

CONTROL CHART TECHNIQUE WHEN MANUFACTURING TO A SPECIFICATION

by

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**Control chart technique when manufacturing
to a specification**

In consequence of the withdrawal of B.S. 600R, Quality control charts, a few modifications to B.S. 2564 have become necessary; these, together with corrections of some small errors in B.S. 2564, are given in the list below. An appropriate revision of the Foreword and the Index to B.S. 2564 will be made when the standard is next reprinted.

Corrigenda

Part One. Introduction. Insert ' B.S. 600 or B.S. 600R ' for ' B.S. 600R '.

Page 9. Insert w for $\bar{w}$ in subsection (iv).

Page 10. Insert w for $\bar{w}$ in the penultimate line.

Insert $\bar{w}$ for $\bar{w}$ in all other places where $\bar{w}$ occurs on page 10.

Pages 11 et seq. Insert $\bar{w}$ for $\bar{w}$ wherever the latter occurs, with the exception of w for $\bar{w}$ in the heading of the Control Chart for Ranges, Fig. 1, page 12, and of Table 3, page 60.

Page 13. In the third column of the table, amend the values to read as follows:

4.12, 2.99, 2.58, 2.36 and 2.22

Page 33. Omit the phrase in parenthesis in line 6 and 7.

Page 59. In the text immediately preceding the table, insert ' σ may be estimated from ' for ' Standard Deviation $\sigma =$ '.

Page 60. Amend the values given under ' $D'_{0.999}$ ' to read as follows:

4.12, 2.99, 2.58, 2.36 and 2.22

Page 62. At the foot of Table 7, insert ' see Fig. 13 ' for ' see B.S. 600R, Fig. 22 '.

Page 65. Insert an additional chart, to be called Fig. 13, reproduced herewith from B.S. 600R, Fig. 22.

Page 70. In Example 1, write $\sqrt{n}$ for $\sqrt{n}$.

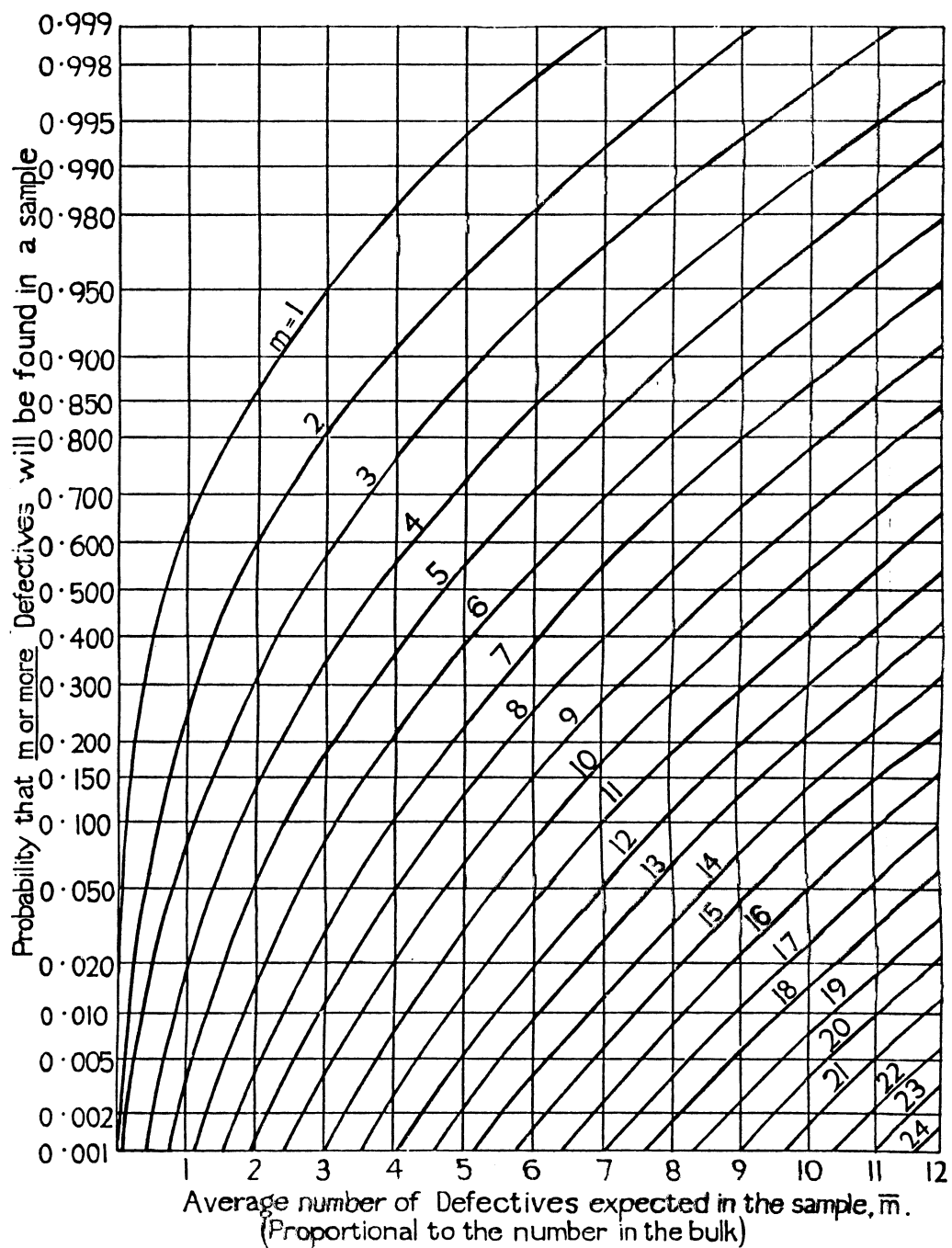


Fig. 13. Curves showing theoretical probability that m or more defectives will be found in a sample, for different values of $\bar{m}$.
The theoretical information given in these curves is sufficiently accurate for those cases where the proportion defective is small, say less than 10 per cent. Higher proportions require different values.

BRITISH STANDARD

CONTROL CHART TECHNIQUE

WHEN MANUFACTURING TO

A SPECIFICATION

**WITH SPECIAL REFERENCE TO ARTICLES
MACHINED TO DIMENSIONAL TOLERANCES**

by

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BRITISH STANDARD

CONTROL CHART TECHNIQUE WHEN MANUFACTURING TO A SPECIFICATION

FOREWORD

The information in this standard was first published in August, 1944, by the General Electric Co. Ltd., of England. With the kind permission of the Director of the Research Laboratories of the General Electric Co. Ltd., and of the Authors, it was re-issued in 1955 under the authority of the Units and Symbols Committee of the British Standards Institution in order to supplement the two British Standards on 'Quality Control':—

B.S. 600. The application of statistical methods to industrial standardization and quality control, by E. S. Pearson.

B.S. 600 R. Quality control charts, by B. P. Dudding and W. J. Jennett.

In B.S. 600 R. 'Quality control charts', graphical methods were described by which results of tests on samples selected regularly from a production line would provide a running commentary on the stability or otherwise of manufacturing processes. The technique described therein now generally known as 'Quality Control', was based on statistical principles and the objects were to give first, instructions to those introducing control charts; secondly, to give examples which would assist in explaining the elementary statistical background on which the technique depends. Whilst the examples given indicated how changes in process variability and changes in process average level influence rejections when the product is examined relative to fixed limits, they did not show in a direct manner how control chart technique could be adapted for use when manufacture is undertaken to a pre-determined specification.

The information contained in this handbook describes a more detailed development of the principles discussed in B.S. 600R (1942). It is written after first-hand experience with the introduction of control charts in collaboration with inspection and production staffs of various grades. The experience has covered a wide range of articles, output per week, size of organization, and varying degrees of development of independent inspection groups within the organizations concerned. It describes procedures which have been found acceptable when introducing and operating control chart technique as a factory routine.

The major aim of the technique described is *to assist production to specification requirements* with the elimination of waste labour and material. The technique has *proved* itself to be an extremely valuable aid to economic production. In many cases the successful operation of the technique results in reduction of inspection, but the success of the methods

should not be judged solely by the economy effected in inspection. The methods tend to increase the value of inspection and to improve the relationships between inspection and production personnel.

Whilst the handbook is intended principally as a guide for those concerned with machine manufacture of articles to dimensional limits, it should be helpful to anyone introducing the technique for use in other types of manufacture. Apart from the direct assistance derived from the use of quality control technique, the user will find that it cultivates an attitude of mind which is peculiarly valuable when considering the daily problems of manufacturing industry. Practice of the technique also provides an excellent training for the application of more advanced statistical methods to investigations designed to disclose the effect of changes in a manufacturing process on subsequent operations.

This book is arranged in Two Parts with Appendices.

Part One is concerned with the utilization of results of measurement (quantitative data).

This is treated first because of its greater technical value in assisting to maintain production in a satisfactory state, because it alone provides a ready means of assessing inherent process variability relative to a specification, and because experience in the interpretation of control chart data for sample range and average will make it easier to apply and understand the instructions given in Part Two.

Part Two is concerned with the utilization of the results of classifying a product into categories; for example 'defective' and 'effective' (qualitative data).

Suggestions are made which will, it is hoped, increase the value of control charts for number defective in those cases where this method has to be adopted or is preferred on grounds of convenience and/or cost.

This method should not be used if measurement can be economically and conveniently incorporated in the system of inspection.

Both parts contain, in addition to an Introduction linking this handbook with the earlier British Standards:

Section A, directions which should be followed when first attempting to introduce control charts;

Section B, directions for constructing control charts when manufacturing to specified limits.

In each part Section B is divided into subsections:

Rules for classifying processes in terms of their inherent variability relative to specification tolerances, including in Part Two rules for determining suitable values for inspection gauges or limits.

Directions for constructing and interpreting control charts appropriate to the various classes of process.

Examples of some of the different cases met in practice.

It is not possible within the compass of a handbook to illustrate every kind of situation that arises in practice, but the scheme of classification described has considerably assisted those who found difficulty in appreciating the practical implications of the manifold variations experienced in the position of control chart limits relative to specification limits and the disposition of the plotted points relative to the chart limits.

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PART ONE

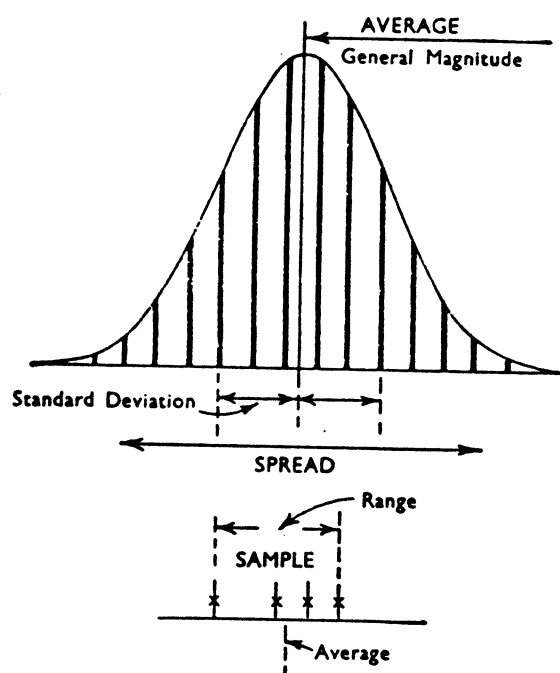
QUALITY CONTROL CHARTS FOR QUANTITATIVE DATA—
MEASUREMENTS MADE ON A CONTINUOUS SCALE

INTRODUCTION

This Part is concerned with utilizing the actual measurements made on samples selected regularly from the production of a machine or process. It is divided into two sections:—

A. Directions when first introducing control charts in order to gain initial experience;

B. Directions for routine application of control chart technique when manufacturing articles to specification or drawing limits, including the classification of processes in terms of process variability relative to specification tolerance.



The users of this handbook may or may not have read B.S. 600 or 600R and it will prove a convenience to tabulate some of the more important principles governing control chart construction for quantitative data which will assist appreciation of the relation between the data plotted on control charts and the proportion of a product which may be rejected by a specification.

(i) When measurements of a number of articles manufactured under *reasonably constant conditions* are grouped into a few subdivisions covering the complete

range of the measurements it is found that they tend to cluster round a central value, the number of measurements found in each group being fewer, the greater the difference between the group measurement and the central value. Such collections of measurements can be described by two values:—

- a. a measure of *general magnitude*, such as the average, and
- b. a measure of *spread, consistency, or variability*. The 'range' is a suitable measure of spread in samples of a few specimens, whilst 'standard deviation' is the most suitable measure for the spread in a considerable number or bulk. (See adjacent diagram and definitions, Appendix A, page 56.)

(ii) The proportion of the articles whose measurements differ from the average of the bulk by more than some given multiple of the standard deviation (or multiple of the average range in small samples of a given size) is generally, for all practical purposes, constant. (See Appendix B, Table 1, page 59 and Tables 1A and 3, page 60.)

(iii) Production is said to be 'stable' or 'in control' if the measured quality or dimension of the bulk product manufactured during different periods of time remains substantially constant with respect to both *general magnitude* (average value) and *variability* (standard deviation).

Stability of production in this sense does not mean that all the product is necessarily satisfactory with respect to some specification. The proportion of rejects may be too large.

(iv) The average value determined from a number of small samples provides an estimate of the *general magnitude*. The average range exhibited in the individual values in small samples of equal size, each sample being manufactured in a relatively short period of time, provides an estimate of the variability *inherent* in the machine or process. That is, this average range is a measure of the minimum variability in the product that can possibly be obtained without fundamentally improving the process. (See also Appendix C, page 66.)

(v) *The proportion of a product rejected by a specification* is determined by both the *general magnitude* and the *variability*, and changes in the proportion rejected are determined by simultaneous or independent changes in the general magnitude and the variability.

Hence, a running commentary is required on the *general magnitude* and the *variability* of a product as manufacture proceeds. This is provided by plotting on control charts the values of the *average* and of the *range* of results obtained by the inspection of small samples selected at intervals from the product, the limits on both control charts being determined by the average range of para. (iv), i.e. by the minimum variability which can be expected in the product.

(vi) *Control chart technique* aims at utilizing the results of patrol inspection of a few specimens examined at or near the machine, in order to obtain the earliest possible information as to whether the production is proceeding satisfactorily or whether preventive action should be taken to keep the production of unsatisfactory articles to the minimum. In this way the technique increases the assistance given by inspectors to the production of satisfactory material.

A. *HOW TO CONSTRUCT THE FIRST CONTROL CHART USING MEASUREMENTS

Procedure. In the early stages of the introduction of control chart technique the normal factory procedure *should not be disturbed*. Hence, the work involved is initially additional and therefore, the first efforts should not be too ambitious. It has been found that a keen junior inspector or foreman can very readily do all that is initially necessary whilst carrying on his normal duties.

The steps advised are as follows:—

(i) Choose a dimension which past experience shows is the cause of rejection on final inspection, but which does not involve too great a difficulty in measurement technique. For example, choose simple diameters and lengths rather than angles of cones, diameters of internal thread, or eccentricities.

(ii) *Sampling.* Visit the machine at intervals of one to four hours if it is a machine with an output of less than, say, about one hundred pieces per hour or each quarter-of-an-hour to one hour if it is one with an output exceeding one hundred per hour. Select for *immediate* measurement a sample of four articles made just previous to the visit, i.e. the last four, or four out of the last dozen or so made in rapid production. This will provide a measure of the minimum variability which can be expected in the bulk. In general, the samples should not be selected from the whole of the product made since the last visit. (See Notes on 'sample size,' 'frequency of sampling' and 'random sampling,' Appendix C, page 66.)

(iii) *Measurement.* Measure the dimension to be studied. (See Notes on 'measurement' in Appendix C, page 66.) Note the four observations on a record card or sheet. (Representative records are illustrated in Fig. 1, etc.) The details recorded at the head of the sheets can be suited to the normal procedures of the factory concerned. A most desirable feature of these records is that of logging only the difference from some fixed standard. If the standard approximates to the lower drawing limit, it is obvious that the maximum convenience in recording and ease of calculation is achieved as the majority of the observations will have the same sign and be small in magnitude. This enables comparatively unskilled inspectors to do the necessary work.

If the actual dimensions are recorded (e.g. 2.5062 in.) then more scrutiny of the work, or more skilled inspectors, may be required to avoid the occurrence of errors.

(iv) Calculate the 'average,' $\bar{x}$ (sum of the four readings divided by four) and the 'range,' w (difference between the extreme readings).

* If the reader has experience in the construction of the usual control charts for range and average and in their interpretation he should nevertheless carefully read this Section before proceeding to Section B.

(v) Arrange two sets of axes on decimal graph paper.* (See Fig. 1, page 12.)

The horizontal ordinate is the same for each set and is numbered to correspond with the serial number of the sample or the time at which the sample is taken. The vertical scales are suitably chosen to cover the individual values and the values of the range respectively.

(For engineering work a maximum vertical width of 2 in. is satisfactory. Recommended scales are 1 in. = 0.001, 0.002, 0.005 in., etc. Avoid difficult scales like 1 in. = 0.003 or = 0.004 in. which are inconvenient to read.)

(vi) Plot on the appropriate charts the values for individuals, average, and range. (The plotting of individuals is not normal but is useful at the start as leading eventually to understanding why, in general, control chart limits for sample average ($\bar{x}$) are narrower than the limits for individual articles.) (See Fig. 1, page 12.)

For a beginner the above is probably sufficient as a first step in the establishment of control charts prior to discussion with someone having more experience. In some cases it has been found that the preparation of the plotted diagrams and the explanation of their implications to all concerned has effected a marked improvement in the uniformity of the product. In such cases it is advisable to delay calculating the control limits until obvious improvement is not occurring. Usually, however, control limits should be placed on the charts for average and range after about ten samples have been taken (40 individual results) and the drawing tolerances indicated on the chart. An example is shown in Fig. 1, where the unit of measurement is 0.001 in. The procedure is then as follows:—

(vii) *Calculation of limits.* Calculate the grand average ($\bar{X} = 2.35$) of all results by dividing the sum of the 10 totals on the record sheet by 40, and calculate the average range ($\bar{w} = 1.35$) by dividing the sum of the 10 sample ranges by 10. The control chart limits are calculated in the following manner:—

For average values: ($\bar{x}$)

Inner limits $\bar{X} \pm 0.48 \times \bar{w} = 2.35 \pm 0.48 \times 1.35 = 3.0$ and 1.7 .

Outer limits $\bar{X} \pm 0.75 \times \bar{w} = 2.35 \pm 0.75 \times 1.35 = 3.4$ and 1.3 .

(The appropriate factors used here are given in Appendix B, Table 2, page 60.)

For range: (w)

Inner limit = $1.93 \times \bar{w} = 2.6$. Outer limit = $2.57 \times \bar{w} = 3.5$.

* Rough squared paper divided in mm or 0.1 in. squares is satisfactory. Millimetre divisions are too fine and often cause confusion. Accurate machine divided paper is unnecessary.

(The appropriate factors used here are given in Appendix B, Table 3, page 60.) Draw these limits on the appropriate charts as broken horizontal lines and include full lines corresponding to $\bar{X}$ and $\bar{w}$.

(viii) *Interpretation of charts.* The patterns of the plotted points relative to the control chart limits indicate for sample average and range whether, as production proceeds, variation is being introduced over and above that occurring in a short period of time; that is, whether or not production can be considered as stable. Whilst it is not possible to give fixed rules to determine in all circumstances whether the pattern of the points on a control chart indicates a state of instability, the following will prove a practical guide until the responsible person is able to form his opinion on the *general appearance of the points relative to the limits*. Manufacture should be considered unstable or out of control when one point appears in close proximity to or outside the outer limits; or two points in any consecutive ten appear in close proximity to and/or outside the inner limits; or three points in any consecutive twenty appear in close proximity to and/or outside the inner limits. (Of course, before taking drastic action, it is desirable to make sure that errors have not been made in the testing, etc., and in some cases it may be useful to take additional samples). In the event of one point in ten being just outside the inner limits but well inside the outer limits, stability can be assumed and the work continued until more data are available, provided that the last few plotted points are well distributed between the limits without undue bias towards one limit.

(ix) Usually the next step is to consider the charts to see whether the requirements of a specification can be met, or if the specification cannot be satisfactorily met, to determine what action should be taken. Detailed guidance is given in the next section together with examples illustrating some typical cases likely to be experienced.

(x) It is recommended that some mark should be made at the appropriate place on the chart, to indicate occurrences of technical importance such as machine adjustment, tool replacements, change of operator, fresh supply of raw material, etc., as these notes will help correct appraisal of the source of trouble when results fall outside the limit values.

NOTE. When the samples are selected in the manner proposed in (ii), page 8, the *range* provides an estimate of process variability, and in industrial examples, lack of control in range generally arises for different technical reasons than lack of control in average level. For example, where skill or care of an operator is concerned in the operation of a machine (or process), the variability, hence range, may show lack of control whereas the setting up of the machine may determine the average value and its stability. Contrarily, in fully automatic machine-shop operations, range or variability may rarely go out of control, whilst the average values of samples may tend to get out of control due to tool-wear.

RECORD SHEET

JOB NO. <u>13</u>		DRG. NO. <u>1721</u>		DIMN REF. <u>K</u>		MACHINE NO. <u>15</u>		OPERATOR <u>H. G.</u>	
DESCRIPTION <u>SHAFT</u>		INSPECTOR <u>J. M.</u>		ZERO OF MEAS. <u>2.506</u>		DRG. DIMN <u>2.506</u>		DRG. TOL. LIMITS ± 0.002	
UNIT OF MEAS. <u>0.001</u>		SAMPLING INTERVAL <u>2 hours</u>							

SPEC. NO.	NO. OF SAMPLE																	
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1	4.0	2.5	2.0	3.5	2.0	2.0	1.5	2.5	1.5	2.0								
2	2.5	2.0	3.0	2.0	3.0	2.5	3.0	2.5	3.5	2.5								
3	1.5	2.5	2.5	2.0	1.0	2.0	2.0	3.0	2.0	3.0								
4	2.0	2.0	2.5	2.0	2.5	3.0	1.5	3.0	2.5	2.0								
5											TOTALS FOR AVERAGES							
6											10 SAMPLES (40 ARTICLES)							
TOTALS	10.0	9.0	10.0	9.5	8.5	9.5	8.0	11.0	9.5	9.5		94.5						
Av.	2.5	2.2	2.5	2.4	2.1	2.4	2.0	2.8	2.4	2.4		2.35	=	$\bar{X}$				
RANGE	2.5	0.5	1.0	1.5	2.0	1.0	1.5	0.5	2.0	1.0		13.5	=	$\bar{\omega}$	=	1.35		

*Measurements made to 0.0005 in. It is generally preferable to estimate measurements to 0.0002 in. unless it is known that the average range ($\bar{\omega}$) is likely to be as large as 0.001 in.

AVERAGE

CONTROL CHARTS

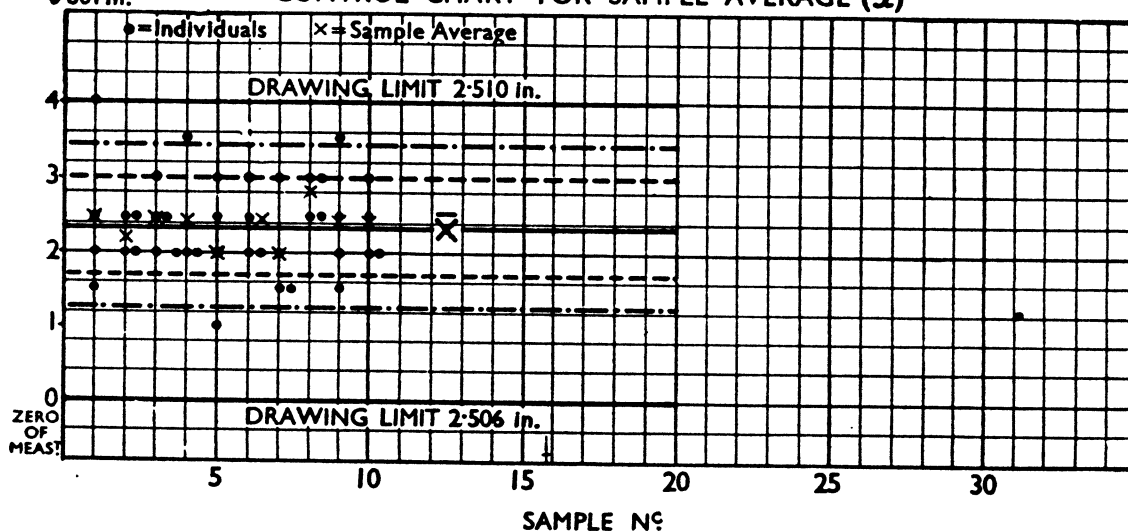
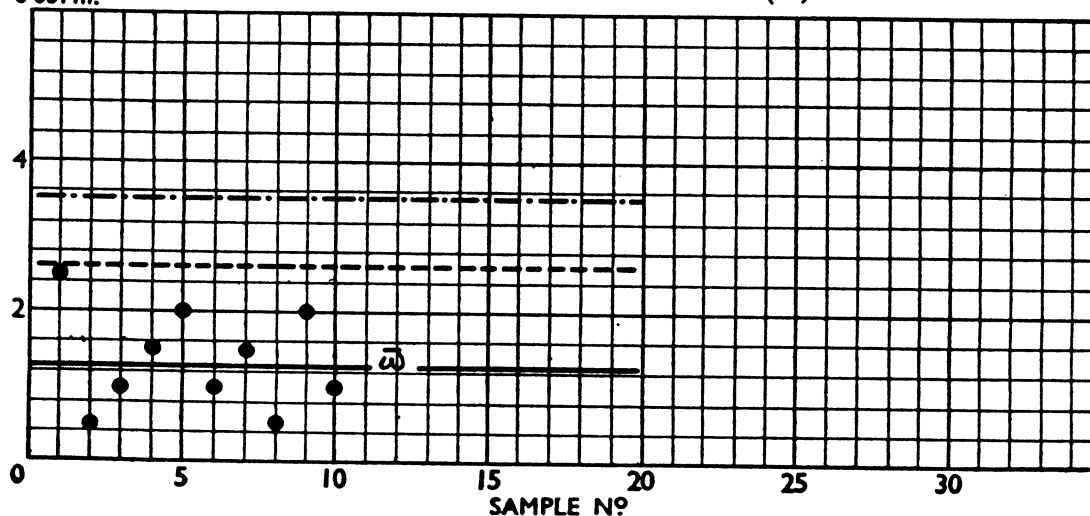
IN
0.001 in.CONTROL CHART FOR SAMPLE AVERAGE ($\bar{X}$)RANGE
IN
0.001 in.CONTROL CHART FOR RANGE (ω)

Fig. 1

B. CONTROL CHART TECHNIQUE WHEN MANUFACTURING TO SPECIFICATION REQUIREMENTS (Quantitative Data)

1. PROCESS VARIABILITY AND CONTROL CHART FOR SAMPLE RANGE

The beginner should follow the instructions given in the preceding Section A, including plotting the results on the charts for sample average and range, calculating the limits and examining the charts for evidence of stability or control. In a short time the plotting of the individual results can cease and the preparation of charts for sample average and range continue as a matter of routine.

Probably the most important characteristic of quality control technique is that it concentrates attention on the *variability* in manufacture. Hence, in all cases *first* determine the magnitude of the *variability inherent in the process*, by calculating the average range ($\bar{w}$) from the results for ten to twenty consecutive samples. (See Section A (ii), page 9.) The average range ($\bar{w}$) also determines the position of the limits on the control charts for sample average and range.

Secondly, construct the control chart for range. The limits on this chart are calculated by multiplying the average range ($\bar{w}$) by the values of the appropriate factors given in the following Table. (See also Appendix B, Table 3, page 60.)

To obtain limits multiply $\bar{w}$ by the appropriate factor given below:—

Sample size	Inner limit	Outer limit
2	2.81	4.12
3	2.17	2.99
4	1.93	2.58
5	1.81	2.36
6	1.72	2.22

The subsequent procedure is given in pages 15 to 30 and summarised for reference in Schedule I, page 16.

The ultimate aim of the technique is to reduce the process variability to a satisfactory value relative to the specification tolerance and to maintain the sample range and average within limits, thereby securing satisfactory production to specification requirements.

Important note. In the illustrations which are discussed later, it is assumed that practically no product (less than 0.5 per cent) outside the specification or drawing limits is acceptable. When concessions on drawing limits are permitted (a not unusual practice) the procedure now to be described should be applied to limits estimated to embrace the effect of these concessions. In the text 'specification limits' and 'drawing limits' are used synonymously.

2. CLASSIFICATION OF MANUFACTURE BY PROCESS VARIABILITY

Having prepared the control chart for range, manufacture should be classified into two broad categories:—

Class (i) Range out of control: Short period (or inherent) process variability *unstable*.

*Class (ii) *Range in control:* Short period (or inherent) process variability *stable*.

(For guidance in judging whether range is in control or not, see Section A (viii), page 11.)

Class (ii) should be sub-divided into three precision classes depending on the relative magnitude of the process variability and the (total) specification or drawing tolerance. This relative magnitude is conveniently expressed by the ratio of the total specification or drawing tolerance to the average range for samples of the same size.

The ratio $\frac{\text{(total) specn. tolerance}}{\text{average range}}$ is referred to as
the **RELATIVE PRECISION INDEX (R.P.I.)**

In the following Table are given suitable values of this ratio to sub-divide Class (ii) into three groups, corresponding to low, medium and high relative precision.† (See also Appendix B, Table 4, page 61.)

Sample size	Value of R.P.I.		
	Low relative precision	Medium relative precision	High relative precision
2	less than 6.0	6.0 to 7.0	greater than 7.0
3	less than 4.0	4.0 to 5.0	greater than 5.0
4	less than 3.0	3.0 to 4.0	greater than 4.0
5 and 6	less than 2.5	2.5 to 3.5	greater than 3.5

For example, when the sample contains four specimens the class into which a particular manufacturing process falls is decided by the following rules:—

- Low relative precision; R.P.I. less than 3.0.
- Medium relative precision; R.P.I. 3.0—4.0.
- High relative precision; R.P.I. greater than 4.0.

* It will sometimes be found that the range remains in control for long periods, e.g. when operating fully automatic machines. In such cases the routine use of a control chart for range need not continue, provided its value is watched against the appropriate inner limit noted on the record sheet.

† A similar classification can be used where the specification involves only one limit. In such a case the R.P.I. is obtained by dividing the difference between the average process level and the specification limit by half the average range.

3. RULES FOR CONSTRUCTION OF CONTROL CHARTS FOR SAMPLE AVERAGE

(For discussion of examples, see Section B.4, page 18.)

Class (i) Range out of control

Defer calculating the limits until the range has been brought into control. Study the position of the points on the average chart relative to the objective dimension in order to determine whether or not the values straddle the objective. If the points tend to lie above or below the objective an adjustment is necessary.

Class (ii) Range in control: *Low and medium relative precision.**

The control chart limits of sample averages are *set outwards from the agreed objective size* ($\bar{X}$), e.g. the dimension mid-way between the drawing limits, by amounts given by multiplying the average range ($\bar{w}$) by the factors given in the following Table. (See also Appendix B, Table 2, page 60.)

To obtain limits multiply $\bar{w}$ by the appropriate factor given below, then add to and subtract from the value mid-way between specification limits:

Sample size	Inner limit	Outer limit
2	1.23	1.94
3	0.67	1.05
4	0.48	0.75
5	0.38	0.59
6	0.32	0.50

E.g. If $\bar{X}$ is the value mid-way between the specification limits and $\bar{w}$ the average range for a sample of four specimens, then

$$\text{Inner limits } \bar{X} \pm 0.48 \times \bar{w}$$

$$\text{Outer limits } \bar{X} \pm 0.75 \times \bar{w}$$

Class (ii) Range in control: *High relative precision.*

Modified control chart limits for sample average are *set inwards from the drawing limits* by amounts given by multiplying the average range ($\bar{w}$) by the appropriate factors given in the following Table. (See also Appendix B, Table 5.)

* *Medium relative precision.* When the value of the R.P.I. is not close to the lower limit value, and experience shows that it is possible to operate the machine or process so that rejections are avoided the procedure for determining the limits for sample average suggested for high relative precision cases can be followed.

SCHEDULE I. CONTROL CHARTS

SCHEME OF OPERATION TO ASSIST

IMPORTANT.—First determine the average range for repeat samples of equal size and		
Classification of manufacture by process variability. (See Section B2)	Control chart limits for sample average. (See Section B3)	Classification of process level by control chart for sample average. See Section A (viii)
Class (i) <i>Range out of control.</i>	Defer calculating until range is in control.	Averages equally distributed about the objective value. Averages distributed asymmetrically about the objective value.
Class (ii) <i>Range in control:</i> <i>a.</i> Low relative precision R.P.I. corresponding to Col. 2, Table 4, Appendix B, page 61.	Calculate R.P.I. (see Section B2, page 14). From Table 2, Appendix B, page 60. Set out from objective size.	Averages in control. Averages out of control.
<i>b.</i> Medium relative precision R.P.I. corresponding to Col. 3, Table 4, Appendix B, page 61.	From Table 2. NOTE. If R.P.I. approaches the higher value given in the Table then use modified limits as from Table 5, Appendix B, page 61.	Averages in control. Averages out of control.
<i>c.</i> High relative precision R.P.I. corresponding to Col. 4, Table 4, Appendix B, page 61.	Use modified limits from Table 5, Cols. 2 and 3, Appendix B, page 61. (Set in from specification or drawing limits.)	Averages within modified limits. Averages outside modified limits.
<i>d.</i> High relative precision R.P.I. greater than the value in Col. 4, Table 4, Appendix B, page 61 by more than about 50 per cent	Use alternative limits from Table 5, Cols. 4 and 5, Appendix B, page 61. (Set in from specification or drawing limits.)	Averages within alternative limits. Average outside alternative limits.

NOTE. When making technical adjustments to bring ranges or averages into control, the

FOR SAMPLE AVERAGE AND SAMPLE RANGE MANUFACTURE TO SPECIFICATION LIMITS

construct control chart for range (see Part One, Section A, page 9 and Section B, page 13).

State of production	Action suggested by control chart	Example and comments
(See Section B4, page 18)		
Unsatisfactory. Unsatisfactory.	Bring range in control, and then examine as directed for Class (ii). Ditto and correct to bring averages about objective.	See page 19 and Fig. 2. In this example the average size is slightly large.
Unsatisfactory. Unsatisfactory.	Increase tolerance or reduce range and then examine for Class (ii) <i>b</i> , <i>c</i> , or <i>d</i> . Bring averages in control then re-examine.	See page 21 and Fig. 3. Border line case illustrated by Fig. 4 and discussed page 22. Averages out of control not illustrated. See Fig. 5.
Satisfactory. Unsatisfactory.	Maintain by taking suitable action early enough if points appear near inner limits. (See Section B.) Bring averages in control and then maintain.	Not illustrated. But Fig. 4 would serve as an example if the tolerance were increased by 0.001 in. See page 25 and Fig. 5.
Satisfactory. Unsatisfactory.	Maintain by taking suitable action early enough. Bring averages in control and then maintain.	See page 26 and Fig. 6. Fig. 7 illustrates the additional latitude in sample average provided by the modified limits over that provided by control limits. See page 30 and Fig. 8. This serves to illustrate the use of an objective set-up size differing from the basic dimension, and also a drift in average size with time.
Satisfactory. Unsatisfactory.	Maintain by taking suitable action early enough. Bring averages in control and then maintain.	See page 30 and Fig. 8. Samples 1-15 illustrate satisfactory production. Samples 17, 18 illustrate the situation where action has been delayed too long to maintain high assurance of absence of rejects.

general practice should be to make one change at the time, see Section 4, General, page 18.

Part 1 : Section B

To obtain limits multiply $\bar{w}$ by the appropriate factor given below, then add to and subtract from the lower and upper specification limits respectively:

Sample size	Inner modified limit	Outer modified limit
2	1.51	0.80
3	1.16	0.77
4	1.02	0.75
5	0.95	0.73
6	0.90	0.71

If H and L are the values of the upper and lower specification or drawing limits respectively, and $\bar{w}$ the average range in a sample of four specimens, then the control chart limits for sample average are given by:

Inner limits $H - 1.02 \times \bar{w}$ and $L + 1.02 \times \bar{w}$

Outer limits $H - 0.75 \times \bar{w}$ and $L + 0.75 \times \bar{w}$

NOTE. For alternative modified limits when desiring greater assurance against manufacture of rejects, see Appendix B, Table 5, page 61.

4. DISCUSSION, WITH EXAMPLES OF MANUFACTURING, PROCESSES CLASSIFIED AS IN SECTION B2 (Also see Schedule I, page 16)

General.

It is good practice, when taking action to improve a manufacturing process, to make one adjustment only at any one time. After making an adjustment to a machine or process, production, sampling, and plotting the results on the control charts should continue. It soon becomes evident what improvement, if any, has been effected.

Sometimes, no obvious fault can be found, and then a general overhaul of the machine has to be made before continuing production, etc.

Class (i) Range out of control

The immediate task is to bring the range into control. The range chart shows that even during the relatively short period occupied in making a few specimens the machine is not repeating itself, but that there is some erratic feature needing correction. Knowledge of the operation characteristics of the machine and of its construction have to be used to seek the source of the trouble.

An example is given in Fig. 2. Although control limits are not determined for the average chart it should be noted that the articles are generally slightly too large, the gross average value being 11 instead of 10.

RECORD SHEET

JOB NO. <u>412</u> DRG. NO. <u>1718</u> DIMN REF. <u>G</u> MACHINE NO. <u>12</u> OPERATOR <u>A.L.</u>																		
DESCRIPTION <u>SPIGOT</u> INSPECTOR <u>M.K.</u> ZERO OF MEAST <u>0.784 in.</u>																		
DRG. DIMN <u>0.285</u> DRG. TOL. LIMITS <u>± 0.001 in.</u> UNIT OF MEAST <u>0.001</u> SAMPLING INTERVAL <u>30</u> min																		
SPEC NO	NO OF SAMPLE																	
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1	14	11	11	15	13	8	25	11	8	10								
2	8	9	13	12	8	9	11	9	10	11								
3	9	12	13	12	9	16	2	14	14	13								
4	12	14	15	3	10	12	13	14	8	9								
5											TOTALS FOR 10 SAMPLES (40 ARTICLES)							
6																		
TOTALS	43	46	52	36	40	45	47	48	40	43		440						
Av.	10 3/4	11 1/2	13	9	10	11 1/2	11 1/2	12	10	10 3/4		11 = $\bar{x}$						
RANGE	6	5	4	18	5	8	27	5	6	4		88 ∴ $\bar{\omega}$ = 8.8						

 AVERAGE
IN
0.0001 in.

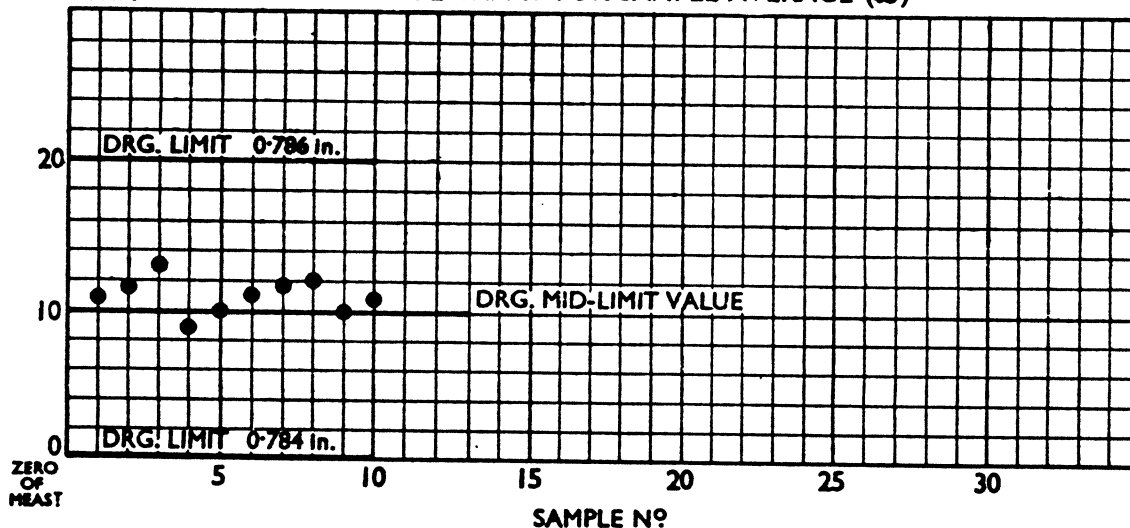
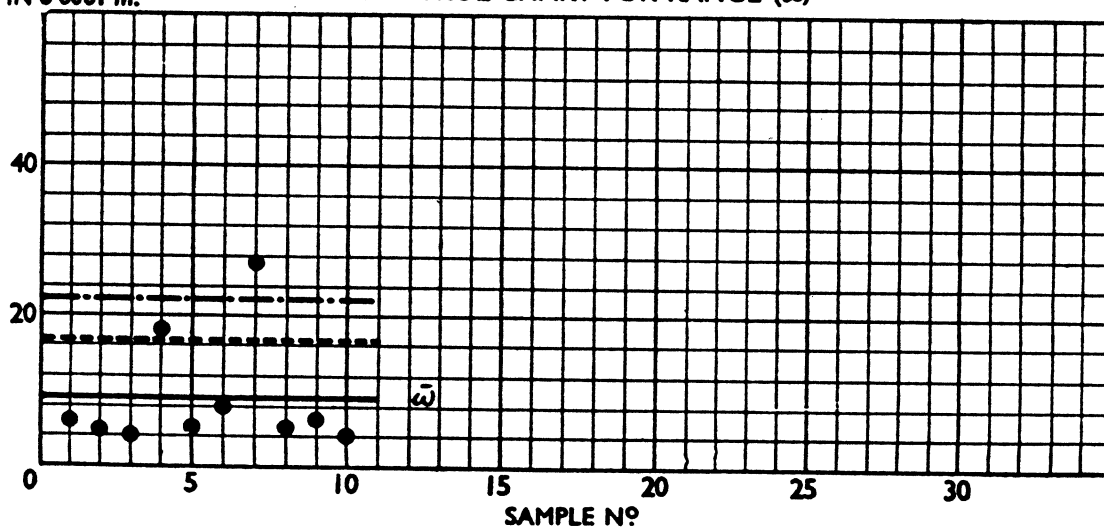
 CONTROL CHARTS
CONTROL CHART FOR SAMPLE AVERAGE ($\bar{x}$)

 RANGE
IN 0.0001 in.
CONTROL CHART FOR RANGE (ω)

Fig. 2

Class (ii) Range in control: *a. Low relative precision.*

Rejections are inevitable in manufactures falling in this class even when the sample average is in control. The rejections will be greater, the smaller the R.P.I. The product can be sorted by 100 per cent inspection but if the manufacture of rejects is to be avoided the R.P.I. must be *increased* either by

increasing the specification (drawing) tolerance, or by
reducing the process variability, i.e. the average range* ($\bar{w}$).

When neither of these alternatives is possible it is sometimes advisable to use an off-centre objective value when setting up the machine or process, thereby reducing the amount of scrap material whilst increasing the amount of correctable work.

Thus in the example discussed below, if the objective set-up size were off-set by 0.0005 in. the scrap would be reduced from about 4 to 1 per cent and the correctable material increased from about 4 to 10 per cent. For further discussion see Appendix E, page 73.

An example is given in Fig. 3. Here the drawing tolerance is 0.0040 in., the average range 0.0023 and the $R.P.I. = 4.0/2.3 = 1.74$. Rejections are inevitable because the limiting value of the R.P.I. for the low precision class is 3.0. (See Appendix B, Table 4, page 61.)

This is illustrated by the full lined curve marked 'bulk' which represents the distribution of the dimensions of the product when manufacture is in control at about mid-tolerance value. (See Appendix E, page 73, for the method of estimating what proportion of the product will be rejected.)

The smaller full lined curve represents the distribution of the average values of samples of four specimens selected from the bulk. This has been drawn to illustrate why, in general, the control chart limits for *sample average* are within the specification (drawing) limits for *individual specimens*. Control chart limits for averages do *not* restrict drawing limits for individuals but in practice are their sample equivalents.

If the drawing tolerance could be doubled or the average range halved the R.P.I. would be large enough to place the process in the medium precision class. (See later example, page 22.)

Class (ii) Range in control: *Low to medium relative precision. (Border-line case)*

Before passing on to consider a medium precision process, it is of interest to examine the border-line case when variability is of such a magnitude that the R.P.I. is equal to the limiting value in Col. 2 of Table 4, Appendix B (e.g. 3.0 for sample of four articles).

* When attention is being given to the process in order to reduce the average range, it is recommended that a satisfactory objective value of the average range be indicated on the control chart.

RECORD SHEET

JOB NO. 1046 DRG. NO. TX 16 DIM. REF. B MACHINE NO. 11 OPERATOR J.K.
 DESCRIPTION BOLT INSPECTOR B.D. ZERO OF MEAS. 0.498 in.
 DRG. DIM. 0.500 in. DRG. TOL. LIMITS $\pm 0.002 in.$ UNIT OF MEAS. 0.001 in. SAMPLING INTERVAL 2 hours

SPEC. NO.	NO. OF SAMPLE																	
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1	3	1	2	1.5	3	2.5	4	3	1.5	4								
2	3.5	3	1	3.5	4	1	2	1.5	1	1.5								
3	1.5	5	1.5	2	1	2	3	1	1	1								
4	5	2.5	3.5	1	3	2	2.5	1	2	1								
5																		
6																		
TOTALS	8.5	7	8	8	11	7.5	11.5	6.5	5.5	7.5								
AV.	2.1	1.8	2.0	2.0	2.2	1.9	2.9	1.6	1.4	1.9								
RANGE	3	2.5	2.5	2.5	3.0	1.5	2.0	2.0	1.0	3.0								

TOTALS FOR 10 SAMPLES (40 ARTICLES)
 $\bar{x} = 2.3$
 $R.P.I. = \frac{4.0}{2.3} = 1.74$

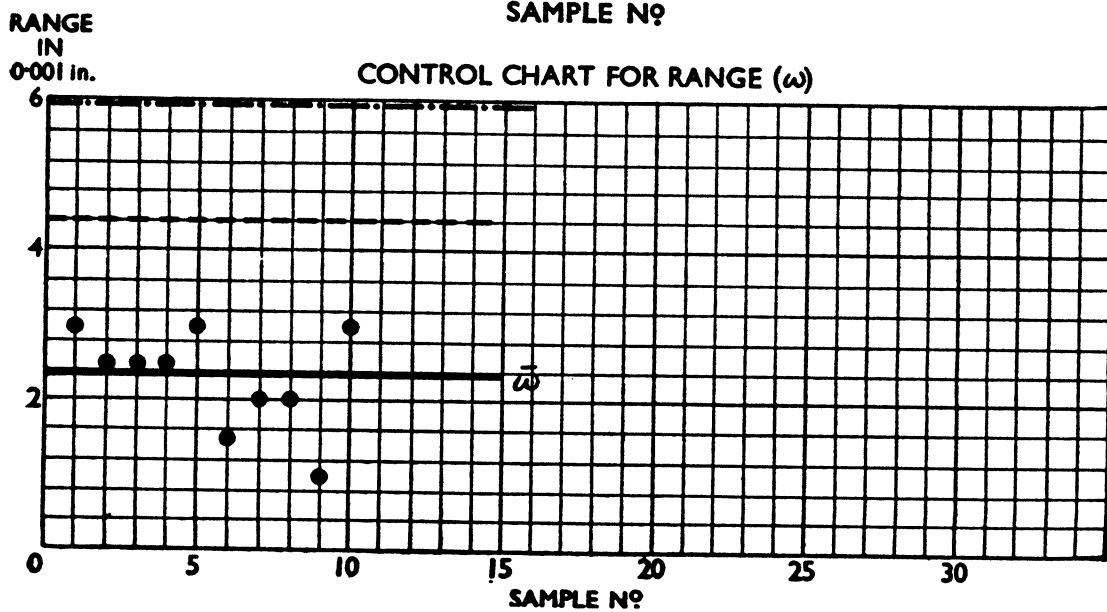
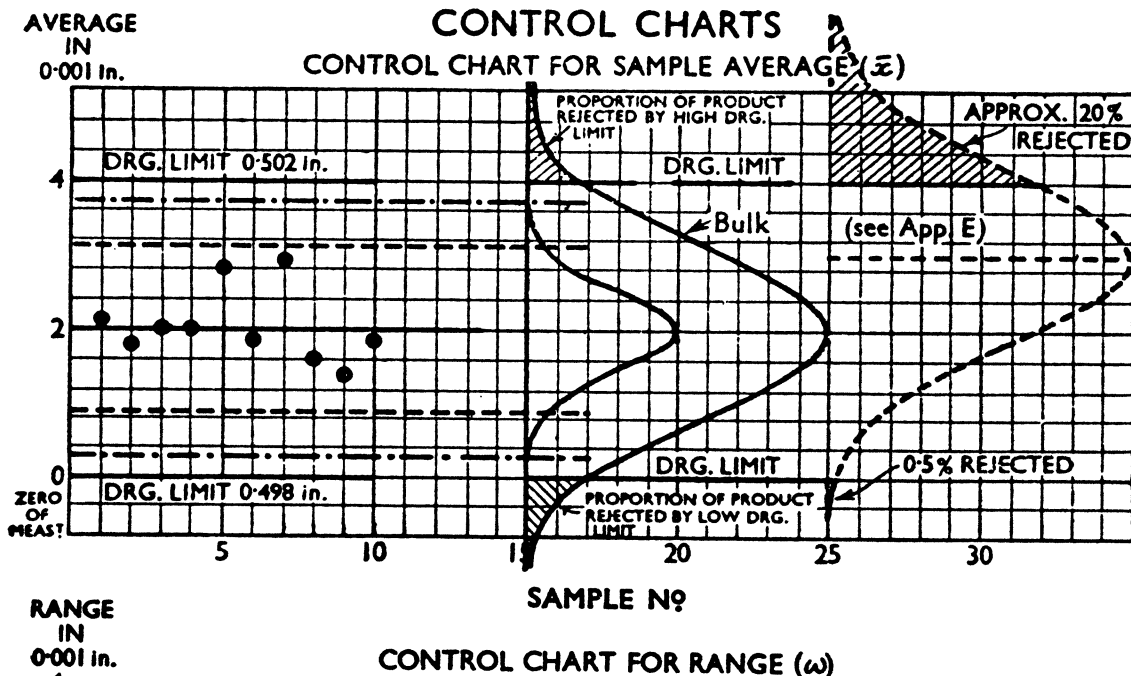


Fig. 3