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Guide to
**Statistical interpretation
of data**

Part 1. Routine analysis of
quantitative data

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Society of Analytical Chemists



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Foreword

The correct interpretation and presentation of test results has been assuming increasing importance in the analysis of data obtained from manufacturing processes based on sample determinations and prototype evaluations in industry, commerce and educational institutions. It was for this reason that subcommittee 2 of Technical Committee 69 'Application of statistical methods' of the International Organization for Standardization (ISO), was charged with the task of establishing a guide to statistical methods for the interpretation of test results. UK delegates have for several years participated actively in this work, with Prof. E. S. Pearson, FRS, in particular providing many valuable contributions. Prof. Pearson, FRS, is also author of BS 600 'Application of statistical methods to industrial standardization and quality control', which was first published in 1935. International agreement has already been reached on the specification of several techniques and it is proposed to publish these as Parts of a revised BS 2846*, as follows.

Statistical interpretation of data

- Part 1 Routine analysis of quantitative data
- Part 2 Estimation of the mean-confidence interval (ISO 2602)
- Part 3 Determination of a statistical tolerance interval (ISO 3207)†
- Part 4 Problems of estimation and test relating to means and variances (ISO 2854)†
- Part 5 The efficiency of tests relating to means and variances (ISO 3494)†
- Part 6 Comparison of two means in the case of paired observations (ISO 3301)†

The above Parts relate to the interpretation of quantitative data and it is recognized that a great amount of the data resulting from manufacturing processes is collected in a *qualitative* form. Accordingly, it is proposed to add further Parts to this standard which will be concerned with the interpretation of qualitative data as well as providing other techniques to those given in Parts 1 to 6 for quantitative data.

The ISO committee is concerned primarily with the publication of a series of documents which present the mathematical calculations of the test statistics without dealing in depth with some of the more fundamental aspects which may need consideration prior to their application. For this reason, as well as to unite the separate ISO publications into a series, the Advisory Committee on Statistical Methods (OC/8) agreed that a revision of BS 2846 : 1957 provided the vehicle for this purpose.

Part 1 is based on BS 2846 : 1957 and is concerned

with the routine analysis, presentation and reduction of quantitative data, such as rounding of test results, which should be considered prior to applying the statistical tests given in later Parts. BS 2846 has accordingly been amended and is superseded by Part 1 of the standard. These amendments can be summarized as follows.

- (a) The examples are now in metric units to conform to later Parts‡.
- (b) The mathematical techniques and tables relating to the estimation of means, variances and confidence limits have been deleted where these are treated in Parts 2 and 4.
- (c) The glossary of statistical terms is now an appendix and takes account of the definitions in ISO 3534 'Statistical terminology and symbols' (to be published as a British Standard).

Part 2 specifies techniques for the estimation of the mean of a normal population on the basis of a series of tests applied to a random sample on individuals drawn from this population, and deals only with the case where the variance of the population is unknown. It is not concerned with the calculation of an interval containing, with a fixed probability, at least a given fraction of the population (statistical tolerance limits).

Part 3 specifies techniques for the estimation of tolerance limits and also includes a brief introduction to distribution-free methods. These latter methods allow for the determination of tolerance limits even when no assumptions regarding the distribution of the underlying population can be made.

Part 4 relates the estimates of the means and variances of a sample to the population parameters and specifies the techniques required to examine certain hypotheses concerning the value of those parameters.

Part 5 on the other hand specifies procedures to ascertain which of the techniques in Part 4 it is preferable to use under certain conditions, that is, which test is most efficient under those conditions. These considerations are based on what are generally referred to as the errors of the first and second kind, or to the related probabilities of drawing wrong conclusions from the sample results by application of the criteria of a given test.

Part 6 deals with the special case known as the method of paired comparisons, which Part 4 touched on briefly. Such cases can arise, for example, when two tests are performed on each specimen in the sample, a first prior to the application of some treatment and the second after, in order to ascertain whether the treatment is effective or not.

* BS 2846 : 1957 'The reduction and presentation of experimental results'.

† In course of preparation.

‡ The factors for this conversion have been extracted from BS 350 'Conversion factors and tables'.

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The committee wishes to call attention to the various assumptions underlying the application of the methods as stated in the text of Parts 1 to 6. These assumptions can be satisfied in almost any situation if the experiment has been designed properly from a statistical standpoint and this standard is concerned with data arising from experiments that are satisfactory in this respect. Although not dealt with in the following pages the design of experiments is an important subject to which experimenters should give careful thought more thought than is sometimes given before an experiment is started.

Most of the techniques for the analysis of quantitative data described in the standard assume that the items in the sample come from a population which has a normal distribution. In certain cases this assumption is not critical but in others unless it can be shown to be at least approximately true the techniques should not be applied. This applies particularly to the estimation of tolerance intervals as specified in Part 3 and to those tests for variances in Part 4. This problem is discussed briefly in Parts 1 and 4 and in more detail in BS * 'Testing for departures from normality'.

This standard does not contain methods for the collection of individuals in the random sample.

In this respect references should be made to the particular British Standards which specify the sampling procedures for a specific product. In the case of discrete items reference to BS 6000 and BS 6002 (see below) contain general descriptions.

The following British Standards also provide practical guidance on the application of statistical methods.

- BS 600 Application of statistical methods to industrial standardization and quality control
- BS 1313 Fraction-defective charts for quality control
- BS 2564 Control chart technique when manufacturing to a specification, with special reference to articles machined to dimensional tolerances
- BS 6000 Guide to the use of BS 6001. Sampling procedures and tables for inspection by attributes
- BS 6001 Sampling procedures and tables for inspection by attributes
- *BS 6002 Sampling procedures and charts for inspection by variables
- *BS Determination and statistical analysis of precision data for a standard test method.

* In course of preparation.

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Statistical interpretation of data

Part 1. Routine analysis of quantitative data

0. Introduction

It is generally agreed that reports of the results of investigations should preferably include the observations that have been made, unless limitations of space preclude this. Even when it is possible to give the observations in full, it is preferable also to give the essential information in a condensed form for ease of discussion. It is desirable, not only to do this in a uniform way to avoid confusion, but also to ensure that the form adopted should follow the methods that have been found most satisfactory in practice.

Section one. General principles**1. Scope**

This standard gives rules which are recommended for use in the condensation of data; it is, however, restricted to the consideration of observations of one variable only.

The standard deals also with the uncertainty of various quantities estimated from the data, arising from the complex of variations which are termed errors. The application of the various measures of uncertainty depends on the validity of certain assumptions, which are referred to in the text, and in general these assumptions are satisfied only if the experiment is designed properly from a statistical standpoint.

2. References

The titles of the British Standards referred to in this standard are listed on the inside back cover.

3. Frequency tables

Although it may be desirable to present the whole of the original data, it is not always convenient to do so if the data are numerous. While a few values may be reported in the text and cause no confusion, such a procedure with a large body of data might cause considerable interruption of the discussion. It is often better, therefore, to give the full results in an appendix, and to include a condensed form of the data in the text. Some condensation is achieved by grouping. Grouped results may be presented in two ways, namely, as group frequency tables or as cumulative frequency tables, and both ways are described in section three. For most purposes grouping is of little value if fewer than thirty results are available.

4. Graphical methods

In general, it is not difficult to grasp the information in a comparatively small number (ten or fewer) of values. It is, however, beyond the ability of most people to visualize the general features of a mass of data, even when they are grouped, and the use of diagrams will usually assist in understanding their implications. The diagrams most frequently used for this purpose are the histogram and the cumulative diagram; these also are described in section three.

5. Representative constants

Even the grouped data cannot be used readily for comparison with other data or information; further condensation is required for this purpose. Two quantities, the arithmetic mean and the standard deviation, are for most purposes adequate to represent the given data; these are described in section four and methods of calculating them are given.

6. Uncertainty of estimates

It is a matter of experience that observations made under apparently identical conditions usually show some variation, because of factors outside the experimenter's control. Such variation in the observations leads to similar, but smaller, variation in means and in standard deviations calculated from the observations. Thus, since experiments when repeated usually give results with slightly different means and standard deviations, the mean and the standard deviation estimated from a single group of results are subject to some uncertainty. Section five discusses this uncertainty and ways in which it may be represented, and Parts 2, 3 and 4 specify techniques for dealing with particular problems.

7. Rounding

It is frequently found that observations are reported to more figures than are required for the purpose in view; this may be corrected by rounding. The subject is discussed in section six, where a rule is given to determine the maximum degree of rounding which may be carried out. Caution must be observed, however, in rounding data, since it may later become desirable to make comparisons which require the greater number of figures originally available.

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8. Comparing estimates

It should be borne in mind that this Part of the standard gives rules only for the condensation and presentation of experimental results. While the purpose of such condensation is usually to permit comparisons to be made, the conditions which must be fulfilled before such comparisons are valid are not discussed here, and reference should be made to texts on the statistical design of experiments. It may be noted, however, that any comparison made should be judged against an estimate of variability, e.g. a standard deviation, which takes account of all sources of error which might affect the comparison. The subject is discussed, in rather more detail, in clause 32 and Parts 4, 5 and 6.

9. General recommendations

This Part of the standard includes a list of general recommendations, which for convenience of reference is given in section two. An appendix of statistical terms used in the standard is also given.

It should be stressed that, although the following sections give some explanation of the reasons for the adoption of the procedures recommended, this explanation is necessarily very brief; those who require a fuller explanation, or wish to pursue the subject further, are advised to consult BS . . . * or one of the standard textbooks on statistical methods†.

Section two. Rules for presentation of experimental results

10. General

10.1 Report experimental observations in full, unless limitations of space preclude this; if the observations are numerous they may be given in an appendix to avoid interrupting the discussion.

10.2 Present the results also as a frequency distribution, unless there are fewer than thirty observations. To facilitate this:

- (a) rules for grouping are given in clause 14;
- (b) a method of classification is described in clause 15;
- (c) presentation of tables is discussed in clause 16.

10.3 Always give the following information:

- (a) the arithmetic mean (see clauses 20 and 22 and Part 2);
- (b) the standard deviation (see clauses 21, 22 and 23 and Part 4);
- (c) the number of observations.

NOTE. This rule should always be observed whether or not a frequency table is given.

10.4 If possible also give:

- (a) 95% confidence limits for the arithmetic mean (see clause 28 and Part 2);

* BS The determination and statistical analysis of precision data for a standard test method.

† BROWNLEE, K. A. Industrial experimentation. The Chemical Publishing Co. Inc.
COCHRAN, W. G. and COX, G. M. Experimental designs. John Wiley and Sons.

(b) 95% confidence limits for the standard deviation (see clauses 29 and 30 and Part 4).

10.5 Give all possible information on the conditions under which the observations were made. If the conditions were not constant for the whole set of observations, it is possible that the observations do not form a homogeneous group; homogeneity is discussed in clause 32. Where there is evidence of heterogeneity the data should be subdivided to give homogeneous subgroups; the conditions ruling for each subgroup should be stated. The mean, the standard deviation, and the number of observations should be given separately for each subgroup.

10.6 Examine whether observations could with safety be rounded (see section six). If there is any likelihood that the data will be used later for purposes other than those intended originally, however, it is best not to round them.

11. Example of presentation

The data on compressive strengths of concrete cubes from table 1 are shown as a frequency table (table 3) and as a histogram (see figure 1). When further condensed these results would be presented as follows.

		95% confidence limits	
		Lower	Upper
Number of observations	400		
Arithmetic mean (N/mm ²)	27.36	27.01	27.71
Standard deviation (N/mm ²)	3.57	3.32	3.86

Some prefer, however, to attach the confidence limits directly to the relevant estimates. This might be done quite conveniently for the mean if the limits were given in brackets as shown below, and would be satisfactory provided a note were added which explained the meaning to be given to the figures in brackets.

Number of observations	Arithmetic mean
400	27.36 (± 0.35)† N/mm ²

† The figures in brackets represent the 95% confidence limits.

Unfortunately, the confidence limits for the *standard deviation* are not usually symmetrical about the estimate, so they cannot be expressed conveniently in this way.

Section three. Frequency distributions

12. Purpose

The simplest method of condensing data is to form groups within which the individual observations differ little from one another. The full set of data

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can then be replaced by a table which shows for each group the number of the original observations which fall within it. This section describes how the groups should be chosen; it also outlines a simple procedure for the preparation of the table, and the way in which the table is best presented.

Table 1. Compressive strengths of concrete cubes (after 7 days)

All quantities are in N/mm².

24.0	30.4	25.3	29.7	27.0	28.5	36.7	24.5
23.8	27.9	21.0	27.7	25.7	23.4	25.5	30.8
28.8	31.1	28.8	26.8	27.2	22.3	33.0	43.6
28.8	27.7	23.6	28.8	28.5	24.3	26.6	33.0
23.2	27.4	29.1	24.7	22.8	26.6	24.0	30.6
32.0	27.2	26.4	31.2	27.9	35.4	24.7	25.2
27.9	22.4	27.6	27.9	28.3	27.0	24.1	28.4
27.6	26.8	31.1	30.3	26.8	27.7	27.4	40.3
28.6	24.6	34.3	25.8	27.2	22.5	27.9	24.0
28.5	28.3	26.3	27.2	25.9	24.5	29.0	17.0
29.3	30.4	28.7	26.2	23.0	27.7	30.4	26.2
28.7	25.8	27.0	28.8	30.7	28.1	28.3	29.0
25.9	25.3	25.8	23.2	27.7	26.6	29.7	31.7
39.1	30.3	31.0	30.1	29.6	25.3	28.1	23.6
30.8	31.2	22.1	18.3	34.8	18.7	25.9	25.8
27.9	29.2	27.9	26.0	26.5	28.1	24.7	23.2
37.3	27.4	25.0	26.8	24.2	27.0	28.8	27.5
37.2	27.2	27.2	24.0	28.9	26.8	33.2	30.5
27.9	27.4	28.7	24.9	27.0	29.0	25.8	28.1
27.6	28.1	29.0	29.4	30.0	28.3	32.8	25.1
28.1	24.9	28.0	25.5	26.5	30.1	28.6	25.1
27.0	23.6	26.5	22.1	27.9	24.5	30.1	25.1
27.0	29.1	27.4	30.2	25.6	28.5	31.3	23.0
32.4	29.8	26.8	24.1	30.5	29.0	28.9	29.4
31.3	24.4	25.5	24.7	22.8	27.4	26.0	26.8
28.8	22.5	26.8	26.9	28.3	28.5	27.8	31.2
31.5	25.3	27.6	32.1	25.1	29.6	31.6	25.1
30.3	25.9	29.4	31.4	23.4	25.1	27.9	31.4
29.0	29.2	29.2	26.4	27.2	25.1	25.4	28.8
32.6	27.0	23.8	25.6	25.9	37.3	23.4	24.5
28.8	22.3	26.4	26.0	19.1	34.1	30.1	32.2
35.5	24.4	27.4	26.3	26.9	19.1	23.9	26.4
32.8	26.8	25.5	27.0	24.5	24.5	24.8	21.9
30.9	27.4	21.9	34.9	24.7	30.1	24.1	25.3
31.5	29.3	22.8	34.2	22.1	22.1	23.5	24.7
30.1	29.3	23.4	25.6	30.1	31.7	27.1	23.8
26.4	27.2	20.8	30.5	21.6	19.3	27.4	24.9
29.6	23.4	32.5	29.2	23.2	29.6	23.4	24.9
27.0	27.9	28.8	31.8	22.1	27.0	31.6	23.9
30.1	30.2	32.5	32.0	26.6	23.4	28.0	26.9
34.6	28.1	33.9	22.5	26.6	25.8	30.4	24.7
26.1	29.4	27.7	23.9	25.3	25.3	19.8	21.2
24.3	31.5	27.4	26.0	25.7	23.2	23.2	24.5
30.7	32.6	26.8	24.8	27.4	25.1	28.5	28.3
30.9	28.5	28.7	29.6	25.9	29.2	23.5	33.3
27.6	25.9	27.6	25.1	25.3	20.4	23.3	31.8
35.1	32.6	27.9	26.4	28.3	27.0	28.8	30.5
28.8	29.4	23.9	24.9	28.5	26.7	26.3	30.0
27.0	26.3	27.6	23.6	21.6	26.7	19.8	29.2
25.5	22.1	25.4	26.0	21.4	25.4	18.3	31.9

The method of grouping the observations is shown in the example given in clause 15.

13. Grouping

When data have been grouped the range of values assigned to a group is referred to as the *group interval*, and the end values which define the interval as the *group limits*. The number of observations which fall into a group is called the *group frequency*, and the way in which the frequencies are distributed over the groups is called a *frequency distribution*.

14. Rules for grouping

It is usual to choose the intervals so that there are about 10 to 20 groups, and preferably the intervals should be equal in width. Where there is a tendency for certain values (e.g. multiples of 10) to occur much more frequently than their near neighbours, the groups should be so arranged that these values occur at about the middle of the corresponding intervals. Group limits should be chosen so that there is no doubt into which group a particular observation should fall; this means that *group limits should never be values which can occur as observations*. The most satisfactory arrangement is to set the limit between two adjacent groups midway between the highest observation which can occur in the lower group and the lowest observation which can occur in the higher group. This usually means that the group limits must be expressed to one more figure than is used in recording the observations. Observations which are to be grouped should not be rounded first (see clause 34), since grouping is a form of rounding, and successive rounding is liable to lead to systematic errors.

15. Example of grouping

Table 1 gives a set of 400 observations of compressive strengths of concrete cubes.

In grouping data such as those given in table 1 the first step is to decide what intervals to use; in this instance, 2 N/mm² is chosen as a convenient common interval, and there are thus 14 groups. As the data are recorded to the nearest 0.1 N/mm² the limits are arranged to end in 0.05; the intervals chosen are shown in table 2 (column 1). For each observation a mark is placed against the corresponding group; for ease of counting the marks are arranged in blocks of five, the fifth mark being drawn across the other four. Table 2 shows the arrangement. When every observation has been allocated to a group, the marks in each group are counted and give the frequency for that group.

16. Presentation of tables

For the convenience of readers each table should be given a title and the headings of the columns should be worded clearly. If this is difficult to achieve in a few words explanatory footnotes can