

Guide to

Data analysis and quality control using cusum techniques —

Part 1: Introduction to cusum charting

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Foreword

It was originally intended that this British Standard should form part of an overall revision of British Standards relating to statistical quality control techniques. However, cusum methods have found applications in fields beyond that of quality control in manufacturing, (e.g. management, government administration, medical science, marketing, commerce, etc.) and comments on the draft standard revealed interest in a more general document. This Part of the standard represents an attempt to meet this demand, and is intended as an introduction to the principles of cusum procedures, and charting.

Part 1 outlines the basic presentation of the cusum method.

Part 2 details decision rules, significance tests for retrospective analysis of accumulated data, and contains a variety of applications of cusum charting, some concerned mainly with data presentation, some with monitoring in quality or budgetary control, and others with retrospective analysis.

Part 3¹⁾ deals with applications of cusum methods to process and quality control by measurement. The techniques cover control of both average level and variability, and may be implemented using charts, tabulations or computers. They are especially appropriate when location of change points and estimation of the corrections required to bring the process on to target are desirable features of the control system.

Part 4¹⁾ deals with control of counts of defectives.

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Summary of pages

This document comprises a front cover, an inside front cover, pages i to iv, pages 1 to 40, an inside back cover and a back cover.

This standard has been updated (see copyright date) and may have had amendments incorporated. This will be indicated in the amendment table on the inside front cover.

¹⁾ In course of preparation.

Section 1. General

0 Introduction

0.1 Basis of cusum chart. The cumulative sum chart (hereinafter referred to by the generally accepted contraction “cusum chart”) is a highly informative means of graphical presentation of data that can be ordered into a logical sequence. Frequently, though not necessarily, this sequence corresponds to the order of observation on a time scale. However, examples will be given where other, but equally logical, ordering is based on separation of the observations into groups having distinctive features.

The essential principle is that a *target* value, T , is subtracted from each observation. This target value is generally a constant but it may be a value that changes for each observation, e.g. a prediction from a forecasting model or a conditional target which may vary depending on the circumstances in which the observation is obtained. The cumulative sum of the deviations from target is formed, and this *cusum* (C) is plotted against the serial number of the observation.

Unlike conventional plotting (where the average level of the values in the series corresponds to height above the serial number axis), the cusum method of plotting results in the representation of average by the local *slope* of the chart. When the average corresponds to the target value, the path of the cusum lies roughly parallel to the sequence axis. When the local average of the series is greater than the target value, the cusum slopes upwards (away from the sequence axis); conversely, when the local average is less than the target value, the cusum slopes downwards. The greater the discrepancy between the local average and the target value, the steeper the slope of the cusum path.

The result of plotting in the cusum mode is that changes in average level over different subdivisions of the sequence of observations are clearly indicated by changes in slope of the chart. The local averages in such subdivisions can be readily calculated, either from the numerical values of the cusum (from which the chart is plotted) or directly, albeit with slight loss in accuracy in some cases, from the chart itself.

0.2 Simple example of cusum chart. The above principles are best appreciated from a simple example. The calculations and plotting procedure will, at this point, be developed without mathematical symbolism; appropriate algebraic formulae will be presented later.

It is supposed that the following data have been obtained, over a time sequence in the order shown, and that a target value of 15 is appropriate.

For a conventional chart, as in Figure 1, the observed values are plotted against their corresponding observation numbers. There is an indication that the last dozen values are clustered around a different mean level from the first 20 or so.

Plotting in the cusum mode gives a much clearer display than the conventional chart. The cusum (column 4 of table) is plotted against the observation number, on conventional axes, using the y (“vertical”) axis for the cusum and the x (“horizontal”) axis for the observation number, as in Figure 2.

The chart clearly separates into three segments. From observations numbered 1 to 7 (inclusive) the cusum path is generally parallel to the observation number axis, i.e., the path, in conventional terms, is roughly horizontal. From observations 8 to 21 inclusive the path is recognizably downward (despite local irregularities such as at observations 14, 20). From observations 22 to 33, the path is upward (again with local irregularities that do not blur the overall pattern).

Thus it could be inferred that

- a) observations 1 to 7 constitute a sample from a “population” whose mean is at or near the target value (15);
- b) observations 8 to 21 appear to have been sampled from a population whose mean is below 15;
- c) observations 22 onward appear to come from a population whose mean is greater than 15.

There are now a number of questions that might be asked.

- a) In the light of the underlying variability (as indicated, for example, by the minor irregularities in the cusum path) can it be concluded that the changes in slope represent real shifts in average rather than merely lucky or unlucky runs of samples from a stable population?
- b) If the changes are real, how should the data be used to estimate local averages?
- c) To what extent might the inferences or estimates be affected by the choice of the target value or the cusum scale factor? Thus Figure 3 and Figure 4 show the same series plotted firstly with the same cusum scale but with a target of 12, and secondly with a target of 15 but a compressed cusum scale.

Table 1 — Data for cusum plotting

Observation	Observed value	Deviation from target (15)	Cumulative sum of deviations	Successive differences (see 9.1)
1	12	− 3	− 3	
2	17	+ 2	− 1	5
3	14	− 1	− 2	− 3
4	14	− 1	− 3	0
5	17	+ 2	− 1	3
6	16	+ 1	0	− 1
7	14	− 1	− 1	− 2
8	11	− 4	− 5	− 3
9	13	− 2	− 7	2
10	14	− 1	− 8	1
11	15	0	− 8	1
12	11	− 4	− 12	− 4
13	14	− 1	− 13	3
14	16	+ 1	− 12	2
15	13	− 2	− 14	− 3
16	14	− 1	− 15	1
17	11	− 4	− 19	− 3
18	12	− 3	− 22	1
19	13	− 2	− 24	1
20	16	+ 1	− 23	3
21	12	− 3	− 26	− 4
22	18	+ 3	− 23	6
23	18	+ 3	− 20	0
24	17	+ 2	− 18	− 1
25	20	+ 5	− 13	3
26	15	0	− 13	− 5
27	14	− 1	− 14	− 1
28	18	+ 3	− 11	4
29	20	+ 5	− 6	2
30	16	+ 1	− 5	− 4
31	18	+ 3	− 2	2
32	14	− 1	− 3	− 4
33	16	+ 1	− 2	2

In Figure 3 and Figure 4, the change in slope around observation number 8 is less apparent, though the change around number 21 is still visible, but less easy to “pinpoint” in Figure 4. Thus the choice of target value and scale factor need careful thought, to avoid either the suppression of useful information or conversely the exaggeration of spurious effects. It is also clear from Figure 3 that use of an inappropriate target value may result in the chart running off the upper or lower edge of the graph paper. However this problem may also be overcome by replotting from a new zero at any point in the sequence; this will be detailed later.

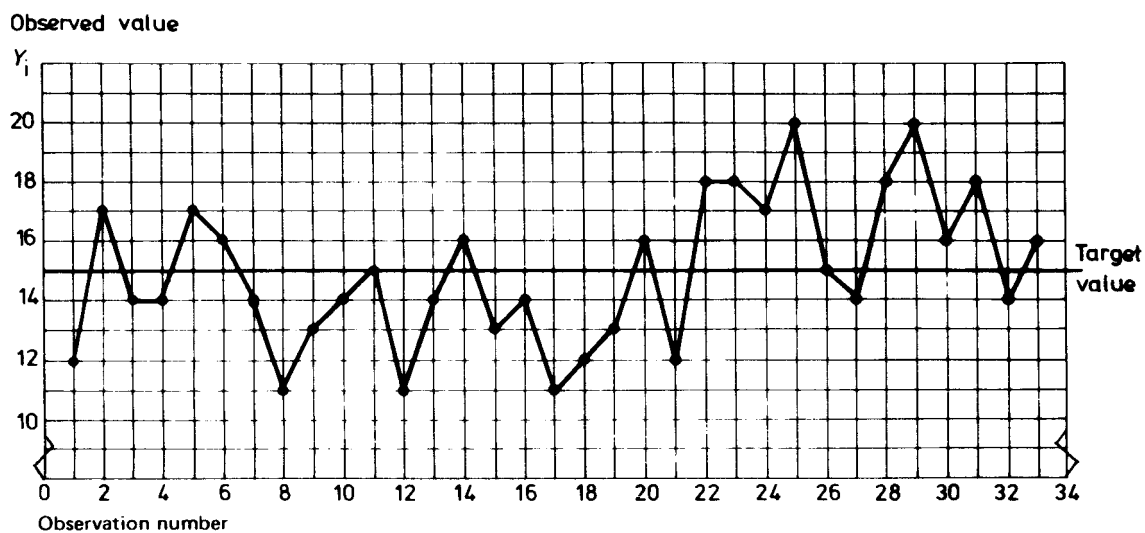


Figure 1 — Conventional chart of data from Table 1

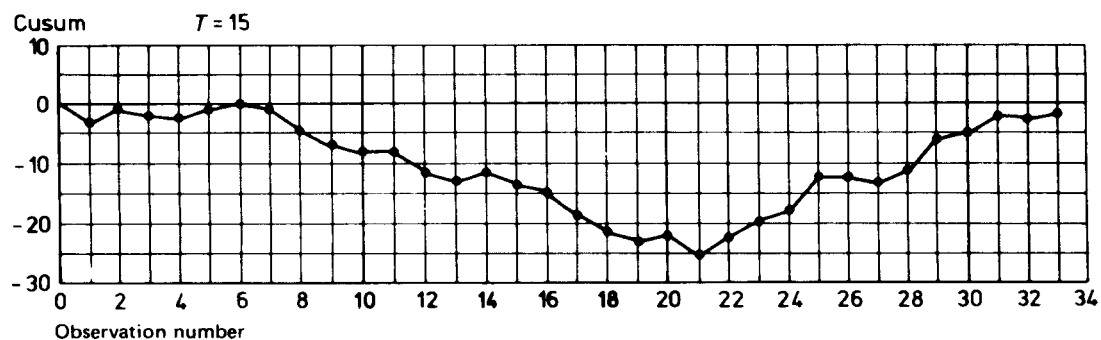


Figure 2 — Cusum chart of data from Table 1

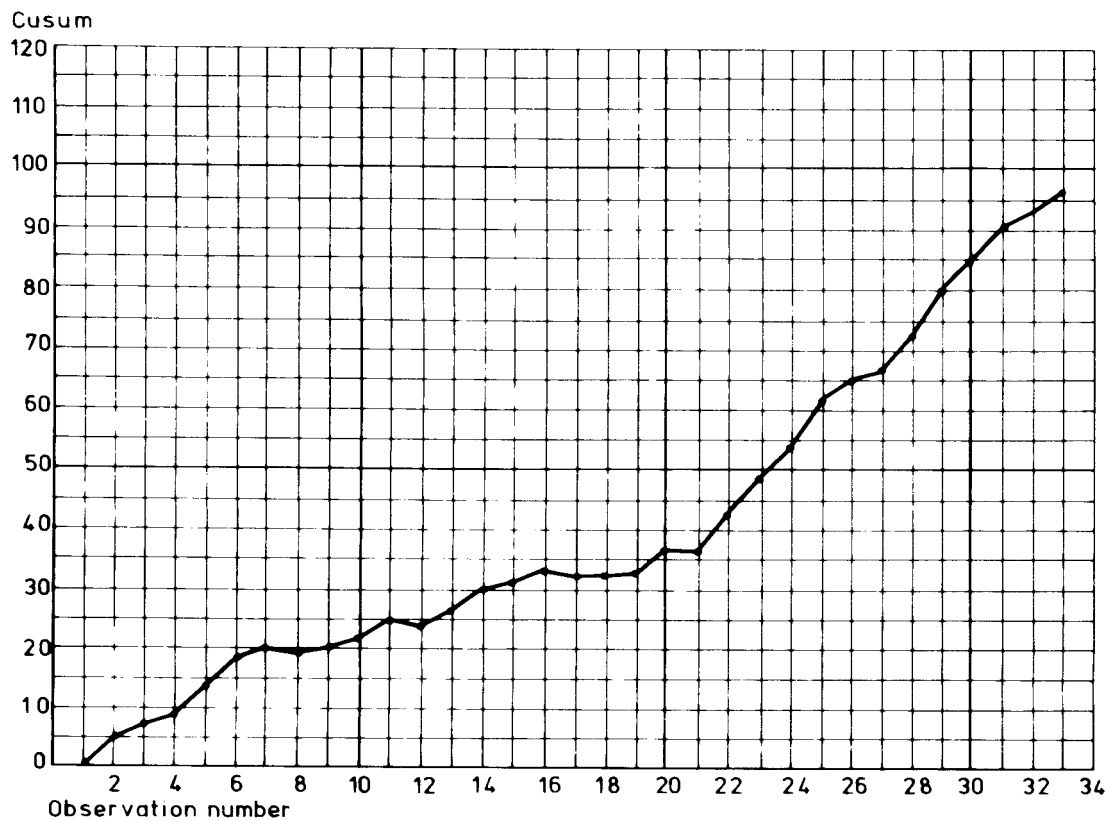


Figure 3 — Cusum chart of data from Table 1 with target value 12

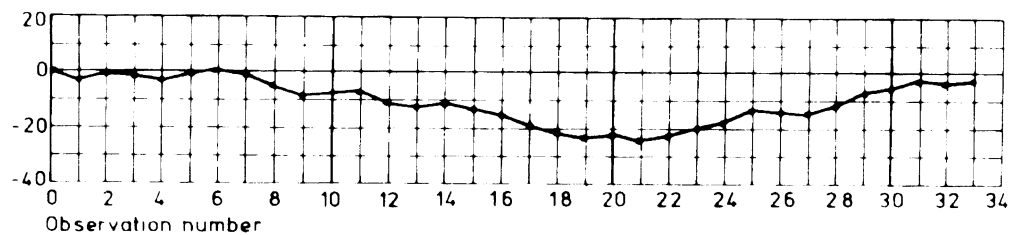


Figure 4 — Cusum chart of data from Table 1, with target value 15 but compressed cusum scale

1 Scope and general principles

1.1 Scope. This Part of this standard introduces the principles of cusum techniques and includes the preparation, charting, presentation and interpretation of data, with worked examples.

1.2 Fundamental requirements. The fundamental requirements for cusum charting are as follows.

- a) The observations should be at least on an interval scale of measurement.
- b) There should be logical grounds for their being sorted into a sequence for plotting.

These requirements are taken in order. The interval properly requires any given numerical difference between two observations to have the same interpretation throughout the range of the variable. Thus a difference of 0.1 mm between the lengths of two objects has the same meaning whether the objects are woodscrews of length 10.1 mm and 10.0 mm or steel girders of length 10 000.1 mm and 10 000.0 mm although the latter difference may be unimportant. Many arbitrary scales do not have this property: defect ratings are an example, where perhaps a serious defect scores 10 points, a moderate defect 5 and a minor defect 1. We cannot then interpret this to mean that the following items are necessarily equally undesirable, although their differences are zero:

Item A	One serious defect	Score = 10
Item B	Two moderate defects	Score = 10
Item C	One moderate, five minor defects	Score = 10
Item D	Ten minor defects	Score = 10

Interpretation of “average” score could be misleading if the *balance* of major, moderate and minor defects, rather than merely their overall frequency, changes.

The logical sequences property may arise in numerous ways. Most obviously, the observations may occur in a time or length sequence, thus forming a natural progression. Monitoring for quality or process control provides many cases of this kind.

Secondly, conditions in the environment from which the observations are taken may vary (either deliberately or fortuitously). The data thus fall into obvious groups, but the ordering of observations within these groups may not be of any particular importance.

Thirdly, the items themselves may provide a basis for ordering or, more commonly, for grouping. Manufactured components may be segregated according to the identity of the machine from which they are sampled, or on the basis of whether or not they possess specified defects or other attributes.

Fourthly, observations may be ordered according to the value of some auxiliary variable measured on the items, the cusum then providing a means of presenting or investigating relationships between variables, augmenting a regression or correlation analysis.

This list of possibilities is not exhaustive, and any kind of ordering or grouping that uses some structural feature of the observations or the background from which they are taken may provide the basis for the cusum sequences.

1.3 Types of data amenable to cusum charting. Many types of data satisfy the fundamental requirements a) and b) of 1.2, and some examples may be useful. Perhaps the most frequent applications of cusum charts have been in quality control, where observations (e.g. sample means or ranges) are plotted in sequence to provide a continuous assessment of the state of a process. It is sometimes assumed that such observations have to be normally distributed for the application of cusum charts. This is not the case, although some distribution models may be necessary when setting up the decision rules. However, when using a cusum chart as a device for effective data presentation, it is not necessary to specify a distribution, nor to require independence between successive observations (again, this condition is important for decision rules, but not for data presentation). Indeed, the cusum chart may assist in the identification of features such as serial correlation or cyclic behaviour.

Thus data involving ranges or sample estimates of standard deviation may be plotted on cusum charts, as well as sample averages. Counts of defects or defectives are also encountered in quality control, and may be monitored by cusum charts.

Outside the quality control area, many applications arise. The consumption of fuel by transport vehicles (even by private cars), the response of patients to changes in treatment, trials of systems or weapons in which each trial has a success or failure (1 or 0) outcome: all provide data amenable to cusum charting.

Some forms of indirect data also occur. In setting up a forecasting system, its performance may be monitored by comparing predicted values with actual values when they become available. The validity or adequacy of a mathematical model for an experiment may be assessed by a cusum of the residual errors. In this instance the various levels of experimental factors will provide the basis for grouping or ordering. Data in the commercial and administrative areas may also be usefully subjected to cusum analysis. Monitoring of absence rates (and examination for day-of-the-week patterns), auditing of frequency of card-punch errors, assessment of labour turnover, are typical examples. Even where the size of the sample or group (for which such rates are calculated) varies, the cusum chart can be simply modified to accommodate this.

1.4 Purpose of cusum charting. Cusum charting has two main purposes, which are related to the way in which the data is collected.

a) *Monitoring or control.* Values are plotted on the cusum chart as they occur, with the aim of detecting any change in the nature of the process generating the observations, such as a change in mean or variability.

Decision rules are necessary to rationalize interpretation of the chart. When an appropriate decision rule so indicates, some action is taken, depending on the nature of the process. Typical actions are as follows:

- 1) in a quality control application, adjustment of the process conditions;
- 2) in a more general technical context, investigation of the underlying cause of the change;
- 3) in monitoring the behaviour of a forecasting system, analysis of and, if necessary, modification to the model or its parameters.

b) *Data analysis.* A complete set of observations is subjected to cusum analysis, the object being to detect whether changes (e.g. in mean level) have occurred at one or more points in time, or between the rational groups into which the data are divided. The objectives are close to those of hypothesis testing, except that the groups or time-segments into which the data are divided may be based on an examination of the cusum pattern, instead of being determined in advance as in conventional hypothesis testing. In essence it is an attempt to detect any change points in the sequence.

Where historical data (i.e. data collected over time, and measuring the past behaviour of systems) are involved, the procedure is often termed “retrospective” or “post mortem” analysis. However, as previously noted, grouping by features other than time may often be appropriate.

For either primary purpose, or in intermediate cases, the changes should be assessed against the intrinsic variations in the data. For time sequences, this means detecting sustained changes in typical level against a suitable measure of short-term variation. For other forms of grouping, the problem (like that in analysis of variance) is in comparing between-group variation with the variation within groups.

2 References

The titles of the standards publications referred to in this standard are listed on the inside back cover.

3 Symbols and terminology

3.1 General. To avoid an unduly lengthy document, it is necessary to assume some familiarity with basic statistical concepts and terminology. In the later Parts, a similar knowledge of the language of quality control will be assumed, but for this Part, the essential requirements are an understanding of the following:

- $\bar{x}$ arithmetic mean of a set of data
- μ arithmetic mean or expected value of the population (real or imaginary) from which the data is a sample
- s sample estimate of the standard deviation
- σ standard deviation of the population
- Σ summation, in particular as used in

$$\bar{x} = \frac{1}{n} \sum_{i=1}^n x_i$$

$$s = \sqrt{\frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{x})^2}$$

$$\text{or } \sqrt{\frac{1}{n-1} \left\{ \sum x^2 - \frac{(\sum x)^2}{n} \right\}}$$

using the abbreviated summation notation, where the range of x is assumed to extend over $i = 1, \dots, n$.

σ_e will be used to denote the standard error of a sample statistic, the conventional measure of variation of a variable calculated from a sample; the standard error of the mean is the most familiar example.

Some notions from elementary probability and distribution theory (independence, correlation) are used in the standard. Where decision rules are discussed, the distribution of the observed variable will be specified. (In this Part this will be almost exclusively the normal distribution.)

An account of this basic material may be found in most elementary statistical text books. The only other requirement of the reader is an appreciation of the terms used in plotting graphs on two axes. The convention adopted is that the sequence in which the data occur are referred to the x-axis (abscissa, or conventional "horizontal" scale), and the cumulative sum is referred to the y-axis (ordinate, or "vertical" scale).

Careful thought has been given to the notation adopted for cusum parameters. In the statistical journals, an informal convention has evolved using the symbols H (or h), K (or k), and θ (theta). There seems to be no reason to depart from the use of H and K , but θ has been adopted as a standard symbol (for another purpose) in BS 5532.

In cusum terms, the required symbol represents an angle; ϕ (phi) has therefore been adopted, leading to f to represent its tangent.

The upper case forms H , K , F are used in conjunction with charts referred to an observed variable, and the lower case (h , k , f) to a standardized variable, e.g. one having a standard normal distribution $\mu = 0$, $\sigma = 1$.

3.2 Symbols and terms

Symbol	Term	Definition
α, A	scale factor	$A = a \sigma_e$, and is the number of units of the plotted variable, measured on the cusum axis, corresponding to one sample interval on the sequence axis of the cusum chart.
AQL	acceptable quality level	The limiting value of the process average regarded as acceptable for process control purposes. NOTE This definition is compatible with those given in BS 4778 and BS 5532, but extends their use to cover measurable characteristics in a manner consistent with current industrial practice.
C	cusum	The cumulative sum of deviations of a variable from a target value
i, j, r		Subscripts used to identify values of a variable, or of the cusum
m	segment length	The number of sample intervals in a cusum sequence for which a test of change in level is carried out
N	sequence length	The total number of sample intervals covered by a retrospective cusum
n	sample size	The number of individual items comprising a sample
p	proportion	The proportion of binary observations scoring one ("successes" in the usual binomial distribution convention)
T	target value	The target or standard value from which departure in average level requires to be detected
x, y, Y	variables	Symbols denoting a random variable
$\bar{x}, \bar{y}, \bar{Y}$	mean values	Arithmetic mean of a set of random variables x, y, Y
RQL	rejectable quality level	In the quality control sense, a level of quality regarded as unsatisfactory, which should be detected quickly by a quality control procedure
RL	run length	At a given quality level, the number of samples taken up to the point at which a signal occurs
ARL or L	average run length	At a given quality level, the average number of samples taken up to the point at which a signal occurs. This is generally related to a stated quality level and the symbol " L " then carries an appropriate subscript, as for example L_a or L_r for average run length at AQL or RQL respectively

Section 2. Preparations for cusum charting

4 Notation

4.1 To simplify the presentation of the cusum procedure, convenient notation is first defined. A target value (whatever its specific interpretation) will be denoted by T . As each observation in the sequence, x_i , is dealt with, the difference $(x_i - T)$ is formed. These differences are summed, so that by the time x_i is reached, the cusum

$$C_i = \sum_{r=1}^i (x_r - T) \quad (1)$$

is formed. The cusum, C_i , is plotted as ordinate ("vertical" axis) against i as abscissa ("horizontal" axis). Assuming for the present that i takes successive integer values 0 (at the origin), 1, 2, ... , the scale factor on the vertical axis will be denoted by A . This is interpreted as meaning that the distance which, on the horizontal scale, corresponds to one plotting interval, represents A on the cusum scale. This scale factor may often be expressed as multiples of the standard error of the plotted values (σ_e), and this standardized scale factor will carry the symbol a (thus $A = a\sigma_e$). The meaning and estimation of σ_e is detailed in clauses 6 and 7.

4.2 It will frequently be useful to calculate a local average for the sequence of points from i to j , or from i to $j - 1$, or perhaps from j to $j + r$, etc. These will be indicated by

$$\bar{x}_{i,j}; \quad \bar{x}_{i,j-1}; \quad \bar{x}_{i,j+r}; \text{ etc.}$$

Other notation will be defined as it is introduced, and all such terms appear in Appendix C.

5 Choice of target value (T)

5.1 Choice of a suitable target value is one of the two most important steps in the preparations (the other being choice of scale factor). An unsuitable target value will cause the cusum to slope persistently up or down, making changes more difficult to observe, and necessitating frequent replotting (see clause 14) when the chart runs off the top or bottom of the graph. Its selection justifies careful thought, and may depend on the purpose of the cusum and the nature of the analysis.

5.2 In certain applications, the target value will be self-selecting. In monitoring a forecasting model, for example, each prediction (for a particular point in the future) becomes the target value for comparison with the actual observation when it becomes available. Denoting the prediction by P_i for the i th (time) point in the series, if x_i is the value observed when time point i is reached, cusum becomes

$$\sum_{r=1}^i (x_r - P_r)$$

Note that in this case the target value may shift for each observation.

5.3 In other simple cases, T may be readily apparent as a specified level of quality (e.g. an acceptable quality level, or a contracted quality). It is wise, of course, to have some assurance that the process can produce this quality, otherwise the cusum chart will merely be a permanent reminder of failure, and in such circumstances control systems tend to be ignored or fall into disuse.

There may not always be a specification value for T , but there may be a desired value. In monitoring fuel consumption of a vehicle, manufacturer's data, reports of motoring organizations or a preliminary assessment may provide a target value. Sometimes the mean level of achievement over a recent (and presumed stable) series of data may be used.

5.4 In cases of monitoring response to a drug, the effects of changes in manufacturing variables, or the reactions of staff to improvements in working conditions, the target value will be based on, for example, the state of the patient, plant under normal manufacturing conditions, or the recent behaviour (absence, productivity) of the staff concerned. In each case, a reliable average measure is required for setting up the target.

5.5 Where a cusum chart is to be used for retrospective examination of a series of historical data, or (for example) of residuals from an experiment, regression analysis, etc., the natural target becomes the arithmetic mean of the complete series. The actual mean may be rounded to avoid the use of $(x - T)$ values with excessive significant figures. Apart from any minor discrepancy arising from rounding, this choice for T will result in the cusum beginning and ending at the same ordinate value. Combined with suitable scaling (see clause 8) the cusum plot may thus be contained within the limits of the graph grid used for plotting.

5.6 Binary data, coded as a sequence of 0's and 1's, require an appropriate estimate of the proportion of responses which are scored as 1's. This proportion (a fraction, suitably rounded to avoid unnecessary digits) is then used as a target value. For quality control applications, a permissible proportion of defectives (AQL) may be specified by contract, or a feasibility study may provide a suitable value. In experiments involving binary response, either the overall proportion for the complete experiment may be used or, in the case of sequential experiments, the target may be set at the proportion observed in the first completed segment of the experiment, modifying this later if it proves unsuitable (see also clause 9).