scientific notation, multiply the numbers but *add the exponents*.

Rule 2

When you *divide* in scientific notation, divide the numbers but *subtract the exponent* in the denominator from the exponent in the numerator.

First let's look at some examples that demonstrate the multiplication rule.

* Mathematics 1, Powers and Scientific Notation.

Example 21

Calculate the following:

$$(30)(600)$$

You can see immediately that the answer is 18,000. But, for practice, put the calculation into scientific notation:

$$(30)(600) = (3 \times 10^1)(6 \times 10^2)$$

Now multiply the numbers, then add the exponents:

$$\begin{aligned}(3 \times 10^1)(6 \times 10^2) &= (3)(6) \times 10^{1+2} \\ &= 18 \times 10^3\end{aligned}$$

Finally, take the answer out of scientific notation by moving the decimal point. The *positive* exponent of 3 indicates that the decimal point should be moved three places to the *right*.

$$18.\underline{000} = 18,000$$

Example 22

Multiply the following whole number and decimal, using scientific notation:

$$(0.0003)(300)$$

In scientific notation this becomes

$$(0.0003)(300) = (3 \times 10^{-4})(3 \times 10^2)$$

Now multiply the numbers and add the exponents:

$$\begin{aligned}(3 \times 10^{-4})(3 \times 10^2) &= (3)(3) \times 10^{-4+2} \\ &= 9 \times 10^{-2}\end{aligned}$$

Finally, take the answer out of scientific notation by moving the decimal point. The *negative* exponent of 2 indicates a decimal point move two places to the *left*.

$$\underline{09.} = 0.09$$

The next example shows how to multiply three decimals and not lose track of the decimal point.

Example 23

Calculate the following, using scientific notation:

$$(0.003)(0.2)(0.0006)$$

First, convert to scientific notation; then multiply the numbers and add the exponents:

$$\begin{aligned}(0.003)(0.2)(0.0006) &= (3 \times 10^{-3})(2 \times 10^{-1})(6 \times 10^{-4}) \\ &= (3)(2)(6) \times 10^{-3-1-4} \\ &= 36 \times 10^{-8}\end{aligned}$$

Finally, take the answer out of scientific notation by moving the decimal point. The *negative* exponent of 8 indicates a decimal point move of eight places to the *left*.

$$\underline{00000036.} = 0.00000036$$

For estimating purposes, a *whole* number between 1 and 9 need not be converted to scientific notation. It would merely be the same as multiplying the number by 10^0 , which is simply 1. Consider how this can happen:

$$\begin{aligned}2 \times 10^2 &\text{ means } (2)(10)(10) \text{ or } 200 \\ 2 \times 10^1 &\text{ means } (2)(10) \text{ or } 20 \\ 2 \times 10^0 &\text{ means } (2)[\text{not multiplied by } 10] \text{ or } 2\end{aligned}$$

However, in using scientific notation to perform a calculation, you may sometimes get an answer where the positive and negative exponents cancel each other and the answer becomes a number multiplied by 10^0 . How do you take such a number out of scientific notation? Since 10^0 means that the number is *not* raised to a power of 10, the answer is, in effect, already out of scientific notation, and the number stands “as is.” Another way to state this is that any number times 10^0 equals the number itself.

Let’s consider an example:

$$(0.002)(3,000)$$