

Methods of

Fatigue Testing —

Part 5: Guide to the application of
statistics

Co-operating organizations

The Mechanical Engineering Industry Standards Committee, under whose supervision this British Standard was prepared, consists of representatives from the following Government departments and scientific and industrial organizations:

Associated Offices' Technical Committee	Institution of Gas Engineers
Association of Consulting Engineers	Institution of Heating and Ventilating Engineers
Association of Mining Electrical and Mechanical Engineers	Institution of Mechanical Engineers
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British Chemical Plant Manufacturers' Association	Institution of Production Engineers*
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British Electrical and Allied Manufacturers' Association*	London Transport Board*
British Gear Manufacturers' Association	Machine Tool Trades Association
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British Iron and Steel Federation*	Ministry of Defence, Navy Department
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Institute of Marine Engineers	National Coal Board
Institution of Civil Engineers	Radio Industry Council
	Royal Institute of British Architects

The Government departments and scientific and industrial organizations marked with an asterisk in the above list, together with the following, were directly represented on the committee entrusted with the preparation of this British Standard:

Aluminium Federation	Ministry of Aviation
British Cast Iron Research Association	Ministry of Technology — Building Research Station
British Non-ferrous Metals Federation	Ministry of Technology — Forest Products Laboratory
British Non-ferrous Metals Research Association	Royal Aeronautical Society
British Welding Research Association	Society of British Aerospace Companies
Copper Development Association	Spring Manufacturers' Research Association
Institute of Welding	Individual manufacturers of testing machines.
Institution of Engineering Inspection	

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Foreword

In order to keep abreast of progress in the industries concerned, British Standards are subject to periodical review. Suggestions for improvements will be recorded and in due course brought to the notice of the committees charged with the revision of the standards to which they refer.

A complete list of British Standards, numbering over 5000, fully indexed and with a note of the contents of each, will be found in the British Standards Yearbook. The BS Yearbook may be consulted in many public libraries and similar institutions.

This standard makes reference to the following British Standards:

BS 600, *Application of statistical methods to industrial standardization and quality control*.

BS 3518, *Methods of fatigue testing — Part 1. General principles*.

This British Standard, which forms part of a series dealing with fatigue testing of metals, is intended as a corollary to Part 1, “General principles” which gives guidance in this specialized form of mechanical testing. In preparing this standard the drafting committee has recognized the progress that has been made in adapting well established statistical techniques to the problem of variability associated with fatigue determinations and the need for a wider appreciation of the usefulness of such techniques in the assessment of scatter.

It has also been necessary to assume that the user of this standard will have some knowledge of statistical techniques. In consequence, although the standard contains symbols and definitions for certain statistical terms which are used, it does not elaborate on the mathematical basis of the techniques described nor on statistical theory which can be acquired from text books on this subject. References to such works are given in Appendix A.

The specific purpose of this standard is to provide guidance in the application of statistical methods to the planning of test procedures and the subsequent analysis of the experimental data obtained by carrying out fatigue tests in accordance with any one of the methods described in Parts 2, 3 and 4 in the series.

The purposes of performing fatigue tests are many, but this standard has been confined to those four types of determination which are considered to be of the most universal interest and are those commonly required in routine testing. In addition these have received considerable attention from statisticians with regard to both the best test procedure to use and the most appropriate methods of analysing the data obtained. The standard has therefore been divided into four sections, each section dealing with a particular type of fatigue determination.

Each of the four sections outlines the test procedure to be followed to provide the maximum amount of information without waste of material or testing time with the available test pieces. Guidance is given in each section as to the methods of analysis appropriate to the type of determination being undertaken. Where more than one method has been given the drafting committee have clearly stated their preference and stated in detail the reasons for their choice.

The scope of the standard has been made sufficiently wide to permit the inclusion of statistical methods and the relevant checks for making comparisons of two or more sets of data obtained from similar series of fatigue tests.

Throughout the standard worked examples have been included for all the methods of analysis and tests, stress having been used as the basis for the determination of fatigue properties. The methods are equally applicable when strain is used as the basis. Guidance regarding the minimum number of test pieces required are given in the description of the techniques, and it should be borne in mind that if few results are obtained margins of error may be greater.

The drafting committee wishes to express its sincere appreciation of the detailed work carried out by its specialist Panel in the preparation of this standard, and in particular to Mr. D. J. Spurrell of the Royal College of Advanced Technology, Salford, whose services in a personal capacity were invaluable as regards the presentation of the statistical data and examples.

A British Standard does not purport to include all the necessary provisions of a contract. Users of British Standards are responsible for their correct application.

Compliance with a British Standard does not of itself confer immunity from legal obligations.

Summary of pages

This document comprises a front cover, an inside front cover, pages i to iv, pages 1 to 36, an inside back cover and a back cover.

This standard has been updated (see copyright date) and may have had amendments incorporated. This will be indicated in the amendment table on the inside front cover.

1 Scope

This British Standard gives guidance in the application of statistical techniques to the planning of experiments for determining the fatigue properties of materials and to the subsequent analysis of the test data.

For simplicity the standard is divided into sections according to the type of information required from the tests, viz.:

- Section 1. The determination of S/N curves for different probabilities ($P/S/N$ curves).
- Section 2. The determination of fatigue strength at N cycles.
- Section 3. The determination of a fatigue limit.
- Section 4. The determination of life at a constant stress amplitude.

The standard also provides guidance on the statistical methods available for making comparisons between two or more sets of similar data. Details of the appropriate tests and checks on the statistical significance of the analytical methods are given throughout the standard, as are worked examples of all the methods and tests.

2 Terms, definitions and symbols

The terms and symbols used, together with the definitions to be attributed to them in this standard are included here for guidance. Distinction is made between terms and symbols which are either basic to fatigue or to statistics and those used in the statistical analysis of fatigue test data.

a) Terms and their definitions.

i) *fatigue terms*

See BS 3518-1, “General principles”.

ii) *Statistical terms*

confidence intervals

the interval between two statistics, t_1 , t_2 , which are functions of the sample values only, is a confidence interval if it is possible to assert that the parameter which is being estimated lies in this interval with a known probability α . This assertion will be true on average in a proportion α of the cases when it is made

confidence limit

the values t_1 and t_2 which form the upper and lower limits to the confidence interval

cumulative distribution curve

a graph of cumulative frequency (or percentage cumulative frequency) as ordinate against the observed value as abscissa

cumulative frequency

the total frequency up to the given magnitude of the observed value, i.e. the total number of occurrences of values up to and including the stated value. The cumulative frequency is often given as a percentage of n , the total frequency over the whole range of the observed values

degrees of freedom

the number of degrees of freedom is a parameter for the χ^2 , F and t distributions. For the F and t distributions, the number is calculated by subtracting one plus the number of parameters estimated from the data from the number of sample observations. For the χ^2 distribution, the number is based upon the number of sets of data which are being compared

deviation

the difference between an observation and the mean of all the observations

error

the difference between an observed value of a variable or of a statistic and its “true” or “expected”, i.e. population, value

The analysis of error and the sources of error is one of the most important objects of statistics.

random error

errors arising by chance events over which the experimenter has no control and for which statistical methods are most appropriate for analysis. It is assumed that each observation is affected by errors which are random samples from a specified distribution

standard error

the standard deviation of a statistic, obtained from repeated samples, as distinct from repeated observations on the test pieces within a given sample. It is a measure of the precision with which a population parameter is estimated by a sample statistic

systematic error

errors which persist irrespective of the samples taken, and which are due to the design of the sample or experiment and not to chance, e.g. a constant error in the calibration of a testing machine. The effect of systematic errors is to influence certain observations differently from others in such a way as to prevent the drawing of inferences until such systematic errors have been eliminated or allowed for

frequency

the number of observations for which the variable has values within an interval having specified limits and width. In the limit as the total number of observations becomes very large, the frequency of occurrence of a given value

frequency distribution

a specification of the relative frequency with which values of the variable occur in a sample, or in the population. This may be expressed in tabular form by grouping the observed data according to magnitude of the variable, and represented either graphically, as a histogram or frequency distribution curve, or as a mathematical function

goodness of fit

the closeness of agreement between an observed set of values and a second set which are wholly or partly derived from hypothesis, i.e. from the “fitting” of a model or curve to the data

histogram

graphical representation of a frequency distribution in which, using each interval of the variable as base, a rectangle is constructed whose area represents the frequency in the interval. If, as is usual, the intervals are equal, the heights of the rectangles will be proportional to the frequencies represented

mean

the arithmetical average of a set of observations

median

a special case of the percentile when $p = 0.5$. It is the value of the variable which divides the total frequency so that 50 per cent exceeds the value

normal (or Gaussian) distribution

a symmetrical, bell shaped frequency distribution with the mean and standard deviation as parameters. The normal distribution is used, often as an approximation, for the distribution of certain statistics, e.g. the mean. The logarithm of the fatigue life of test pieces is often assumed to follow the normal distribution.

parameter

a constant which defines some property of the frequency distribution. The constant is usually unknown, and the object of the experiment is frequently to estimate it, e.g. mean, standard deviation

percentile

a $100p$ percentile is the value of the variable which divides the frequency distribution into two parts. The part below the value corresponds to $100p$ per cent of the total frequency; the part above the value corresponds to $100(1 - p)$ per cent of the total frequency

population

a large collection of specimens from one definite source, each perhaps possessing a number of different and variable characteristics. A population may form the actual total of a batch of specimens produced under nominally constant conditions, e.g. a batch of bolts from production, or it may represent the hypothetical collection of all possible test specimens that could be prepared in a specified way from the material under consideration, as would apply for example when preparing a set of standard fatigue test pieces from a large bar of material to be otherwise used for making machine components

probability

a numerical measure of the frequency with which a value is expected to occur if a very large number of observations were to be made under the same conditions

probit analysis

probit analysis is applied in situations where the individuals in a population require a varying stimulus to produce a response. The test is designed so that groups of individuals are tested at different levels of the stimulus, and the number which respond are counted. The statistical objective is to estimate the parameters of the distribution of critical stimuli from the experimental data (*see response curve*)

randomization (random selection)

a process of selecting test pieces (or treatments or testing conditions, etc.) from a large group such that there is an equal chance of each test piece, etc. being selected at each stage of the selection process. Tables of random numbers are often employed to simplify the process

ranks

the ordinal numbers given to sample observations when they are arranged systematically in ascending order of magnitude

regression line or equation

the line, or equation which is fitted to sample observations on two or more variables to describe the relationship between one of the variables and the others. In this standard, the relationship between only two variables is required

replication

the carrying out of an experiment more than once so as to increase the precision of estimation of a population parameter and to reduce the corresponding standard error

sample

test piece selected for a test programme. The selection is made in such a way as to be representative of the material under consideration, or population

NOTE The method of selecting the sample defines the characteristics of the material of which it is representative. Unless due care is exercised, the sample may be representative of something other than was intended i.e. the material is restricted to certain characteristics. For example, the sample may be representative of the material at the ends of bars, rather than the bar material as a whole.

significance

tests of hypotheses usually take the form of assuming that a given effect is not present, and that variation in the sample results is due to random variation. Using an appropriate test statistic, an assessment is made of the probability that the variations occurred by chance without the presence of the effect. If the probability is small enough (in the context of the test and hypothesis) then the result is said to be significant, and the presence of the effect admitted as a likely explanation of the “larger than expected” differences. If the probability is not small, then the presence of the effect is not admitted

standard deviation

a measure of the dispersion of a frequency distribution, i.e. the square root of the variance

standardized normal variable

the value of a variable, measured in units of the standard deviation from the mean, which gives a stated percentile of the normal frequency distribution

statistic

a summary value calculated from the observed values in a sample. Under this term come both parameters, and special statistics for tests of significance

transformed values (transformation of values to standardised normal variables)

in the analysis of data to obtain a response curve, the assumption is made that the percentage survival value has a cumulative normal distribution with respect to the applied stress, i.e. that the number of test pieces having the given life at N cycles is normally distributed with respect to the applied stress. The analysis therefore requires that the present survival values should be transformed to a value of a normal variable with zero mean and unit standard deviation for which the cumulative frequency equals that given by the experimental results. The rest of the analysis is then completed in terms of the transformed variable because this bears a linear relationship to the applied stress if the basic assumptions hold

variation

the difference between two or more sets of data which include all sources of error

variance

the mean of the squares of the deviations

χ^2 , F, Student's t distribution

these are important distributions used in testing the significance of test statistics. The parameter in these distributions is the number of degrees of freedom

iii) ***Terms used in the statistical analysis of fatigue experiments.*****cut off**

the number of cycles, decided before testing commences, at which the testing will be terminated if the test piece is still unbroken. Thus one refers to the "fatigue strength at (a cut-off of) N cycles"

run out

a test piece which has reached the cut-off life without breaking and for which no complete life is available

P/S/N curve

a curve fitted to the fatigue lives giving an estimated 100 p per cent failure at each of several stress levels, in order to estimate the relationship between applied stress and the number of cycles to failure that 100 p per cent of the population would not survive

NOTE In some cases the $P/S/N$ curve is taken to represent the relationship between applied stress and the life that 100 p per cent would survive. One definition can be converted to the other by taking 100 p per cent failure as equivalent to 100(1 - p) per cent survival.

the Median S/N curve

the curve fitted to the median values of fatigue life at several stress levels

fatigue strength at N cycles

the stress value at which a test piece would have a life of exactly N cycles

distribution of fatigue strength at N cycles

the frequency distribution of the stress at which test pieces fail at N cycles. This distribution is usually estimated by response curve methods

response curve for N cycles

a curve fitted to the observed values of percentage survival at N cycles for several stress levels. The method used is that of probit analysis (q.v.) where the stimulus is applied stress, and the response is the percentage survival at N cycles

staircase estimation of mean fatigue strength at N cycles

a sequential method of testing pieces at equally spaced stress levels to estimate the mean fatigue strength without investigating the whole distribution.

b. Symbols.

i) ***Fatigue symbols.*** See BS 3518-1, "General principles".

ii) ***Statistical symbols.***

χ^2	Chi-squared variable, extensively tabulated.
$\chi^2_{\alpha(r)}$	the 100 α percentage point of a chi-squared variable with r degrees of freedom.
$F_{r,s}$	the F variable, or variance ratio, with r and s degrees of freedom. Also extensively tabulated.
t	Student's t statistic, extensively tabulated.
s	standard deviation.
q	the studentized range test statistic.
Y	the standardized normal variable.
a	the intercept of the regression line fitted in probit analysis.
b	the slope of the regression line fitted in probit analysis.

iii) ***Statistical symbols applied to fatigue tests.***

n	the number of test pieces.
θ	the mean of exponentially distributed lives, the parameter of the exponential distribution.
r	the number of failed test pieces.
X	the stress applied in a probit experiment.
$\bar{X}$	the mean stress applied.

Y_f	the standardized normal variable predicted from the probit regression line for a given stress level.
$X(Y)$	the predicted stress level to give a stated percentage of test pieces surviving. The percentage is first changed into the equivalent Y variable.
N	the life of a test piece, or the life at which testing is terminated, usually the number of cycles.
$L(100p)$	the percentile of a life distribution where $100p$ per cent fail and $100(1 - p)$ per cent survive.
T	the total time of all specimens on test.

Many of the above symbols are written with subscripts i or j indicating that they are one of a set of similar quantities, e.g. χ_j^2 is the χ^2 calculated for the j -th stress level and N_i is the fatigue life of the i -th test piece.

3 Design of experiments and the control of error

In fatigue testing a number of test pieces are selected to be representative of a much larger group of possible test pieces. Since it is not possible to eliminate error completely from all stages of sampling, test piece preparation and testing, precautions must be taken to control the effect of error on the test material and on comparisons to be made in the tests.

Errors in the selection and testing procedures produce variation in the test results. It is the object of statistical techniques to analyse and draw valid conclusions from situations showing variation, and to apportion variation to the various sources.

Broadly the errors are of two kinds, namely systematic errors, and random errors. Random errors are those arising by chance events over which the experimenter has no control, and for which statistical methods are most appropriate. Systematic errors are those which persist irrespective of the number of samples, e.g. when material A is given treatment 1 and material B receives treatment 2, it is not possible to establish whether differences in the two sets of results are due to differences in the materials or the treatments. Such errors can, however, be avoided by a suitably designed programme. The two kinds of error occur in varying degrees at all stages of selection, heat treatment, processing of blanks, and the preparation and testing of the test pieces.

For good design in the experiment it is necessary to assess the relative importance of these various sources of variation. It is rarely adequate to attend to one source of variation only and to ignore the remainder.

Two important methods of controlling the effects of error are randomization and replication.

Randomization is used to allocate treatments to test pieces and to arrange the order in which material is prepared or machined. Its use is to safeguard against unsuspected patterns of variation, e.g. along a bar of material, wear of machine tools, etc. If randomization is employed it is recommended that published tables of random numbers be used. However, if randomization leads to very complicated schedules and to the possibility of serious mistakes in carrying them out, it should be used with caution.

Replication reduces the effect of random variability and permits the estimation of the random error from the comparison of different test pieces exposed independently to the same conditions.

It is emphasized that the treatment of data by the application of statistical techniques cannot give results any more accurate than the original accuracy of the test data. It is, therefore, necessary to decide the accuracy required from the results when planning the tests. The examples shown in the various sections of this standard should not be regarded as implying any specified degree of accuracy.

Section 1. Determination of *S/N* curves for different probabilities

4 Procedure

S/N curves can be most accurately determined when groups of test pieces are tested at each of a series of different stress levels. The number of test pieces in each group should not be less than five, and if some indication of the shape of the distribution at each stress level is required the number should be increased to at least ten. It may be adequate to use the minimum number at each stress level but it is preferable to increase the number per group at the lower levels where an increase in variability occurs.

The results obtained may be regarded as samples from a life distribution at particular stress levels, and it is then possible to make estimates of life for a given probability. It is recommended that a given probability be chosen and appropriate estimates of life made for each stress level as an *S/N* curve can then be fitted to such estimates. If several probabilities are required, a diagram consisting of curves which resemble a “contour map” can be derived. These are commonly referred to as *P/S/N* curves.

5 Analysis

The determination of a *P/S/N* curve may be made by assuming a particular distribution for lives at a given stress level and determining the percentage failure points, but it is usually sufficient to employ a simple graphical analysis. For this graph paper which will permit stress against log life or log stress against log life to be plotted is required.

The numerical values of the lives of test pieces tested at each individual stress level require to be tabulated in ascending order of magnitude, and the median life calculated. A minimum of five test results should be used at each stress level for the determination of median life, but if probability levels of failure other than 50 per cent are desired the number of test results should be increased proportionately.

The median life, i.e. 50 per cent chance of failure, is given by the $\frac{n+1}{2}$ life when n , the number of test results at the stress level (and perhaps including run outs) is odd. When n is even the estimate of the median is midway between a life of $\frac{n}{2}$ and $\frac{n+2}{2}$. In general the probability of failure for the i -th life in the list of ordered lives is best given by $\frac{i}{n+1}$.

Once the probability level for lives has been chosen, these lives should be estimated from the data at each stress level and plotted on the graph. Thus the points may be plotted for say the lives by which 10 per cent (or 50 per cent) of the test pieces may have failed. A line representing the points over several stress levels may be sketched in as an approximate *P/S/N* curve. This line will in no sense be a “best fit” but arbitrary; also because the scale is logarithmic for one or both axes the errors are not equal everywhere and this will deceive the eye. In some cases it will be possible and preferable to fit a convenient function by mathematical methods.

*Example of empirical determination of *P/S/N* curve when $P = 50$ per cent:*

Table 1 gives the lives of four groups of eight test pieces tested at four different stress levels.

Table 1

	Stress (tonf/in ²)			
	26	24	22	20
Lives (10 ⁶ cycles)	0.62	0.20	1.86	3.32
	3.56	1.24	11.58	15.44
	4.00	1.66	13.38	16.00
	4.62	15.54	22.02	35.68
	5.11	17.84	29.04	36.64
	6.16	34.18	30.66	37.68
	12.73	35.44	31.50	46.84
	13.47	48.16	38.19	49.92

The 50 per cent life is estimated by the average of the $\frac{n}{2} = 4\text{th}$ and $\frac{n+2}{2} = 5\text{th}$ life. Thus the 50 per cent estimate for the 26 tonf/in² stress level is the mean of 4.62 and 5.11, which is 4.865.

The estimates for the four stress levels are given in Table 2.

Table 2

Stress tonf/in ²	50 per cent lives
26	4.865
24	16.690
22	25.530
20	36.160

6 Comparison of *P/S/N* curves

Although it is not possible to give a general method of fitting a mathematical function to the data, it is possible to devise a test which will measure the agreement of several sets of the data with each other. This test does not assume any particular distribution of lives at each stress level.

For each stress level j , a test is made as to whether the unspecified life distribution is the same for each set of data. For this purpose calculate:

$$\chi_j^2 = \left[\frac{12}{n_j(n_j + 1)} \sum_{i=1}^k \frac{(R_i)^2}{(r_i)} \right] - 3(n_j + 1)$$

where k = number of sets of data tested at stress level, j . ($j = 1, 2, \dots, c$);

r_i = number of lives, obtained in the i -th set of data, less than a common terminating life applied to all data;

R_i = sum of rankings of the i -th set of results when all the results are arranged in ascending order and numbered 1, 2, 3, 4, ... etc.;

$$n_j = \sum_{i=1}^k r_i$$

All the r_i must be greater than 4 otherwise the test is not valid, and all the data must have the same cut off point to maintain strict comparability. If the data has been collected from different sources each with different cut off points, then the lowest cut off point is employed, and the observed lives which exceed this are ignored. If less than five lives are given in any set as a result of this procedure, then the set giving the lowest cut off should be removed, and the next highest cut off should be employed. If the set with the lowest cut off is not the set with less than five lives, this appears to be strong evidence that there are different distributions of life at this stress level.

Values of the statistic χ_j^2 which are exceeded only a given proportion of the time in random sampling from the same population are given in tables, and depend upon the number of degrees of freedom ($k - 1$). If the value of χ_j^2 calculated from the test results is less than the tabulated value with ($k - 1$) degrees of freedom at an appropriate significance level, it is reasonable to conclude that the arrangement of the test results is random, and that there are no differences between the k life distributions. If the value of χ_j^2 is greater than the tabulated value, then the arrangement is not random but systematic. This means that at this stress level, the k sets of results show different life distributions.

This technique assesses the results at only one stress level, but a comparison is possible between the c stresses which were used in the tests. The technique described is applied to each stress level, at each of which there are k sets of results. To test whether the life distributions are the same for all sets of results over all the stress levels, the statistic χ^2 is calculated.

$$\chi^2 = \sum_{j=1}^c \chi_j^2$$

The value of χ^2 can be compared with tabulated values at a significance level, but the degrees of freedom are now given by $c(k-1)$ when there are k sets of results at each stress level. If there are k_j sets of results at the j^{th} stress level, and these differ from stress to stress, it is still possible to use the test with

$\sum_{j=1}^c (k_j - 1)$ degrees of freedom. However, the interpretations of significant results will be more difficult than in the case of k sets of results at each stress level.

In this case, a value of χ^2 less than the tabulated significance point means that the arrangement of the test results is random at all stresses, and that the same $P/S/N$ curve will apply to all the sets of results. If the result is significant on the other hand, the arrangement is not random, indicating that there are differences between the sets of results. Further analysis is required to determine $P/S/N$ curves for the sets of results.

Example of comparison of life data at a single stress:

Four sets of results are considered A, B, C and D, with a life cut off at 51×10^6 cycles.

The lives are first listed and ranked as in Table 3.

Table 3

A		B		C		D	
Life ^a	Rank	Life ^a	Rank	Life ^a	Rank	Life ^a	Rank
1.24	(2)	2.34	(4)	13.86	(12)	0.60	(1)
2.10	(3)	5.22	(6)	15.54	(13)	6.28	(7)
4.96	(5)	6.84	(8)	25.44	(17)	10.90	(10)
17.34	(14)	9.24	(9)	28.54	(19)	17.36	(15)
25.46	(18)	11.60	(11)	28.56	(20)	32.18	(21)
U.B. ^b	—	22.18	(16)	U.B. ^b	—	50.50	(22)
^a $\times 10^6$ cycles ^b unbroken							