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Guide to

Statistical interpretation of data —

Part 3: Determination of a statistical tolerance interval

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- Electronic Components Board
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- Institute of Statisticians
- Institution of Production Engineers
- Ministry of Defence
- National Coal Board
- Post Office
- Royal Statistical Society
- Society of Analytical Chemists
- University of Essex

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Foreword

The correct interpretation and presentation of test results has been assuming increasing importance in the analysis of data obtained from manufacturing processes based on sample determinations and prototype evaluation in industry, commerce and educational institutions. It was for this reason that sub-committee 2 of Technical Committee 69, Application of Statistical Methods, of the International Organization for Standardization (ISO), was charged with the task of establishing a guide to statistical methods for the interpretation of test results. International agreement has already been reached on the specification of several techniques and it is proposed to publish these as Parts of a revised BS 2846¹⁾, as follows:

Statistical interpretation of data

- *Part 1: The routine analysis of quantitative data;*
- *Part 2: Estimation of the mean — Confidence interval (ISO 2602);*
- *Part 3: Determination of a statistical tolerance interval (ISO 3207);*
- *Part 4: Techniques of estimation and tests relating to means and variances (ISO/DIS 2854)²⁾;*
- *Part 5: Efficiency of tests relating to means and variances (ISO/DIS 3494)²⁾;*
- *Part 6: Comparison of two means in the case of paired observations (ISO/DIS 3301)²⁾.*

There is often confusion over the terms “statistical tolerance interval” and “tolerance zone”. Although closely linked they are not synonymous. The first term relates to a statistical concept outlined on the next page. The latter term, however, relates generally to a limiting (physical) requirement specified for, or imposed on, a particular characteristic (e.g. a dimension of an item may be specified as 50 ± 2 mm, giving a tolerance zone of 48 mm to 52 mm).

A “statistical tolerance interval”, on the other hand, is a function of inherent variability the limits of which are often referred to as “natural limits of a process”. As such it relates directly to “process capability”.

In formal terms a “tolerance zone” is the zone of values in which a measurable characteristic is in conformity with its specification. A “statistical tolerance interval” is an interval for which it can be stated with a given level of confidence that it contains at least a specified proportion of the population (totality of items).

This Part of the standard thus provides techniques by which process capabilities can be established to enable tolerance zones to be specified objectively (e.g. if a tolerance zone is specified which lies inside a statistical tolerance interval then out-of-specification commodities are unavoidable).

Statistical tolerance intervals can be established by either assuming or not assuming a particular distributional form for the characteristic being studied. In the latter case the intervals are often called “distribution-free intervals”. The main advantage of taking into account the distribution of a characteristic is that the number of determinations (sample size) required to provide an interval similar to the distribution-free interval is, in general, significantly smaller. The disadvantage is that the statistical tolerance interval is often critically dependent on the distribution assumed (i.e. gross errors can result in the size of the interval and level of confidence if the assumption is false).

¹⁾ BS 2846:1957 “*The reduction and presentation of experimental results*” by J T Richardson, BSc.

²⁾ In course of preparation.

Section 1 provides procedures for determining statistical tolerance intervals based on the assumption that the distribution of the characteristic under study is “normal”. This subject is discussed briefly in Parts 1 and 4 of this standard and in more detail in BS . . . ³⁾ “*Tests for departure from normality*”. Following the publication of Addendum 1 to ISO 3207 in 1978 which provided graphical representations of Table 9 and Table 10 in the form of two nomographs, Amendment No. 1 was issued to this British Standard to incorporate the additional information.

Annex A provides procedures for the determination of statistical tolerance intervals which may be utilized irrespective of the type of distribution applicable. This Part of this British Standard is identical to ISO 3207 “*Statistical interpretation of data — Determination of a statistical tolerance interval*”.

Because the text of an International Standard has been copied, it includes some minor material not appropriate to a British Standard.

For the purposes of this British Standard the text of ISO 3207 given in this publication should be modified as follows:

Terminology. The words “British Standard” should replace “International Standard” wherever they appear.

The decimal point should replace the decimal comma wherever it appears.

Cross reference. The references to other International Standards should be replaced by references to British Standards as follows.

Clause	Reference to ISO Standard	Appropriate British Standard
General remarks	ISO/DIS 2854 “ <i>Statistical interpretation of data — Techniques of estimation and tests relating to means and variances</i> ”	BS 2846 “ <i>Statistical interpretation of data</i> ” Part 4 ³⁾ “ <i>Techniques of estimation and tests relating to means and variances</i> ”
	ISO . . . ³⁾ “ <i>Statistical interpretation of data — Normality tests</i> ”	BS . . . ³⁾ “ <i>Tests for departure from normality</i> ”

The following British Standards also provide practical guidance on the application of statistical methods.

BS 600, *Application of statistical methods to industrial standardization and quality control*.

BS 1313, *Fraction-defective charts for quality control*.

BS 2564, *Control chart technique when manufacturing to a specification, with special reference to articles machined to dimensional tolerances*.

BS 6000, *Guide to the use of BS 6001. Sampling procedures and tables for inspection by attributes*.

BS 6001, *Sampling procedures and tables for inspection by attributes*.

BS 6002, *Sampling procedures and charts for inspection by variables for per cent defective*³⁾.

BS . . . , *Determination and statistical analysis of precision data for a standard test method*³⁾.

³⁾ In course of preparation.

A British Standard does not purport to include all the necessary provisions of a contract. Users of British Standards are responsible for their correct application.

Compliance with a British Standard does not of itself confer immunity from legal obligations.

Summary of pages

This document comprises a front cover, an inside front cover, pages i to iv, pages 1 to 17 and a back cover.

This standard has been updated (see copyright date) and may have had amendments incorporated. This will be indicated in the amendment table on the inside front cover.

Section 1. Formal presentation of results

General remarks

1) This International Standard specifies methods enabling a sample to be used as the basis for determining a statistical tolerance interval, i.e. an interval such that there is a fixed probability (confidence level) that the interval will contain at least a proportion p of the population from which the sample is taken. The statistical tolerance interval may be two-sided or one-sided. The limits of the interval are called “statistical tolerance limits”; they are also called “natural limits of the process”.

2) These methods are applicable only where it may be assumed that in the population under consideration the sample units have been selected at random and are independent.

3) The methods described below apply also only on condition that the distribution of the characteristic being studied is normal. The requirement of normality is more important here than for the inferences on means and differences between means in ISO 2854, *Statistical interpretation of data — Techniques of estimation and tests relating to means and variances*.

4) In order to check the hypothesis of normality, the methods laid down in ISO . . . , *Statistical interpretation of data — Normality tests*⁴⁾, are used.

5) Where the hypothesis of normality has to be rejected or where there is some reason to doubt its validity, one may envisage transforming the variate to make it normal or applying the method described in the introductory remark of Annex A of this International Standard.

It is also possible to apply now methods which allow the determination of statistical tolerance intervals for other distribution forms than normal distributions. The description of these methods has not been considered in this International Standard.

6) In determining a statistical tolerance interval, it is desirable in connection with the origin or the method of collection of data to give all information that may assist in their statistical analysis, in particular the smallest unit or fraction of a measurement unit having practical significance.

7) No elimination or potential correction of individual data that are doubtful shall be carried out unless there are experimental, technical or obvious reasons to provide circumstantial justification of such elimination or correction.

In every instance, mention shall be made of the data eliminated or corrected.

8) As stated in 1), the confidence level $1 - \alpha$ is the probability that the statistical tolerance interval will contain at least a proportion p of the population. The risk of this interval containing less than a proportion p of the population is α . The most usual values of $1 - \alpha$ are 0,95 and 0,99 ($\alpha = 0,05$ and 0,01).

This means that if statistical tolerance intervals are determined for a large number of samples at the confidence level 0,95 for example, the proportion of those intervals which will contain at least the desired fraction of the population will be close to 95 %.

9) Table 1 and Table 2 are applicable to the case where the standard deviation for the population is known (the mean being unknown); Table 3 and Table 4 to the case where the mean and the standard deviation are unknown.

Where the mean and the standard deviation having respectively the values m and σ are known, the distribution of the characteristic under investigation (assumed to be normal) is fully determined; there is exactly a proportion p of the population:

- on the right side of $m - u_p \sigma$
 - on the left side of $m + u_p \sigma$
 - between $m - u_{(1+p)/2} \sigma$ and $m + u_{(1+p)/2} \sigma$: two-sided interval
- } one-sided intervals

where u_p is the fractile of order p of the standardized normal variate.

Numerical values of u_p are in these cases to be read on the bottom line of Table 5 and Table 6.

10) The calculations can often be very much simplified by making a change in origin and/or in unit.

⁴⁾ In preparation.

Table 1 — One-sided statistical tolerance interval (known variance)¹⁾

Technical characteristics of the population under investigation ²⁾	
Technical characteristics of the sample units ²⁾	
Eliminated observations ³⁾	
Statistical data Sample size: $n =$ Sum of the observed values: $\Sigma x =$ Known value of the variance of the population: $\sigma^2 =$ whence the standard deviation: $\sigma =$ Proportion of the population selected for the statistical tolerance interval ⁴⁾ : $p =$ Chosen confidence level ⁵⁾ : $1 - \alpha =$ $k_1 (n, p, 1 - \alpha) =$	Calculations $\bar{x} = \frac{\Sigma x}{n} =$ $k_1 (n, p, 1 - \alpha) \sigma =$ 6)
Results a) One-sided interval “to the left” There is a probability $1 - \alpha$ that at least a proportion p of the population is above: $L_s = \bar{x} + k_1 (n, p, 1 - \alpha) \sigma =$ b) One-sided interval “to the right” There is a probability $1 - \alpha$ that at least a proportion p of the population is above: $L_i = \bar{x} - k_1 (n, p, 1 - \alpha) \sigma =$	
¹⁾ A numerical example is given in section 2 of this International Standard: example No. 1 ²⁾ See paragraph 6 of General remarks. ³⁾ See paragraph 7 of General remarks. ⁴⁾ See paragraph 1 of General remarks. ⁵⁾ See paragraph 8 of General remarks. ⁶⁾ The values of $k_1 (n, p, 1 - \alpha)$ can be read directly from Table 5 for different values of n , and for $p = 0,90; 0,95; 0,99$ $1 - \alpha = 0,95 \text{ and } 0,99$	