

Spur and helical gears —

Part 3: Method for calculation of contact and root bending stress limitations for metallic involute gears

UDC 621.833.1

NO COPYING WITHOUT BSI PERMISSION EXCEPT AS PERMITTED BY COPYRIGHT LAW

BSI
Documents

AVA

Document Sharing Hub

Committees responsible for this British Standard

The preparation of this British Standard was entrusted by the Machinery and Components Standards Committee (MCE/-) to Technical Committee MCE/5 upon which the following bodies were represented:

- Association of Consulting Engineers
- British Clock and Watch Manufacturers' Association
- British Gear Manufacturers Association
- British Horological Institute
- British Railways Board
- Engineering Equipment and Materials Users' Association
- Federation of Manufacturers of Construction Equipment and Cranes
- Gauge and Tool Makers' Association
- Institution of Mechanical Engineers
- Institution of Production Engineers
- Lloyds Register of Shipping
- Machine Tool Industry Research Association
- Milling Cutter Association
- Ministry of Defence
- National Coal Board
- Society of Motor Manufacturers and Traders Limited

This British Standard, having been prepared under the direction of the Machinery and Components Standards Committee was published under the authority of the Board of BSI and comes into effect on 30 September 1986

© BSI 06-1999

The following BSI references relate to the work on this standard:
Committee reference MCE/5
Draft for comment 84/73219 DC

ISBN 0 580 15227 8

Amendments issued since publication

Amd. No.	Date of issue	Comments
5797	May 1988	Indicated by a sideline in the margin

Contents

	Page
Committees responsible	Inside front cover
Foreword	ii
Section 1. General	
1 Scope and field of application	1
2 Definitions and symbols	2
3 Guide to using the calculation procedure	7
Section 2. Contact stress calculations	
4 Basic equations for contact stress calculations	8
5 Nominal tangential force for contact stress, F_{Ht}	8
6 Zone factor for contact stress, Z_H	9
7 Contact ratio factor for contact stress, Z_ϵ	9
8 Elasticity factor for contact stress, Z_E	10
9 Basic endurance limit for contact stress, $\sigma_{H \text{ lim}}$	10
10 Material quality for contact stress, Z_M	12
11 Lubricant influence, roughness and speed factors for contact stress, Z_L , Z_R and Z_v	13
12 Work hardening factor for contact stress, Z_W	13
13 Size factor for contact stress, Z_X	13
14 Life factor for contact stress, Z_N	13
15 Application factor, K_A	16
16 Dynamic factor, K_v	18
17 Load distribution factors, $K_{H\alpha}$ and $K_{H\beta}$	22
18 Minimum demanded and actual safety factors on contact stress, $S_{H \text{ min}}$ and S_H	26
Section 3. Bending stress calculations	
19 Basic equations for tooth root bending stress	27
20 Nominal tangential force for bending stress, F_{Ft}	28
21 Geometry factors for bending stress, Y_F , Y_S , Y_β	28
22 Basic endurance limit for bending stress σ_{F0}	31
23 Material quality factor for bending stress, Y_M	32
24 Sensitivity factor for bending stress, Y_σ	33
25 Surface condition factor for bending stress, Y_R	34
26 Size factor for bending stress, Y_X	34
27 Life factor for bending stress, Y_N	36
28 Load factors for bending stress, $K_{F\alpha}$ and $K_{F\beta}$	36
29 Minimum demanded and actual safety factors on tooth root stress, $S_{F \text{ min}}$ and S_F	36
Appendix A Variable duty calculations	39
Appendix B Gearing equations	41
Appendix C Design guidance on tooth modifications	42
Appendix D Typical residual stresses	44
Appendix E Tooth and mesh stiffness c' and c_y	45
Appendix F Definition of material quality	46
Appendix G Examples of calculations	47
Appendix H Equations of graphs	52
Figure 1 — Yield strength for contact stress, σ_{HY}	10
Figure 2 — Values of σ_{HD}	11
Figure 3 — Values of Z_{G2}	12

	Page
Figure 4 — Casedepth factor, Z_c	12
Figure 5 — Combined speed and lubricant factors, $Z_L Z_v$	14
Figure 6 — Roughness factor, Z_R	14
Figure 7 — Values of Z_W	15
Figure 8 — Life factor for contact stress, Z_N	16
Figure 9 — Constituent parts of typical gear load	17
Figure 10 — $K_{v350\beta}$ for helical gears, $\epsilon_\beta \geq 1$	21
Figure 11 — $K_{v350\alpha}$ for spur gears	21
Figure 12 — Constant K for calculation of f_{sh}	24
Figure 13 — Values of q_y	25
Figure 14 — Dimensions of the basic rack of the gearing	29
Figure 15 — Values of σ_{F0}	32
Figure 16 — Values of Y_R	35
Figure 17 — Values of Y_X	35
Figure 18 — Y_N for through hardened steel	37
Figure 19 — Y_N for thick case surface hardened steel and cast iron	37
Figure 20 — Y_N for thin case surface hardened steel, grey cast iron and bronze	38
Figure 21 — Typical S/N curve	39
Figure 22 — Height and length of end relief	42
Figure 23 — Height of crowning	43
Table 1 — Value of Z_E for some material combinations	10
Table 2 — Limiting casedepth	11
Table 3 — Values of Z_M	13
Table 4 — Values of application factor, K_A	16
Table 5 — Examples of prime mover with different working characteristics	16
Table 6 — Examples of driven machines with different working characteristics	17
Table 7 — Values of X	19
Table 8 — Auxiliary value, A	23
Table 9 — Minimum and maximum values of $K_{H\alpha}$	26
Table 10 — Value of Y_M	33
Table 11 — Values of ρ'	33
Table 12 — Default values for contact stress S/N curve parameters	40
Table 13 — Typical values of residual stress, σ_R	44
Table 14 — Change in residual stress due to post-hardening operations	45
Table 15 — Variable duty calculation example	52
Table 16 — Values of $K_{v350\beta}$ at discontinuities	55
Table 17 — $K_{v350\beta}$ termination points	55
Table 18 — Values of $K_{v350\alpha}$ at discontinuities	55
Table 19 — $K_{v350\alpha}$ termination points	56
Publication referred to	Inside back cover

Foreword

This Part of BS 436 has been prepared under the direction of the Machinery and Components Standards Committee. It is a general application standard for spur and helical external and internal gears.

The standard follows the principles developed by the International Organization for Standardization (ISO) in that the stress levels in the tooth flank and in the tooth root are calculated and compared with basic permissible stress levels derived from tests on simple test specimens.

Modifying factors used to calculate stress levels are based on the ISO proposals but have been adjusted to avoid the step functions which occur in these proposals.

This Part of BS 436 together with BS 436-1 "*Basic rack form pitches and accuracy (diametrical pitch series)*" and BS 436-2 "*Basic rack form, modules and accuracy (1 to 50 metric module)*" supersede BS 436:1940 which is therefore withdrawn.

BS 436-1 is retained solely for the purpose of supplying replacement gears designed in accordance with the imperial system of units.

To assist in the data processing of the calculations given in this standard, FORTRAN sub-routines can be obtained from the British Gear Association¹⁾.

Procedures for some factors are extracted or derived from Draft International Standard ISO/DIS/6336/1, 2 and 3, "*Calculation of load capacity of spur and helical gears*".

A British Standard does not purport to include all the necessary provisions of a contract. Users of British Standards are responsible for their correct application.

Compliance with a British Standard does not of itself confer immunity from legal obligations.

Summary of pages

This document comprises a front cover, an inside front cover, pages i to iv, pages 1 to 58, an inside back cover and a back cover.

This standard has been updated (see copyright date) and may have had amendments incorporated. This will be indicated in the amendment table on the inside front cover.

¹⁾ British Gear Association, c/o Institution of mechanical Engineers, Birdcage Walk, London SW1 H9JJ.

Section 1. General

1 Scope and field of application

1.1 This Part of BS 436 is a general application standard for spur and helical external and internal gears of accuracy grades 3 to 10 operating at any pitch line speed. The calculations given in this standard are not applicable to the prediction of gear damage caused by scuffing, wear, welding or fracture of the gear rims, web, or hub.

The standard covers methods for determining the actual and permissible contact stresses and bending stresses in a pair of involute gears. Stress levels on the tooth flank and in the tooth root are calculated and compared with basic permissible stress levels derived from simple test specimens. Modification factors are given to take account of:

- a) the effects of dimensional variations arising from manufacture and assembly;
- b) vibrations arising from sources internal and external to the gears;
- c) the effect of the lubricant film and gear flank roughness;
- d) stress concentration effects in the tooth root;
- e) the effect of different depths of case or surface hardened gears;
- f) the effect of bending stress on the tooth flank stress cycle;
- g) the effect of residual stress in the tooth root.

Procedures are included for calculating the peak load capacity and for taking account of variable duty.

1.2 The gear type and qualifications in respect of the gear design are as follows:

- a) *Types of gears*: internal and external spur, helical and double helical gears.
- b) *Range of speed*: no restriction but note that at pitch line speeds less than 1 m/s the load capacity is often limited by wear.
- c) *Gear accuracy*: grade 3 to 10 of BS 436-2. The calculation of load modifying factors are based on the largest deviation allowed for the particular manufacturing grade.
- d) *Range of transverse contact ratio*: $1.2 \leq \epsilon_\alpha \leq 1.9$.
- e) *Range of helix angle*: $\beta \leq 45^\circ$.
- f) *Basic racks*: no restriction.
- g) *Pinion and pinion shaft*: solid or hollow pinion with $d_{i1}/d_{f1} \leq 0.5^{2)}$.
- h) *Gear blank and rim*: solid gear blanks and fabricated or cast wheels with rim thickness under the root greater than $3.5m_n^{2)}$.
- j) *Material*:
 - 1) through hardened steel;
 - 2) surface hardened steel;
 - 3) cast iron;
 - 4) bronze.

Three grades of material and material production quality are specified (see Appendix F). The permissible stresses are reduced by a factor Z_M on contact stress and Y_M on bending stress for lower quality materials (see clauses **10** and **23**).

The effect of residual stress at the tooth root is included in this standard. Surface hardening processes, e.g. carburizing, nitriding and induction hardening, induce beneficial compressive residual stress at the surface balanced by tensile residual stress in the region of the case/core junction. Grinding the tooth surface after hardening can reduce the compressive stress and may leave a tensile stress at the surface. A compressive stress can be introduced (or re-introduced after grinding) by means of controlled shot peening. Typical values of residual stresses resulting from good heat treatment practice are included in Appendix D. Appendix H gives the equations and data from which the graphs in the appropriate figures are derived.

²⁾ If the gears have dimensions outside these limitations then additional calculations are necessary to check the stress levels at the root of the teeth.

Where no in-house data are available for the calculation of the endurance limit for contact stress, suitable fatigue factors can be determined from the information contained in clause 9.

NOTE The titles of the publications referred to in this standard are listed on the inside back cover.

2 Definitions and symbols

2.1 Definitions

For the purposes of this Part of BS 436 the definitions given in BS 2519-1 apply together with the following.

2.1.1

effective casedepth, c_{eff}

the depth at which the hardness falls below 500 HV for carburized and nitrided cases or below 450 HV for induction hardening

2.1.2

endurance limit for contact stress, $\sigma_{\text{H lim}}$

the maximum contact stress that may be sustained for an infinite number of cycles without the occurrence of progressive fatigue damage (pitting)

2.1.3

limiting casedepth, c_{lim}

that effective casedepth beyond which a further increase in casedepth does not produce a further increase in failure load

2.1.4

nominal tangential force for bending stress, F_{Ft}

the force tangential to the reference cylinder and perpendicular to its straight generators

2.1.5

nominal tangential force for contact stress, F_{Ht}

the force tangential to the reference cylinder and perpendicular to its straight generators

2.1.6

peak torque capacity for bending stress, $T_{\text{F max}}$

that torque which may be transmitted for up to 1 000 tooth cycles during the design life of the gears without causing failure due to bending stress

2.1.7

peak torque capacity for contact stress, $T_{\text{H max}}$

that torque which may be transmitted for up to 1 000 tooth cycles during the design life of the gears without causing failure due to contact stress

2.1.8

tooth stiffness constant, c' and c_{v}

that force which will deform one or several meshing gear teeth having a facewidth of 1 mm by an amount of 1 μm

2.2 Symbols

For the purposes of this British Standard the following symbols apply.

NOTE This subclause is based on the symbols of BS 2519-2:1976.

Symbol	Designation	Units
a	centre distance ^a	mm
b	facewidth	mm
c_{eff}	effective casdepth	mm
c'	maximum tooth stiffness of one tooth pair in normal section	N/(mm μm)
c_{v}	mean value of total tooth stiffness (or mesh stiffness) per unit facewidth	N/(mm μm)
d	reference diameter	mm
d_1	reference diameter of pinion	mm
d_2	reference diameter of wheel	mm
d_{a}	tip diameter	mm
d_{an}	virtual tip diameter	mm
d_{b}	base diameter	mm
d_{bn}	virtual base diameter	mm
d_{en}	virtual diameter to highest point of single tooth pair contact	mm
d_{f}	root diameter	mm
d_{f2}	root diameter of internal gear	mm
d_{fn2}	virtual root diameter of internal gear	mm
d_{i}	internal diameter	mm
d_{m}	mean diameter $[= (d_{\text{a}} + d_{\text{p}})/2]$	mm
d_{n}	virtual reference diameter	mm
d_{w1}	pitch diameter of pinion	mm
d_{w2}	pitch diameter of wheel	mm
f	individual deviation	μm
f_{f}	profile tolerance (maximum of pinion and wheel)	μm
f_{ma}	mesh misalignment due to manufacturing tolerance	μm
f_{p}	permissible single pitch deviation	μm
f_{pe}	tolerance on pitch	μm
f_{sh}	mesh misalignment due to shaft deflections	μm
g	grinding allowance	mm
g_{α}	length of path of contact	mm
h	tooth depth	mm
h_{a0}	addendum of the basic rack of the tool	mm
h_{f2}	dedendum of internal gears	mm
h_{fp}	dedendum of the basic rack of the gearing	mm
h_{pr}	height of protuberance	mm
h_{F}	bending moment arm	mm
l	bearing span	mm
l_{c}	length of end relief per flank	mm
m	module	—
m_{n}	normal module	mm

^a a , u and z_2 are negative for internal gears.

Symbol	Designation	Units
n	rotational speed	r/min
p	pitch	mm
p_{bn}	normal base pitch	mm
p_{bt}	transverse base pitch	mm
p_r	protuberance of tool	mm
q	combined flexibility of a pair of teeth	mm $\mu\text{m}/\text{N}$
q_s	notch parameter	—
q_y	reduction factor on misalignment due to running-in	—
s_{Fn}	thickness of virtual tooth at critical section	mm
s_{pr}	residual undercut left by the protuberance	mm
u	gear ratio ^a	—
v	pitch line velocity	m/s
w	specific loading F_t/b	N/mm
w_m	mean load intensity	N/mm
x	addendum modification coefficient	—
y	running-in allowance (only with subscript α or β)	μm
y_α	running-in allowance for $K_{H\alpha}$	μm
z	number of teeth ^a	—
z_v	virtual number of teeth	—
B	parameter for effect of load	—
C_a	height of tip or root relief	μm
C_b	height of end relief	μm
C_c	height of crowning	μm
D	tool diameter	mm
E	Young's modulus of elasticity	MN/m^2
F_m	mean tangential force	N
F_t	nominal tangential force at reference circle	N
F_{Ft}	nominal tangential force for bending stress	N
F_{Ht}	nominal tangential force for contact stress	N
$F_{\beta x}$	mesh misalignment prior to running-in	μm
$F_{\beta y}$	effective mesh misalignment	μm
(HV)	Vickers hardness value	—
I	polar moment of inertia	kg m^2
K_v	dynamic factor	—
K_{v350}	dynamic factor for $F_t K_A/b = 350$	—
$K_{v\alpha}$	dynamic factor for spur gears	—
$K_{v\beta}$	dynamic factor for helical gears ($\epsilon_\beta \geq 1$)	—
K_A	application factor	—
$K_{F\alpha}$	transverse load factor for bending stress	—
$K_{F\beta}$	face load factor for bending stress	—
$K_{H\alpha}$	transverse load factor for contact stress	—

^a a , u and z_2 are negative for internal gears.

Symbol	Designation	Units
$K_{H\beta}$	face load factor for contact stress (Hertzian pressure)	—
M_{red}	mass per unit facewidth of the gear pair referred to the line of action	kg/mm
P_F	actual power capacity based on bending stress	kW
P_{FP}	permissible power capacity based on bending stress	kW
P_H	actual power capacity based on contact stress	kW
P_{HP}	permissible power capacity based on contact stress	kW
R	roughness	μm
R_a	arithmetic average roughness (CLA value)	μm
R_z	mean roughness	μm
S	safety factor	—
S_F	actual safety factor for bending stress (against breakage)	—
$S_{F\min}$	minimum demanded safety factor for bending stress (against breakage)	—
S_H	actual safety factor for contact stress	—
$S_{H\min}$	minimum demanded safety factor for contact stress	—
T	torque	N m
T_1	pinion torque	N m
T_2	wheel torque	N m
T_F	actual torque based on bending stress	N m
$T_{F\max}$	peak torque capacity for bending stress	N m
T_{FP}	permissible torque based on bending stress	N m
T_H	actual torque based on contact stress	N m
$T_{H\max}$	peak torque capacity for contact stress	N m
T_{HP}	permissible torque based on contact stress	N m
Y	factor for bending stress	—
Y_F	tooth form factor for bending stress	—
Y_M	material quality factor for bending stress	—
Y_N	life factor for bending stress	—
Y_R	surface condition factor for bending stress	—
Y_S	stress correction factor for bending stress	—
Y_X	size factor for bending stress	—
Y_β	helix angle factor for bending stress	—
Y_δ	sensitivity factor for bending stress	—
Z	factor for contact stress	—
Z_c	casedepth factor for contact stress	—
Z_v	speed factor for contact stress	—
Z_E	elasticity factor for contact stress	—
Z_G	disc/gear correlation factor for contact stress	—
Z_H	zone factor for Hertzian pressure at pitch point for contact stress	—
Z_L	lubricant factor for contact stress	—

Symbol	Designation	Units
Z_M	material quality factor for contact stress	—
Z_N	life factor for contact stress	—
Z_R	roughness factor for contact stress	—
Z_W	work hardening factor for contact stress	—
Z_X	size factor for contact stress	—
Z_ϵ	contact ratio factor for contact stress	—
α_{en}	pressure angle at highest point of single tooth pair contact	radians
α_n	normal pressure angle at reference cylinder	radians
α_t	transverse pressure angle at reference cylinder	radians
α_{tw}	transverse pressure angle at pitch cylinder	radians
α_{Fen}	angle for application of load at highest point of single tooth pair contact	radians
β	helix angle (without subscript: at reference cylinder)	radians
β_b	base helix angle	radians
ϵ	contact ratio	—
ϵ_α	transverse contact ratio	—
$\epsilon_{\alpha n}$	virtual transverse contact ratio	—
ϵ_β	overlap ratio	—
ϵ_γ	total contact ratio	—
ν	Poisson's ratio	—
ρ_2	root radius of internal gear	mm
ρ_{a0}	tip radius of the basic rack of the tool	mm
ρ_f	tooth root fillet radius	mm
ρ_{fp}	root fillet radius of the basic rack of the gearing	mm
ρ_{rel}	radius of relative curvature	mm
ρ_F	root fillet radius at critical section	mm
σ_B	ultimate tensile strength	MN/m ²
σ_F	actual tooth root bending stress	MN/m ²
σ_{F0}	basic endurance limit of a polished specimen under a reversing bending load	MN/m ²
σ_{FP}	permissible tooth root bending stress	MN/m ²
σ_{FY}	yield strength for bending stress	MN/m ²
σ_H	actual contact stress (Hertzian pressure)	MN/m ²
$\sigma_{H \text{ lim}}$	endurance limit for contact stress for gears	MN/m ²
σ_{HD}	endurance limit for contact stress for discs	MN/m ²
σ_{HP}	permissible contact stress (permissible Hertzian pressure)	MN/m ²
σ_{HY}	yield strength for contact stress	MN/m ²
σ_R	residual stress	MN/m ²

Subscripts

1	pinion
2	wheel
core	core material properties of surface hardened steels
eff	effective values, real stress
est	estimated value
lim	value of endurance limit
min	minimum
max	maximum
red	reduced
sh	shaft
stat	static load

3 Guide to using the calculation procedure

3.1 Calculation procedure

The calculation procedure is used to calculate gear load carrying capacity either:

- a) in terms of stresses; or
- b) in terms of power.

In the case of a) the permissible stress is calculated from the material endurance limits modified by the stress modifying factors. This has to exceed the actual stress which is calculated from the nominal tangential force, modified by the load modifying factors and the gear geometry.

In the case of b) the power capacity of the gear pair is calculated from the permissible stress, the load modifying factors and the gear geometry. This has to exceed the power required of the gear pair.

In either case the calculation is performed separately for four cases:

- 1) pinion contact stress (see section 2);
- 2) wheel contact stress (see section 2);
- 3) pinion bending stress (see section 3);
- 4) wheel bending stress (see section 3).

Illustrative examples are given in Appendix G.

NOTE The values of some factors (K_v , $K_{H\alpha}$, $K_{H\beta}$, $K_{F\alpha}$, $K_{F\beta}$) depend on the value of the nominal tangential force, F_t . When calculating the maximum rating of a pair of gears the value of F_t has to be estimated in order to calculate these factors. It is recommended that if the final rated value of F_t differs from the estimated value by more than 10 % then the load dependant factors are re-calculated using the rated value of F_t .

3.2 Lubrication

The procedure is valid for gears having adequate lubrication.

NOTE At slow speed, particular care is required to ensure an adequate supply of lubricant at the mesh. It should also be ensured that the lubricant will not cause corrosion of the gears or any other parts of the gear unit. Corrosion is not covered by this procedure.

3.3 System dynamics

The values of application factor provided for design purposes do not apply if the driving or driven machinery causes an excitation at a frequency at or close to one of the system's natural frequencies.

NOTE In such cases it is recommended that the designer of the system supplies a value of the application factor, based on calculation or measurements on similar systems.

3.4 Safety factors

The use of this procedure requires a realistic appraisal of the influencing factors. When experience has been gained by running other similar gears in similar environments such experience can be used in choosing appropriate values of the safety factors.

Section 2. Contact stress calculations

4 Basic equations for contact stress calculations

4.1 Permissible contact stress σ_{HP}

The permissible contact stress, σ_{HP} , is calculated separately for pinion and wheel from the equation:

$$\sigma_{HP} = \sigma_{Hlim} Z_L Z_V Z_R Z_M Z_W Z_X Z_N / S_{Hmin} \quad (1)$$

4.2 Actual calculated contact stress σ_H

The actual calculated contact stress, σ_H , is calculated separately for pinion and wheel from the equation:

$$\sigma_H = Z_H Z_E Z_c \sqrt{\frac{F_{Ht}}{bd_1} \times \frac{(u+1)}{u} \times K_A K_V K_{H\alpha} K_{H\beta}} \quad (2)$$

where the value of F_{Ht} is the nominal tangential force at the reference circles appropriate to pinion or wheel calculations respectively from equation (6) or (7).

4.3 Permissible power capacity and torque based on contact stress P_{HP} and T_{HP}

The permissible power capacity, P_{HP} , based on contact stress is calculated separately for pinion and wheel from the equation:

$$P_{HP} = \frac{bd_1^2 \pi n_1}{60 \times 10^6} \times \frac{u}{(u+1)} \times \frac{1}{Z_H^2 Z_c^2} \times \frac{1}{K_A} \times \frac{1}{K_V} \times \frac{1}{K_{H\alpha} K_{H\beta}} \times \left(\frac{\sigma_{HP}}{Z_E} \right)^2 \quad (3)$$

The permissible torque, T_{HP} , based on contact stress is calculated separately for pinion and wheel from the equation:

$$T_{HP} = \frac{60 \times 10^3}{2\pi} \times \frac{P_{HP}}{n_1} \quad (4)$$

4.4 Peak torque capacity for contact stress, T_{Hmax}

The peak torque capacity for contact stress expressed as a multiple of T_{HP} is given by the equation:

$$\frac{T_{Hmax}}{T_{HP}} = K_A K_V \left(\frac{\sigma_{HY} Z_M}{\sigma_{HP} S_{Hmin}} \right)^2 \quad (5)$$

where σ_{HY} is obtained from Figure 1.

NOTE The lines for surface hardened steels in Figure 1 are to be extended downwards as far as the σ_B line corresponding to the condition of the core.

5 Nominal tangential force for contact stress, F_{Ht}

The nominal force for contact stress, F_{Ht} , is calculated from either equation (6) or (7):

$$F_{Ht} = \frac{2000 T_{H1}}{d_1} \quad (6)$$

$$F_{Ht} = \frac{1000 P_H}{v} \quad (7)$$

For information:

$$d_1 = \frac{z_1 m_n}{\cos \beta} \quad (8)$$

If either

- a) the gear pair is subject to variable duty; or
- b) the gear pair is subject to intermittent high loads (for instance at start-up) greater than the nominal running torque

then the pinion torque, T_{H1} , in equation (6) is calculated in accordance with the variable duty procedure in Appendix A for $T_{\infty H1}$.

6 Zone factor for contact stress, Z_H

6.1 Purpose of Z_H

The zone factor, Z_H , accounts for the influence of tooth flank curvature at the pitch point on Hertzian stress and converts the tangential force at the reference cylinder to a normal force at the pitch cylinder.

6.2 Calculation of Z_H

The zone factor, Z_H , is calculated from the equation:

$$Z_H = \sqrt{\frac{2 \cos \beta_b \cos \alpha_{tw}}{\cos^2 \alpha_t \sin \alpha_{tw}}} \quad (9)$$

For gears at standard centres this simplifies to:

$$Z_H = 2 \sqrt{\frac{\cos \beta_b}{\sin(2\alpha_t)}} \quad (10)$$

7 Contact ratio factor for contact stress, Z_ϵ

7.1 Purpose of Z_ϵ

The contact ratio factor accounts for the load sharing influence of the transverse contact ratio and the overlap ratio on the specific loading.

7.2 Calculation of Z_ϵ

7.2.1 For spur gears, the contact ratio factor, Z_ϵ , is calculated from the equation:

$$Z_\epsilon = \sqrt{\frac{4 - \epsilon_\alpha}{3}} \quad (11)$$

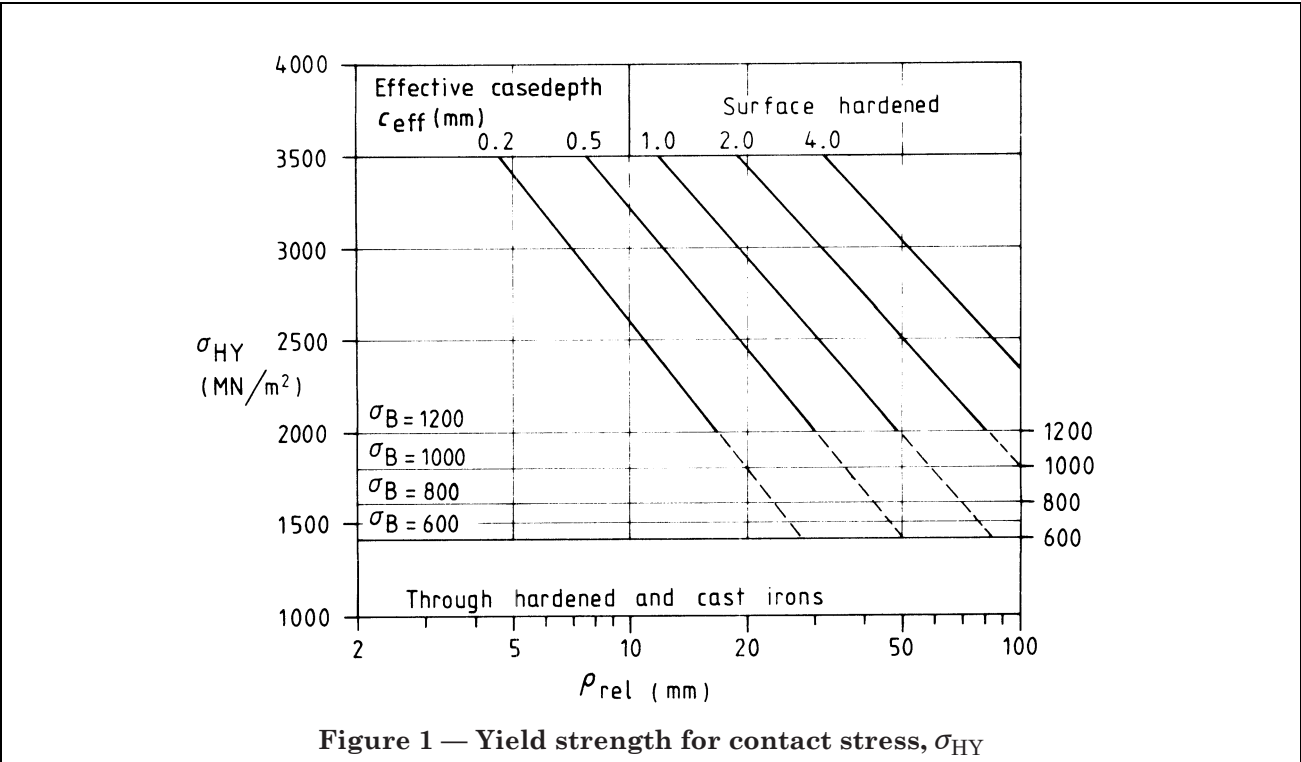
7.2.2 For helical gears with $\epsilon_\beta < 1$, the contact ratio factor, Z_ϵ , is calculated from the equation:

$$Z_\epsilon = \sqrt{\left[\frac{(4 - \epsilon_\alpha)}{3} \times (1 - \epsilon_\beta) \right] + \frac{\epsilon_\beta}{\epsilon_\alpha}} \quad (12)$$

7.2.3 For helical gears with $\epsilon_\beta \geq 1$, the contact ratio factor, Z_ϵ , is calculated from the equation:

$$Z_\epsilon = \sqrt{\frac{1}{\epsilon_\alpha}} \quad (13)$$

For information, equations for ϵ_α and ϵ_β are given in B.8 and B.9, respectively.



8 Elasticity factor for contact stress, Z_E

8.1 Purpose of Z_E

The elasticity factor accounts for the influence of the specific material properties E (modulus of elasticity) and ν (Poisson’s ratio) on the Hertzian stress.

8.2 Calculation of Z_E

The elasticity factor Z_E is calculated from the equation:

$$Z_E = \sqrt{\pi \left(\frac{1 - \nu_1^2}{E_1} + \frac{1 - \nu_2^2}{E_2} \right)}$$
 (14)

This is tabulated for some gear materials in Table 1. For properties of bronzes see BS 1400.

Table 1 — Value of Z_E for some material combinations

Gear materials	Z_E
Steel/steel	189
Steel/SG cast iron	181
SG cast iron/SG cast iron	174
Grey iron/grey iron	146

9 Basic endurance limit for contact stress, σ_{Hlim}

The basic endurance limit for contact stress, σ_{Hlim} , is calculated separately for pinion and wheel from the equation:

$$\sigma_{Hlim} = \sigma_{HD} Z_G Z_c$$
 (15)

where

σ_{HD} is obtained from Figure 2;

Z_c for through hardened steels = 1.0;

Z_c for surface hardened steels is obtained from Figure 4 and the limiting casedepth from Table 2;

Z_{G2} is obtained from Figure 3;

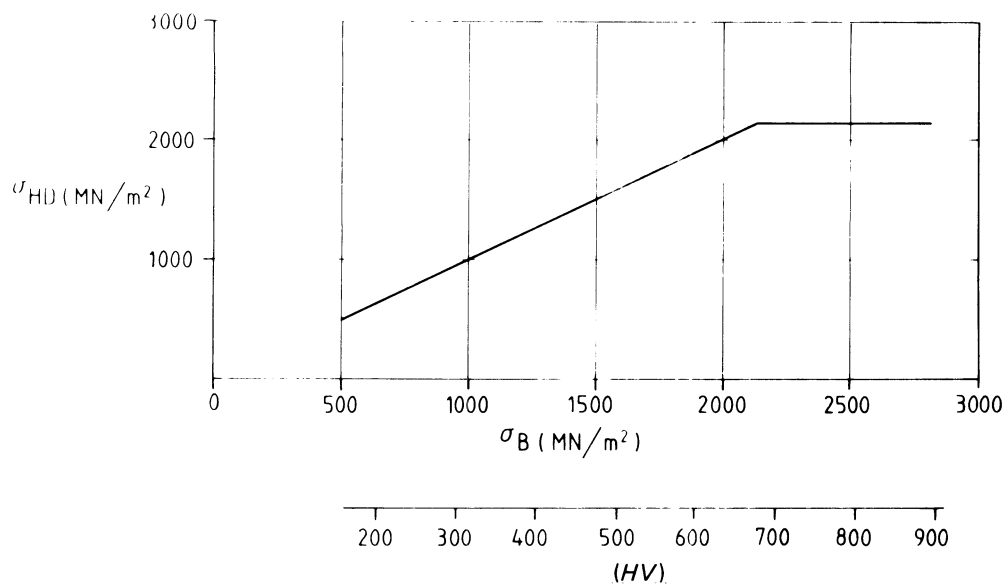
Z_{G1} is the greater of:

$$Z_{G1} = (1.02 - 0.02 |u|) Z_{G2} \text{ and} \quad (16)$$

$$Z_{G1} = 0.9 Z_{G2} \quad (17)$$

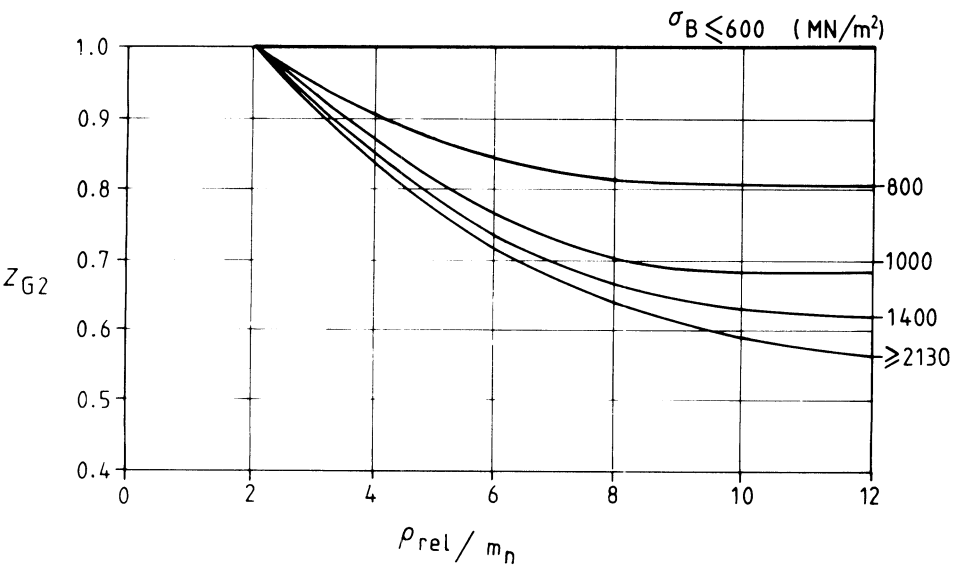
Table 2 — Limiting casedepth

Hardening process	Limiting casedepth c_{lim}
Carburizing and hardening	$0.16 m_n$
Nitriding	$0.16 m_n$
Induction hardening	$0.32 m_n$



NOTE Values of σ_{HD} in this figure are derived from disc tests performed under the auspices of the Admiralty Vickers Gear Research Association and the Navy and Vickers Gear Research Association

Figure 2 — Values of σ_{HD}



NOTE For internal gears the value of ρ_{rel} for an equivalent external gear should be used, i.e. the value of ρ_{rel} obtained from B.11 should be multiplied by $(|u| - 1)/(|u| + 1)$.

Figure 3 — Values of Z_{G2}

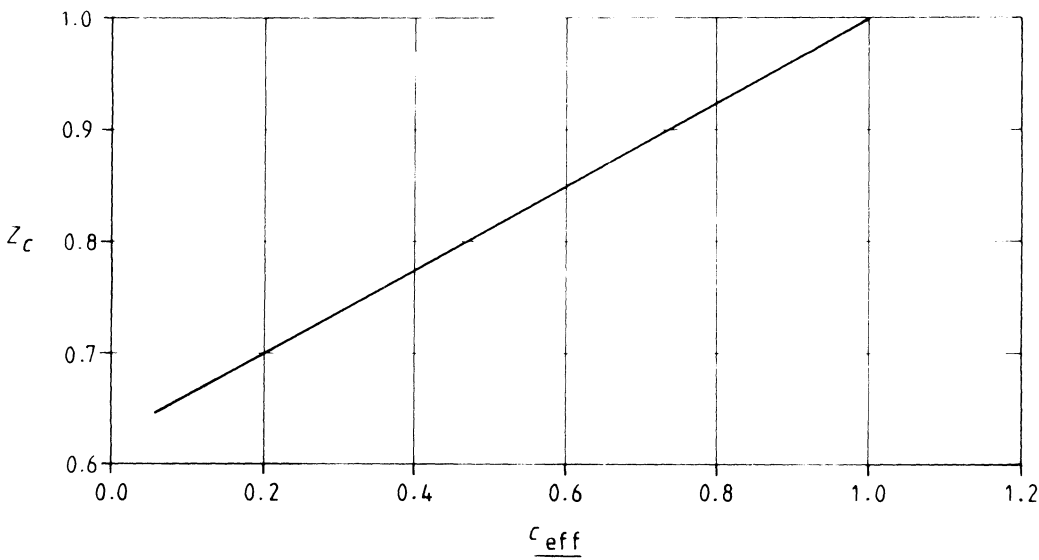


Figure 4 — Casdepth factor, Z_c

10 Material quality for contact stress, Z_M

10.1 Purpose of Z_M

Better quality control exercised in the manufacture of a material results in less scatter on the mechanical properties of the finished material. Hence, for a given confidence level, better quality materials have a higher permissible stress and, conversely, lower quality materials a lower permissible stress. Material qualities are defined in Appendix F.

10.2 Values of Z_M

The value of Z_M is obtained from Table 3.

Table 3 — Values of Z_M

Material	Z_M		
	Quality A	Quality B	Quality C
Surface hardened steels	1.0	0.9	0.8
Through hardened or normalized steels	1.0	0.9	0.8
Through hardened or normalized cast steels or bronze.	0.9	0.8	0.7
Nodular cast iron	0.9	0.8	0.7
Other cast irons	0.7	0.5	0.5

11 Lubricant influence, roughness and speed factors for contact stress, Z_L , Z_R and Z_v

11.1 Purpose of Z_L , Z_R , Z_v

The lubricant viscosity, surface roughness and pitch line speed affect the lubricant film thickness which in turn affects the Hertzian component of the total stress at the pitch cylinder.

11.2 Calculation of Z_L , Z_R and Z_v

The value of the product ($Z_L Z_v$) is obtained from Figure 5.

The roughness factor Z_R is obtained from Figure 6. If the pinion and wheel roughnesses are different, then:

$$R_a = (R_{a1} + R_{a2})/2 \quad (18)$$

If the roughness is measured in terms of R_z , then the value of R_a is calculated from the equation:

$$R_a = R_z/6 \quad (19)$$

NOTE The values of R_{a1} and R_{a2} relate to the flank roughness in the finished condition after completing any running-in treatment or other manufacturing process which may improve the roughness of the flanks. This includes running-in during commissioning, when it is specified.

12 Work hardening factor for contact stress, Z_W

12.1 Purpose of Z_W

The work hardening factor accounts for the increase of surface durability due to meshing a through hardened steel wheel with surface hardened pinion. In all other cases, $Z_W = 1.0$.

12.2 Calculation of Z_W

For pinions, $Z_W = 1.0$.

For wheels of hardness less than 400 HV, the value of Z_W is obtained from Figure 7.

If the roughness is measured in terms of R_z then calculate R_a from equation (19).

For wheels of hardness greater or equal to 400 HV, $Z_W = 1.0$.

13 Size factor for contact stress, Z_x

The size factor is included to take into account possible influences of size on material quality and its response to heat treatment and other manufacturing processes. The value is taken as $Z_x = 1.0$.

14 Life factor for contact stress, Z_N

14.1 Purpose of Z_N

The life factor for contact stress takes account of the increase in permissible stress if the number of stress cycles is less than the endurance life.

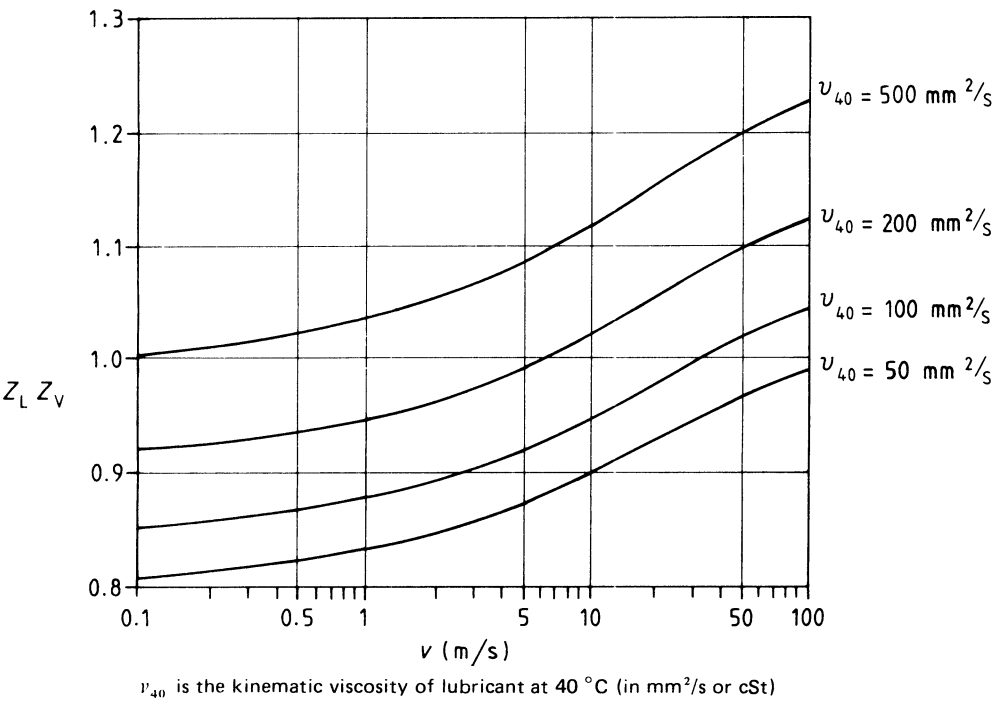


Figure 5 — Combined speed and lubricant factors, $Z_L Z_v$

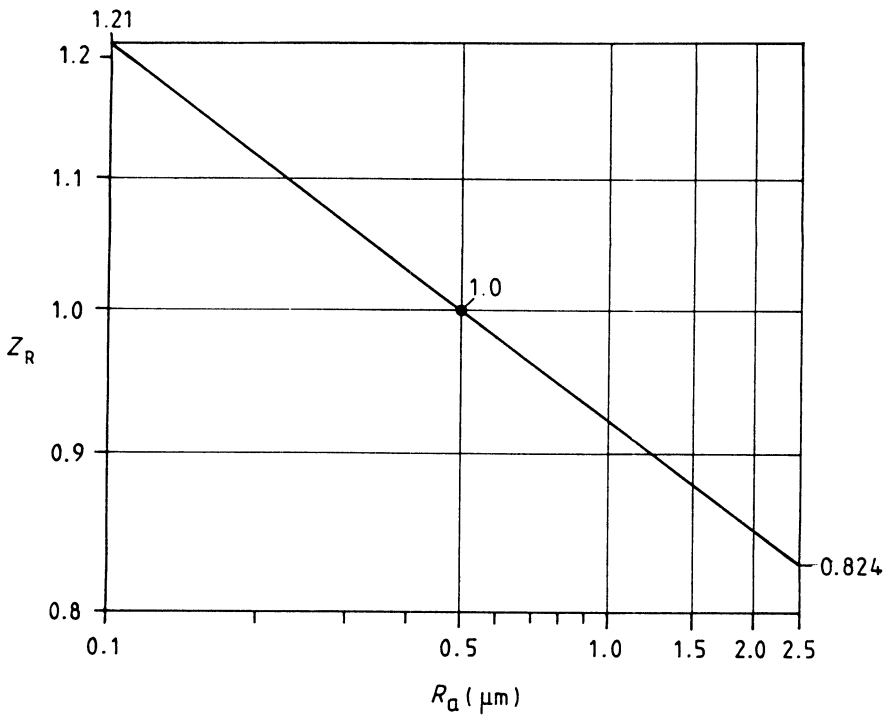


Figure 6 — Roughness factor, Z_R