

Guide to

Process control using quality control chart methods and cusum techniques

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Foreword

This British Standard has been prepared under the direction of the Quality, Management and Statistics Standards Committee and is complementary to the following British Standards on control charting methods:

BS 5701, *Guide to number-defective charts for quality control*.

BS 5702, *Guide to quality control charts for measured variables* (revision of BS 2564:1955)¹⁾.

BS 5703, *Guide to data analysis and quality control using cusum techniques* (published in four Parts).

All these standards are “free standing” in that any one can be implemented without reference to this guide, or to the other standards. This guide gives sufficient information to clarify the choice between these standards for any particular application. Further, BS 5701 and BS 5702 are deliberately based on clear-cut and simple sets of rules. This guide explains the reasoning behind the rules chosen and gives some insight into situations in which complicating factors in a particular process may put the procedures in these two standards in need of modification.

In order to give a logical sequence to this guide, it has been necessary occasionally to duplicate or paraphrase material that is contained within BS 5701, BS 5702 and BS 5703. This is considered a preferable alternative to repeated cross-referencing between the standards and this guide.

It should be noted that BS 5701 and BS 5702 are effectively revisions of previous standards BS 1313 and BS 2564. These revisions had the twin objectives of simplifying the procedures used and separating “control by attributes” (BS 5701) and “control by variables” (BS 5702). BS 5703 is entirely new and is a world leader in its field.

Acknowledgement is made to the authors of papers in the statistical journals whose work has been drawn upon, and to Mr. D. W. Newton (University of Birmingham) who was primarily responsible for the development work on this guide, carried out on behalf of BSI committee QMS/1/4.

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Summary of pages

This document comprises a front cover, an inside front cover, pages i to vi, pages 1 to 66, an inside back cover and a back cover.

This standard has been updated (see copyright date) and may have had amendments incorporated. This will be indicated in the amendment table on the inside front cover.

¹⁾ In course of preparation.

Section 1. General

1.0 Introduction

1.0.1 Description. A control chart is a graphical method for evaluating whether or not a process is “in a state of statistical control”. The values of some statistical measure(s) on an ordered series of samples or subgroups are compared with control limits to determine whether the process is “in statistical control”.

NOTE 1 The emphasis is on the process being “in statistical control” rather than being acceptable with respect to any specified tolerance. The control chart can, however, be used in an “acceptance” sense calling for action or investigation when a process is deemed to have changed from its standard level.

NOTE 2 The term “chart” is conventionally used to describe the control procedures discussed in this guide and used in BS 5701, BS 5702 and BS 5703. However, it should be noted that the use of non-graphical forms of tabular presentation is an accepted alternative.

1.0.2 Objectives. The basic objective of any form of control is to detect changes in some aspect of a process or its product. Information about the current state of the process is obtained by inspecting a sample drawn from recent production. From this sample is extracted information that is recorded and, where necessary, acted upon. Some value(s) calculated from the sample (such as its average, its range or the number of defective items, depending on the type of chart) is (are) plotted on a graph. The positions of these plotted points give rise to decisions about the process. This guide is largely concerned with explaining the statistical reasoning underlying the rules associated with these decisions and giving guidance for the choice of control chart method for particular applications.

The various control charting methods described in this guide are designed to indicate any lack of control in a process resulting from some change or deterioration, thus allowing an early decision to be made about re-setting or correcting the process. It has to be recognized, however, that in many cases the major impact and benefits of control charting do not arise only from the statistical interpretation of the plotted points, but more because someone is seen to be taking an interest in the process or product, and is extracting information that is being recorded and used. If this is viewed in contrast to the usual situations of either no monitoring at all, or of inspection (be it 100 % or sampling) on a batch basis at some stage removed from the process, it is clear that the motivational effects of simply instituting any form of sampling and recording close to the process should not be underestimated.

It should also be recognized that the title “control chart” is itself a misnomer to the extent that the chart is an historical record and does not of itself “control” the process. The essential action of a chart is to sound an alarm when attention to the process is required, and to give as much indication as possible of the nature of the change that has occurred so that remedial action can be taken to restore the process to its acceptable state (or to retain the process in its new state if this is more advantageous).

Charts should be related, where possible, to individual machines, operators, shifts or other production units and should be marked to identify any significant changes in material, shift or any other external factor.

Finally, it should be noted that a process can be stable and “in statistical control” and yet be producing outside specification requirements. For instance it may consistently produce 6 % of items above some specified value whereas 1 % is the most that can be tolerated. Thus it is apparent that an understanding of the relationship between the variability of the process and the specification of the resulting product is required both to judge the acceptability of the process and to provide information for subsequent control charting.

1.0.3 Assignable and unassignable causes of variation. The characteristics of any realistic process will be subject to variability. In control charting the assumption is made that there is a “natural (inherent) variability” that is typical of the process when it is “in control”. This inherent variability is called the *process capability*; it will be reflected in variations in observations in samples drawn from the process. For example, a measurement that has a stable underlying mean will show variation in averages of small samples taken from the process. Similarly, a process that is assessed in terms of the proportion defective produced will show variation in the number of defectives observed in a series of samples of the same size, even though the underlying proportion defective is stable.

The process capability is usually taken to be an irreducible level that is simply typical of the process, and is said to be due to random effects or *unassignable causes* of variability. This term is to some extent misleading in that in some circumstances it would no doubt be possible to carry out investigations to determine some of these causes. A more realistic view is that these causes are unassignable not because it is impossible to assign them, but because it is uneconomic (because the process capability is acceptable) or pointless (because even if the cause of the variability could be identified, it can not always be removed).

The purpose of a control chart is to detect any departure of the process from the “in control” state; any such movement is said to be due to *assignable causes* of variation. The decision-making aspect of control charts is centred on the problem of deciding whether a sample observation is due only to the expected, inherent unassignable causes of variation (in which case no action is required) or whether it is due to the effect of some additional, assignable variation (in which case corrective action is usually required).

1.0.4 Decision rules and average run lengths. The effectiveness of the decision rules associated with a control chart is best described by the *average run length* (ARL). This is defined as the expected value of the number of samples plotted on the chart up to and including the point that gives rise to an action decision that some assignable cause of variation is present.

If no assignable cause is present, one would ideally wish for an ARL of infinity; in other words one would wish that an “action” decision is never reached. In the practical circumstance of operating a control chart using sample data it is not feasible to achieve an infinite ARL, but the objective instead is to make the ARL when no assignable cause is present (usually denoted L_0) large, usually in the range 200 to 1 000.

Conversely, when an assignable cause is present, the ideal value for the ARL is 1, i.e. one would expect an “action” decision on the next sample taken. How close one comes to achieving this depends on the magnitude of the assignable cause, and on the design of the control chart and its decision procedures. For any particular magnitude of assignable cause of variation the objective in designing control charts is to make the ARL (usually denoted L_1) as small as possible.

When designing decision procedures for control charts based on sampling, a satisfactory compromise has to be achieved between L_0 and L_1 values; other things (including sample size) being equal, small L_1 values can be obtained by a willingness to tolerate a small L_0 . Conversely, a large L_0 necessitates tolerating large values of L_1 .

The BS 5700 series of control charting methods has aimed for a satisfactory compromise between these conflicting requirements. The average run length behaviour of the various types of chart is discussed in greater depth in section 5.

1.0.5 Sampling. A feature common to all control charting is that it involves sampling from the process as distinct from 100 % inspection (or no inspection at all).

The samples are taken at intervals from the process and the resulting measurements or inspection results are used to calculate statistics (typically average value, range between largest and smallest value, number of defective items in the sample) which are plotted on the control chart. Decisions about the state of the process are made based on the positions of these plotted points.

Aspects of sampling that merit general consideration before dealing with specific types of control chart are as follows.

a) *Frequency of sampling.* This is determined by the economics of the process, involving such factors as the stability of the process, the cost of inspection, costs incurred by defective production etc., and by the frequency of changes (materials, tools etc.) that might affect quality. Many formal economic models for control charting have been proposed, but a useful rule-of-thumb is to ensure that the sampling interval does not exceed the maximum time that the process could be allowed to operate if it was “out of control” and producing defective output.

In the case of processes where operator control is a principal influence on the quality level, the sampling times should not be too predictable, for obvious reasons.

b) *Structure of sample.* The objective of control charting is usually to assess the performance of the *process*, and to do this effectively the sample should be of the most recent output possible. In some circumstances, however, the requirement may be to evaluate the *product*, and in such cases a random sample of production since the previous sampling is called for.

It has to be noted, however, that any decision as to sampling strategy is subject to many practical considerations; there may be circumstances where adherence to one or other principle may be impractical or undesirable, as for instance in the presence of cyclical variation, or in the need to disguise the moment of sampling from production under direct human control.

c) *Sample size and run lengths.* The value of the average run length when the process is in control (as described in 1.0.4) can be decided arbitrarily in the design of control chart procedures. BS 5701, BS 5702 and BS 5703 contain such arbitrary decisions which are explained in the particular standards.

Having made this choice, the average run length when an assignable cause of variability is present will depend on the magnitude of the resulting change in the process, and the size of the sample. For a given chart and a particular magnitude of change due to an assignable cause, an increase in sample size will give a reduction in average run length.

d) *“Holding” of production.* When it is essential that items made when the process was out of control are not despatched, production has to be held in “quarantine” until results on a later sample are available. Because the control chart cannot be guaranteed to detect a change when a sample is taken, holding of goods produced even earlier is advisable. If the next sample indicates that the process is “in control”, the held goods can be released. Otherwise, any appropriate inspection and rectification is restricted to the goods that were held.

When loss of control is related only to the economics or subjective acceptability of the process (for example a loss of control which results in the manufacture of overweight packages), decisions on holding production can be made on estimated economic costs and penalties.

e) *Sample size.* In general terms, the bigger the sample sizes the lower the risks of coming to incorrect decisions, but there are other statistical considerations affecting sample size. In general, sample sizes for attributes charts (BS 5701) need to be considerably larger than those for variables charts (BS 5702).

The sample size, and the frequency of inspection, have to be a practical compromise between the costs of inspection and the costs of making incorrect decisions.

1.1 Scope

This British Standard provides guidance on the selection and application of quality control chart methods for process control complementary to BS 5701, BS 5702²⁾ and BS 5703 (which provide detailed operational procedures based on the use of either traditional control charts or cusum techniques).

This guide explains how control charts work, the types of risk involved, the reasoning behind the decision rules, and how the average run length gives a measure of their effectiveness. Other aspects covered are frequency of sampling, sample sizes and methods for identifying types of variation. Further to this, the guide also provides an itemized procedure for the determination of process capability, which is an essential prerequisite to the achievement of process/quality control.

The systematic approach given in this standard has the principal objective of assisting development within industry of a sound appreciation of the relative merits of the many and various control methods currently available.

As such, BS 5700 provides a basic reference standard for both industrial training and technical educational purposes irrespective of whether the method of presentation desired is by charts or by computer tabulations.

NOTE The titles of the publications referred to in this standard are listed on the inside back cover.

1.2 Terminology and symbols

In general, for the purposes of this British Standard, the definitions and symbols given in BS 4778 and BS 5532 apply. However, the terminology used in quality control is subject to a continuing process of development in response to market demand. Product liability considerations currently indicate that the terms “defect” and “defective” are only appropriate where a product, process, or service is evaluated in terms of actual usage.

In some situations, specified requirements coincide with market usage requirements. In other situations they may not coincide, being either more or less stringent, or the exact relationships between the two is not fully known or understood. When a technical/quality characteristic of a product, process or service is evaluated in terms of *conformance to specified requirements*, the use of the term “nonconformity” is more appropriate.

Contractual obligations, stated or implicit, may be involved in certain instances, but in others the specified requirements may be purely internal and set deliberately tighter than the market or customer requirements.

²⁾ In course of preparation.

Consequently, pending revision of the terminology given in BS 5532 (ISO 3534), it is suggested that the terms “defect” and “defective” be replaced by “nonconformity” or “nonconforming unit” as appropriate, when using this British Standard.

NOTE Where examples have been extracted from related standards, e.g. BS 5703, the special symbols defined in that standard apply.

Section 2. The choice of British Standard for control charting

2.1 General

There are two decisions to be made in choosing the British Standard appropriate to a particular situation:

- a) variables or attributes?

and then, whichever is chosen:

- b) traditional or cusum?

BS 5701 deals with traditional control charts for attributes.

BS 5702 deals with traditional control charts for variables.

BS 5703 deals with cusum control charts for both attributes and variables.

2.2 Measured variables or counted attributes?

A “measured” variable is used in this standard to refer to a quality that is measured on a continuous scale, whereas a “counted attribute” is used to refer to a quality that is recorded as either “good” or “defective”. In many situations either technique may be used. The factors that influence the choice between them are as follows.

- a) *Type of characteristic.* A characteristic may not be measurable on a continuous scale. In this case counted attributes are the only possibility.
- b) *Provision for adjusting the process.* If a process contains little or no provision for adjustment, there may be no benefit in using measured variables.
- c) *Number of variables.* Measured variables require one chart for each variable, so if it is necessary to check several variables it may be better to use counted attributes, in which case the several variables can be monitored on one chart.
- d) *Cost of inspection.* Attributes can be recorded using visual inspection or by gauges, templates etc. Measurement on a variable scale requires more sophisticated equipment, which will be slower and require higher skilled inspection staff.
- e) *Sample size.* Measured variables require smaller sample sizes than counted attributes; BS 5701 recommends samples of 25 or 50 whereas BS 5702 recommends samples of 5.
- f) *Long-term control.* Measured variables are good at providing a basis for specification, negotiation, and analytical evaluation of effects of actions. Counted attributes are good at exposing underlying weaknesses and improvement opportunities as a basis for quality plans or quality improvement programmes.
- g) *Capability of process.* Measured variables are suitable for processes of high, medium, or low relative capability. Counted attributes can only be applied directly to processes of low relative capability; for other processes it is necessary to apply “compressed” limits.
- h) *Sensitivity.* For a given sample size, measured variables give a much higher probability of detecting changes, and smaller risks of wrong decisions, than counted attributes.
- i) *Critical variable.* A variable will usually be measured if the consequences of a change in it are severe, or if the process is operating close to the specified limits, or if it is a key variable in that changes in one or more of several other variables will be reflected in a change in it.
- j) *Legal requirements.* There may be a legal requirement to maintain a control chart for a particular measured variable, e.g. EEC directive “Council Directive 76/211/EEC relating to the making up by weight or by volume of certain pre-packaged products”.

BS 5701 and BS 5702 contain more detailed discussions and examples of the circumstances in which the two techniques are used. Most of the above factors are of a practical nature; the essentially statistical questions of the choice of sample size and of sensitivity are covered in more detail in section 5.

2.3 Traditional or cusum?

For this choice to be meaningful it will be helpful to understand the principles of the two types of chart. These are explained in 3.3 and 3.6 respectively for control by attributes, and in 4.5 and 4.7 respectively for control by variables.

The advantages of cusum charts are explained in detail, and illustrated by examples in BS 5703-1. Their advantages can be briefly summarized as follows.

- a) For the same sample size, a cusum chart will give a much more vivid illustration of any changes that are occurring (purely by visual examination of the plot, without recourse to decision rules).
- b) The cusum chart uses data more effectively, thereby giving cost savings.
- c) The cusum gives clear indication of the location and magnitude of change points in a process.

Possible disadvantages are as follows.

- d) The procedure is comparatively less well known, and there is a problem of re-education of those used to traditional methods. In particular it is sometimes difficult to establish the basic principle that the control parameter is the *slope* of a plot rather than its vertical ordinate.
- e) It is sometimes maintained that the calculations for the cusum plot are more complex than those for the traditional plot. This is arguable, particularly when associated with the use of microcomputers.
- f) Because average is indicated by slope, staff operating the charts require training to achieve effective interpretation.

Section 3. How a control chart works: charts for number defective

3.1 Introduction

3.1.1 General. Traditional charts for number defective are considered first, then, in 3.6, cusum charts are introduced and compared with traditional charts.

3.1.2 Example. The traditional control chart for number defective is simply a graphical record of the number of defectives observed in samples taken from the process, as shown in Figure 1.

3.1.3 Applications. The situations in which control charts for number defective (as in BS 5701) can be applied, can be described as follows.

Items are being produced that can be categorized as “good” or “defective”, or defects (within defective items) are being counted and, under normal production conditions, the following apply.

- a) A small proportion of defective items is allowable (or “synthetic” defectives can be produced by using compressed limits or by tightening the standards for visual inspection).
- b) A “target” proportion defective (greater than zero) can be defined. The process is deemed to be acceptable when operating at this level or below.
- c) An “action” decision is required when the process proportion defective increases above the allowable or target value.
- d) Inspection of the product when the process is operating satisfactorily may be on a sampling basis.

3.1.4 Procedure. Under BS 5701, samples are taken from the process at regular intervals. On the basis of the number of defective items observed in the sample, two possible conclusions can be reached.

Either

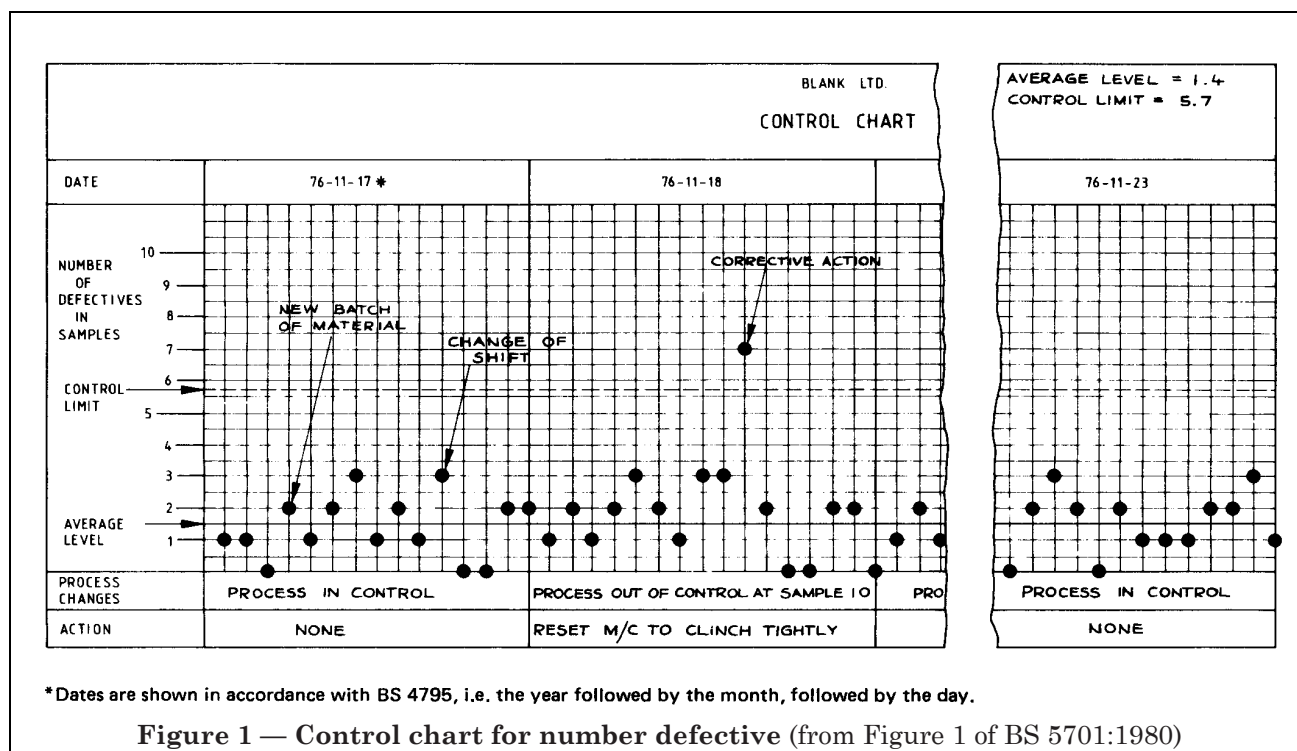
- a) the process is satisfactory in that the process proportion defective is not greater than the expected (or target) value, and production can continue,

or

- b) the process proportion defective is greater than the expected (or target) value.

In case b), action will be required as follows.

- 1) Decide if the new, higher proportion defective is acceptable. If so, production can continue.
- 2) If not, action is required to reduce the effective proportion defective. In the short term this can be achieved by instituting 100 % inspection instead of sampling control, but in the longer term, investigation and corrective action will be necessary.



3.1.5 Sampling risks. Because of the variation that is inherent in the sampling process, the number of defective items occurring in a sample will vary from sample to sample even if the underlying proportion defective is constant. The pattern of this variation is explored in more detail in 3.2, but its presence gives rise to two dangers when the number defective in a sample is used as a criterion for controlling the proportion defective in the underlying process.

These dangers are as follows.

- Type I risk.* The risk that an “unfortunate” sample with a high number of defectives will give a spurious “action” decision when no deterioration has in fact occurred.
- Type II risk.* The risk that a “fortunate” sample with a low number of defectives will fail to signal a deterioration in the process.

The design of any control charting scheme is a compromise between these two opposing risks. BS 5701 uses a type I risk of about 1 in 200, a value that experience has shown to be satisfactory.

3.2 The probability models used in BS 5701

3.2.1 General. To understand how the control limits given in BS 5701 are calculated, and when they may not be appropriate, it is necessary to consider the sampling variability of the observed number of defectives.

In an attributes control chart as described in BS 5701, the charted variable is the observed number of defectives in a sample. To understand how such a control chart works it is first of all necessary to understand the sampling variability of such observations.

3.2.2 The binomial and Poisson distributions. Two probability distributions can be used to model the sampling variability. The binomial distribution is used when a sample consists of a defined number of items that can be classified as “good” or “defective”. The Poisson distribution is used when the sample consists of a defined “sample space” (such as a length of wire, area of fabric, operating time of a machine) within which a number of defects is counted. By regarding the number of defective items in a sample as being equal to the number of defects in the sample, one can also apply the Poisson distribution to circumstances when the binomial distribution might at first sight appear to be the natural choice. In deriving the control limits in BS 5701, the Poisson distribution has been used throughout. This allows the standard to be simplified; the conditions in which the Poisson distribution can be applied are discussed in the following paragraphs. Examples of processes which give rise to defects are given in 3.5.

3.2.3 The binomial distribution

3.2.3.1 If a sample of n items is drawn at random from a process where the proportion of defective items is p [and the proportion of good items is therefore $(1 - p)$], the probability of obtaining x defective items in the sample, denoted $P(x)$, is given by:

$$P(x) = \frac{n!}{x!(n-x)!} p^x (1-p)^{(n-x)}$$

NOTE The symbol “!” denotes factorial where:

$$n! = n(n-1)(n-2) \dots (3)(2)(1)$$

e.g. $6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$

Example. The probability of obtaining two defectives in a sample of 25 drawn from a process with a proportion defective p equal to 0.04 (4 %) is:

$$\begin{aligned} P(2) &= \frac{25!}{2! 23!} \times 0.04^2 \times 0.96^{23} \\ &= \frac{25 \times 24}{2} \times 0.04^2 \times 0.96^{23} = 0.188 \end{aligned}$$

3.2.3.2 For any situation defined by sample size n and proportion defective p , the probability of x defectives can be shown pictorially as in Figure 2, the values of the probabilities being calculated as shown in 3.2.3.1.

The mean of the binomial distribution = np ; in the example in 3.2.3.1 the mean of the number of defectives is $25 \times 0.04 = 1.0$. This is the expected number of defectives in the sample.

The variance of the binomial distribution is $np(1 - p)$; in the example in 3.2.3.1 the variance is $25 \times 0.04 \times 0.96 = 0.96$. (The standard deviation is $\sqrt{0.96} = 0.98$.)

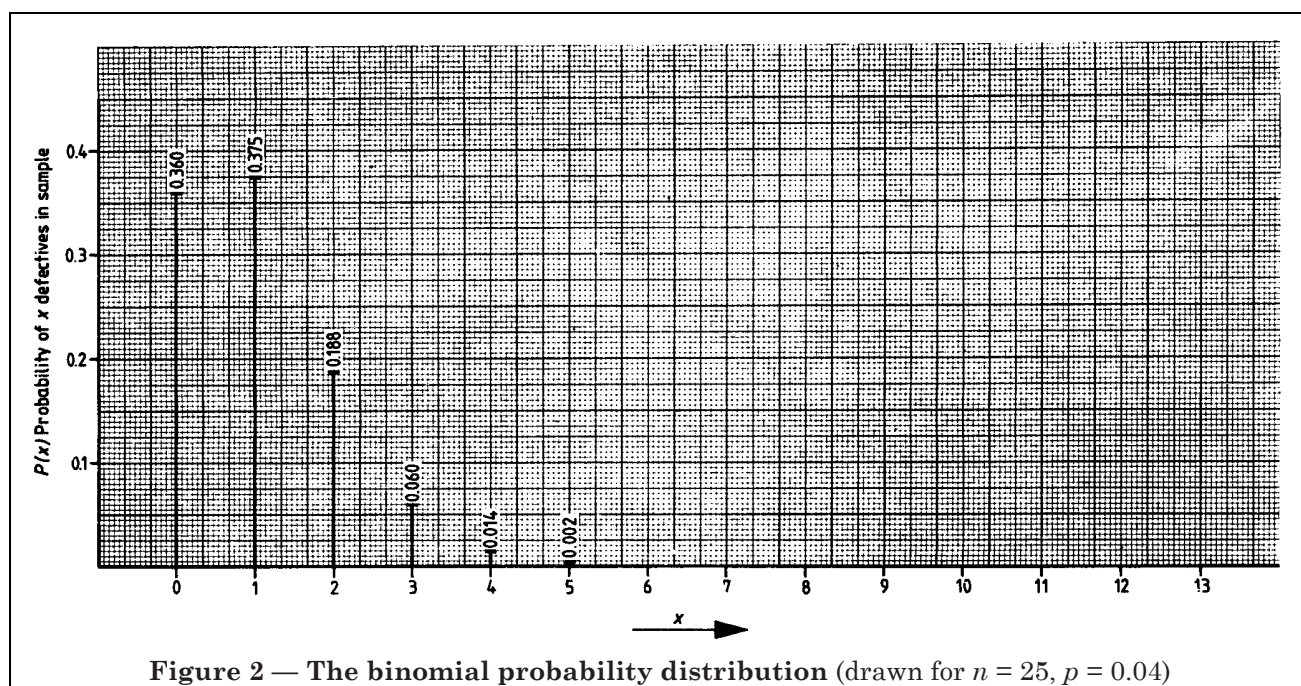


Figure 2 — The binomial probability distribution (drawn for $n = 25$, $p = 0.04$)

3.2.4 The Poisson distribution. Whereas the binomial distribution defines the number of defective items in a sample consisting of a finite number of items, the Poisson distribution defines the number of defects in samples where each item may have zero, one, or more defects and the possible number of defects may have no upper limit. The Poisson distribution is characterized by “ m ”, the expected number of defects in the sample (i.e. the average number of defects found per sample in the long run). For example, a process painting car bodies might, on average, produce one blemish for every six bodies produced. On a particular body we therefore expect $1/6$ of a blemish. We obviously cannot obtain this particular result, but the Poisson distribution will enable us to calculate the probability of obtaining no blemishes, one blemish, two blemishes and so on, on a particular body.

If a sample of n items is drawn at random from a process where the expected number of defects in the sample is m , then the probability of obtaining x defects in the sample is given by:

$$P(x) = e^{-m} m^x / x !$$

where e is the base of natural logarithms, $= 2.71828$ approximately.

NOTE e^{-x} is tabulated in most mathematical tables and is also available as a pre-programmed function on most scientific calculators.

This formula applies when the defects occur independently of each other, so that each item may have more than one defect, and the number of defects in the sample has no upper limit. The parameter m can be interpreted as the average number of defects found, per sample, in the long run (i.e. it is the total number of defectives found divided by the number of samples tested).

As an example, the probability of obtaining $x = 2$ defects in a sample, drawn from a process such that the expected number of defects per sample is $m = 1$, is:

$$P(2) = e^{-1} 1^2 / 2 ! = e^{-1} / 2 = 0.368 / 2 = 0.184$$

Note that the probabilities calculated using the Poisson distribution depend on only one parameter, m .

If the situation is one in which the binomial distribution is appropriate, the expected number of defects (i.e. defective items) per sample can be calculated as:

$$m = np$$

Using this relation it can be shown that the binomial distribution can be approximated by the Poisson distribution, i.e.:

$$P(x) = \{n! / x! (n-x)!\} p^x (1-p)^{n-x} \approx e^{-m} m^x / x !$$

This approximation is valid for small values of p , roughly for p less than 0.1 (i.e. 10 %). As an example, the values obtained for $P(2)$ in this clause and in 3.2.3 can be compared. Figure 3 compares the two distributions for proportions defective of $p = 0.05$, 0.10 and 0.25, using a sample size of $n = 25$.

3.2.5 Calculation of the probabilities for a range of outcomes. From the calculations of probabilities of x defectives, it is then possible to further calculate the probability of x or more defectives in the sample:

$$P(x \text{ or more}) = P(x) + P(x+1) + P(x+2) + \dots \text{etc.}$$

or, more simply, particularly for small values of x :

$$\begin{aligned} P(x \text{ or more}) &= 1 - P(x-1 \text{ or fewer}) \\ &= 1 - \{P(0) + P(1) + \dots + P(x-1)\} \end{aligned}$$

Reverting to the example in 3.2.4, it was calculated that $P(2) = 0.184$. $P(0)$ and $P(1)$ are calculated using the same formula:

$$P(0) = e^{-1} = 0.368$$

$$P(1) = e^{-1} = 0.368$$

Thus:

$$P(1 \text{ or fewer}) = P(0) + P(1) = 0.368 + 0.368 = 0.736$$

$$\begin{aligned} P(2 \text{ or fewer}) &= P(0) + P(1) + P(2) \\ &= 0.368 + 0.368 + 0.184 = 0.920 \end{aligned}$$

from which can be derived:

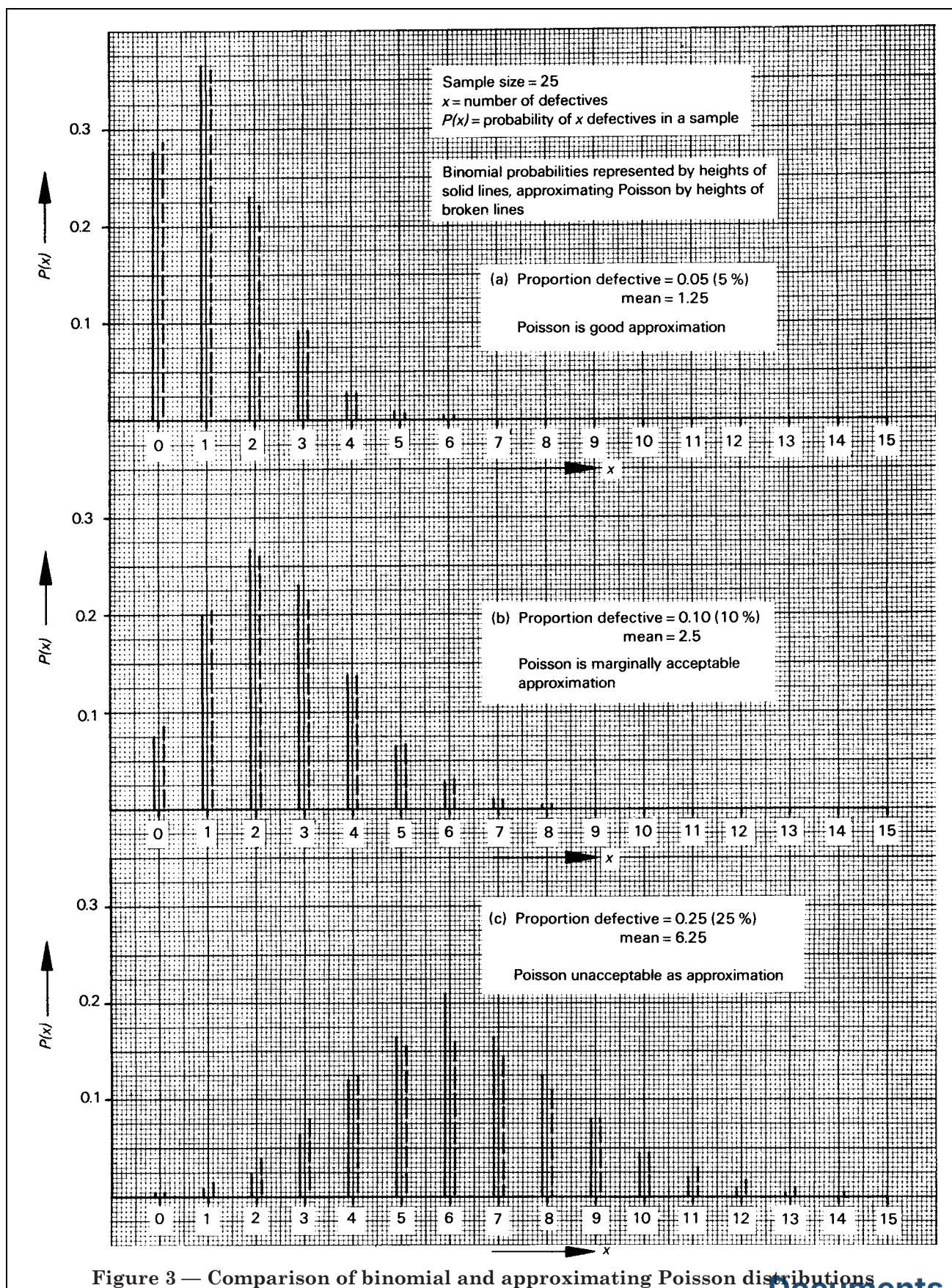
$$P(1 \text{ or more}) = 1 - P(0) = 0.632$$

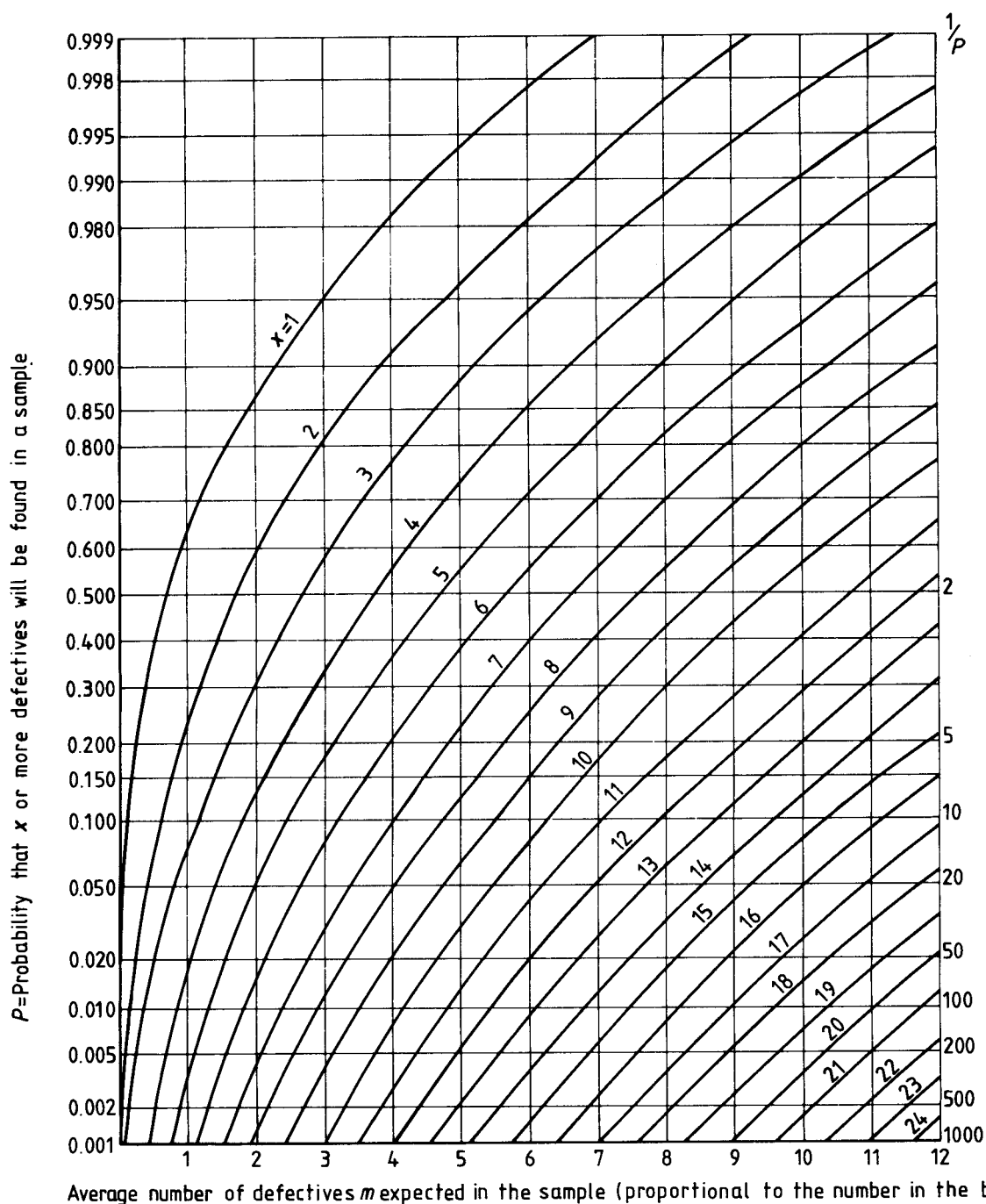
$$P(2 \text{ or more}) = 1 - P(1 \text{ or fewer}) = 0.264$$

$$P(3 \text{ or more}) = 1 - P(2 \text{ or fewer}) = 0.080 \text{ etc.}$$

To avoid the necessity for these calculations, a chart is available as shown in Figure 4 (which also appears as Figure 3 of BS 5701:1980).

By entering this for $m = np = 1$, the values obtained in this clause are confirmed.





The theoretical information given in these curves is sufficiently accurate for those cases where the proportion defective is small, say less than 10 %. Higher proportions require different values.

Figure 4 — Curves showing theoretical probability that x or more defectives will be found in a sample (from BS 5701)

3.2.6 Application to BS 5701. The criterion for validity of the approximation (proportion defective less than 10 %) should be satisfactory in most quality control situations, for which reason BS 5701 bases its procedures upon the Poisson distribution. This has the advantage of greatly simplifying the calculations and permitting the use of “expected defectives per sample” as a basis for producing control chart limits irrespective of sample size.

Using the Poisson distribution, for any value of the expected number of defectives in a sample, the probability of actually observing x defectives, where x is any number between 0 and the sample size, can be calculated as shown in 3.2.4. It will be noted that use of the Poisson approximation has the great advantage that these probabilities depend upon one parameter only, the *expected* number of defective items in the sample.

The probabilities are correct for any combination of proportion defective and sample size which, when multiplied together, give the expected number defective in the sample. Use of this principle in BS 5701 permits flexibility in the choice of sample size without the necessity for extensive tabulations of control limits.

3.3 Basis for decision rules

3.3.1 Basic principle of control charts for number defective. The objective of an attributes control chart is to detect an increase of the process proportion defective above some predetermined value.

This value can be either:

- a) the existing level of quality; or
- b) the specified level of quality.

Whilst the interpretation of a decision that an increase has occurred may be different in the two cases, there is no difference in the way in which decision rules are derived.

3.3.2 Application to control limits

3.3.2.1 Control limit. Figure 1 shows a control limit drawn on a number-defective control chart. This control limit is an action line.

The associated decision rule is: “Take action if a plotted point occurs above an action line.”

As one can observe only an integer number of defective items in a sample, the action line is positioned between integer values to avoid the ambiguity of a point being plotted exactly on an action line.

The convention adopted in BS 5701 is to find the value of the integer x such that, for a given expected number of defectives in a sample, the probability of x or more defectives is not greater than one in 200. The action line is then drawn at the value $(x - 0.3)$.

As an example, consider the situation where the sample size is 40 and the proportion defective is 0.05 (5 %). The expected number of defectives in the sample is $40 \times 0.05 = 2.0 = m$. Reference to Figure 4 shows that in this case the probability of 7 or more defectives is 0.005 (1 in 200). The control limit is therefore placed at $7 - 0.3 = 6.7$. This accords with the value given in Table 1 of BS 5701:1980.

All the values in Table 1 of BS 5701:1980 are calculated in a similar way.

Under these rules, the probability of obtaining a plotted point above the control limit when the proportion defective is at the expected value is never greater than one in 200. On this basis the type I risk will never exceed this value.

3.3.2.2 Warning limits and critical gap. As a way to increase the sensitivity of the chart without markedly increasing the risk of spurious action decisions when no change has occurred, additional decision criteria may be invoked.

The recommendation in BS 5701 is that a “warning limit” line be drawn one unit below the control limit (i.e. if the control limit is 6.7, the warning limit is 5.7). The process is deemed to have deteriorated if the interval (measured in samples) between two successive points outside the warning limit is less than the “critical gap” in Table 1 of BS 5701:1980.

3.3.3 The choice of sample size and type II risks. BS 5701 suggests a sample size of 25 as an initial trial. Once familiarity with attributes control charts is established, the sample size chosen should be such that the average number of defects expected per sample is in the range 1 to 3, preferably between 2 and 3. Any sample size to meet this requirement can be chosen, as the limits in BS 5701 are based directly on the number of defects expected per sample.

When deciding upon the sample size, the effect on the type II risk (failure to detect an increase in process proportion defective) should be taken into account. In general, the larger the sample size, the smaller the type II risk.

This can be illustrated by considering the example in 3.3.2.1 and examining the risk of failing to detect an increase in the proportion defective to 15 % (compared with the target value of 5 %). When $p = 15\%$ (0.15), $m = 40 \times 0.15 = 6.0$, and, from Figure 4, the probability of 7 or more defectives in the sample is 0.40. This means that the type II risk at 15 % defective is $(1 - 0.40) = 0.60$ (60 %). Should this be considered unacceptably high, an increase in the sample size will reduce it. For example, an increase from 40 to 60 will give an expected number of defectives at 5 % defective of $60 \times 0.05 = 3.0$. The control limit given in Table 1 of BS 5701:1980 is now 8.7, i.e. an action decision is reached for 9 or more defectives in the sample. When $p = 15\%$, $m = 60 \times 0.15 = 9$ and, from Figure 4, the probability of 9 or more defectives is 0.54 which means that the type II risk at 15 % defective is now reduced to $(1 - 0.54) = 0.46$ (46 %).

3.3.4 Example. The following are 20 observations of the number of defective components in daily samples of 50 taken from a high-speed manufacturing process.

Day	1	2	3	4	5	6	7	8	9	10
No. of defectives in sample	1	0	0	2	1	0	1	2	0	3
Day	11	12	13	14	15	16	17	18	19	20
No. of defectives in sample	1	1	3	0	0	2	2	1	0	1
Total defectives = 21										

The observed average number of defectives found per sample is:

$$\frac{21}{20} = 1.05$$

Before using this value as the basis for subsequent control charts, it is necessary to check that no apparent assignable cause is at work. This can be done by simply plotting the results on a provisional control chart (without limits) as in Figure 5(a), from which it will be seen that no obvious trends, cyclic movements, step changes etc. are present.

This may be confirmed by examining the variance of observed results. If the variance is numerically close to that expected for a binomial distribution with a *constant* proportion defective $[np(1-p)]$, then it can be assumed that a change in the proportion defective did not occur during the period of observation.

Observed proportion defective:

$$p = \frac{21}{50 \times 20} = 0.021$$

Expected variance of number defective = $np(1-p) = 50 \times 0.021 \times 0.979 = 1.028$.

Observed variance:

$$s^2 = \frac{1}{g-1} \left[\sum x_i^2 - \frac{(\sum x_i)^2}{g} \right]$$

where x_i is the number of defectives in the i th sample and g is the number of samples (= 20).

$$s^2 = \frac{1}{19} \left[41 - \frac{21^2}{20} \right] = 0.997$$

As the two values are approximately equal it can be said that no assignable cause of change was present during this preliminary sampling.

NOTE The comparison between expected and observed variances in the case of Poisson and binomial distributions is further discussed in 8.3 of BS 5703-4:1982.

On the assumption that, for subsequent control charting purposes, a sample size of 50 will continue to be used, the observed average number defective in the sample can be used to obtain a scheme from BS 5701.

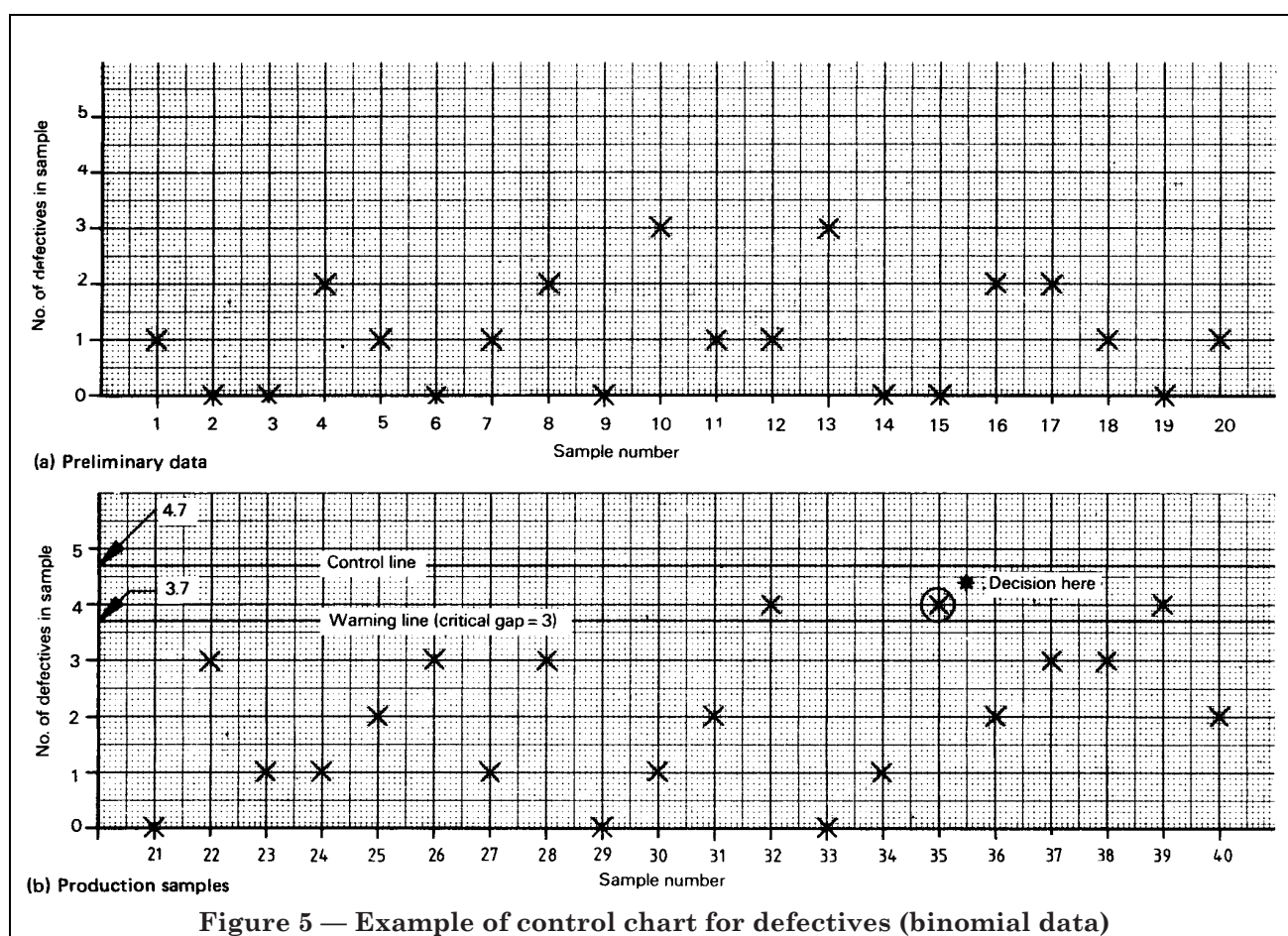
Reference to Table 1 of BS 5701:1980 shows that the action line would be at 4.7 and the warning line at 3.7 with a critical gap of 3.

Subsequently, the first 20 production samples gave the following results:

Sample no.	21	22	23	24	25	26	27	28	29	30
No. of defectives in sample	0	3	1	1	2	3	1	3	0	1
Sample no.	31	32	33	34	35	36	37	38	39	40
No. of defectives in sample	2	4	0	1	4	2	3	3	4	2
			Gap = 2							

Figure 5(b) shows a plot of these data on the control chart.

An action decision (that the proportion defective has increased above 0.021) is taken at sample no. 35 because the gap between two samples, between warning and action lines, is less than the critical gap of 3.



3.4 Using attributes charts for controlling measured variables

3.4.1 General. An attributes chart can be used to control a measured variable by deeming an item to be defective when the dimension or property of interest is outside its specified limit. The advantages and disadvantages of this are discussed in section 2.

3.4.2 Control of more than one dimension or property. Attributes charts can be used for this purpose by considering an item as being defective if one (or more) of the dimensions or properties is outside its specified limit. This allows a single chart to be used instead of having to maintain a chart for each dimension or property individually (but see 3.4.3).

3.4.3 Control of a single dimension or property (small proportion outside limits acceptable)

3.4.3.1 Single-sided (unilateral) limit. The items are measured or gauged for the dimension or property in question. Any item that has the dimension or property outside the specified limit is deemed defective. The number defective in the sample is charted in the usual way.

3.4.3.2 Two-sided (bi-lateral) limit. In this case there are two possibilities.

- Consider all items where the dimension or property is outside either limit as defective, and plot as for the case in 3.4.3.1.
- Plot separate charts for each of the specification limits, i.e. one chart where the number defective is the number in the sample above the upper limit and one chart where the number defective is the number below the lower limit.

The information displayed in these two charts will give much more information as to the state of the process than the single chart as in (a) above.

3.4.3.3 For further details and an example see 6.2 of BS 5701:1980.

The approach to one-way and two-way charts is shown graphically in Figure 6.

3.4.4 Compressed limits (for control of a single dimension or property: defective items not produced or not acceptable)

3.4.4.1 General. There are many situations where it would be convenient to use an attributes control chart for a measured variable which, when the process is “in control”, does not produce any defective item outside the specification limits, as shown in Figure 6. A straightforward number-defective chart cannot be used in such circumstances because the “expected number defective” is zero, however large the sample size. Furthermore, the observation of any defective items would occur too late, in that the process would already be in an unacceptable state. There are other situations where the proportion defective is not zero, but is still so small that a conventional attributes chart would be of very low sensitivity. In both these circumstances it is possible to use “compressed limits” that produce “synthetic defectives” for control purposes that are nonetheless acceptable with respect to the specification limits.

The procedure for compressed limits is illustrated in Figure 7. The objective of the procedure is to provide artificial gauging limits such that 10 % to 15 % of production will be outside each compressed limit.

3.4.4.2 Two-sided specification limits. If the process capability (see 4.3 and 4.6) is unknown, the compressed limits should be set ($0.3 \times$ gross specification tolerance) inside the specification limits. “Defectives” outside these limits should be plotted on a two-way chart as described in 3.4.3.2 b).

If “action” decisions are reached on one chart only, this indicates a change in the process average.

If “action” decisions arise on both charts, this indicates an increase in the dispersion of the process.

If, with the process operating normally, the proportion outside a compressed limit is much less than 10 %, that limit should be adjusted closer to the process average.

3.4.4.3 Single specification limit. When the process standard deviation is known, the compressed limit is set approximately 2 standard deviations inside the specification limit. If the standard deviation is not known, it is difficult to give a hard-and-fast rule; the limit should be such as to give 10 % to 15 % outside it. Alternatively, if it is known that a particular average value of the process is satisfactory, the compressed limit should initially be set $0.6 \times$ (distance between specification limit and satisfactory average) inside the specification limit.

For more details on how compressed limits are derived, and methods of application, reference should be made to Appendix A of BS 5701:1980.

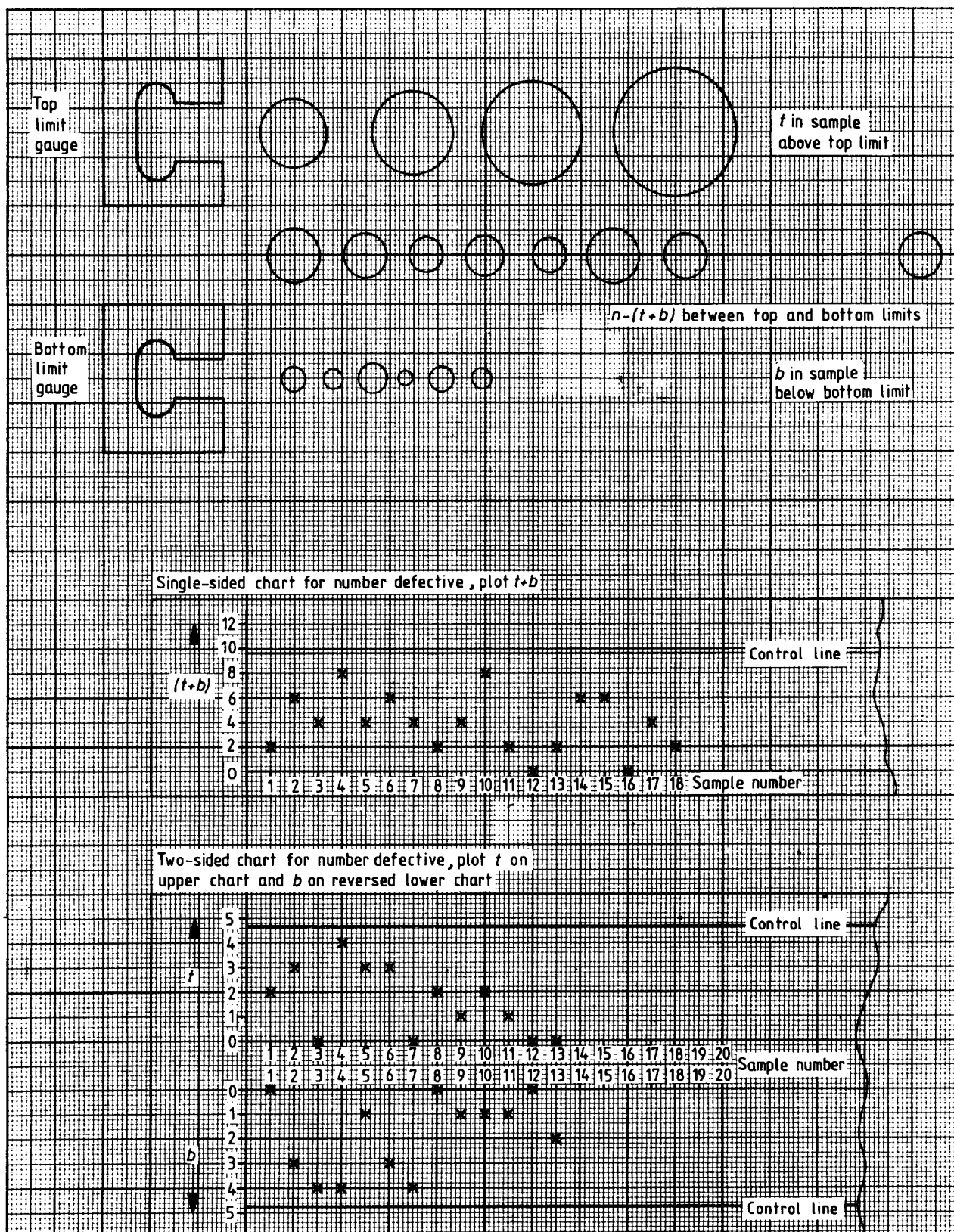


Figure 6 — One-way and two-way charts for two-sided specification limits