

Guide to data analysis, quality control and improvement using cusum techniques —

**Part 1: Uses and value of cusum charts
in business, industry, commerce and
public service**

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- BAE Systems
- British Standards Society (BSS)
- Clay Pipe Development Association (CPDA)
- Federation of Small Businesses (FSB)
- Gauge and Tool Makers Association (GTMA)
- General Domestic Appliances Ltd.
- Institute of Quality Assurance
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Foreword

This part of BS 5703 was prepared by Technical Committee SS/4. It supersedes BS 5703-1:1980, which is withdrawn.

It was deemed necessary to revise BS 5703 for the following reasons:

- to present the text from the ordinary users view point rather than that of the statistician;
- to incorporate modern cusum computer software routines;
- to reduce the quantity of text and eliminate repetition;
- to improve the diagrams;
- to bring statistical symbols into line with standard practice.

BS 5703 is published in four parts:

- *Part 1: Uses and value of cusum charts in business, industry, commerce and public service;*
- *Part 2: Introduction to decision-making using cusum techniques;*
- *Part 3: Cusum methods for process/quality control using measured data;*
- *Part 4: Cusum methods for discrete (count/attribute) data.*

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Summary of pages

This document comprises a front cover, an inside front cover, pages i to iv, pages 1 to 29 and a back cover.

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Introduction

This first part of BS 5703 demonstrates the versatility and usefulness of a very simple, yet powerful, pictorial method of interpreting data arranged in any meaningful sequence.

These data can range from overall business figures such as turnover, profit or overheads to detailed operational data such as stock outs and absenteeism to the control of individual process parameters and product characteristics. The data can either be expressed sequentially as individual values on a continuous scale (e.g. 24.60, 31.21, 18.97...), in yes/no, good/bad, success/failure format, or as summary measures (e.g. mean, range, counts of events).

The method has a rather unusual name, cumulative sum, or, in short, “cusum”. This name relates to the process of subtracting a predetermined value, e.g. a target, preferred or reference value from each figure in a sequence and progressively cumulating (i.e. adding) the differences. The graph of the series of cumulative differences is known as a cusum chart. Such a simple arithmetical process has a remarkable effect on the visual interpretation of the data as will be illustrated.

The cusum method is already used unwittingly by golfers throughout the world. By scoring a round as plus 4, or perhaps, even, minus 2, golfers are using the cusum method in a numerical sense. They subtract the par value from their actual score and add (cumulate) the resulting differences. This is the cusum method in action. However it remains largely unknown and hence is a grossly underused tool throughout business, industry, commerce and public service. This is probably due to cusum methods generally being presented in statistical language rather than in the language of the workplace.

The aim of this part of BS 5703 is, thus, to be readily comprehensible to the extensive range of prospective users and so facilitate widespread communication and understanding of the method. As such, it focuses on a practical non-statistical treatment of cusum and presents examples using a simple pictorial approach.

While the treatment of cusum in this part of BS 5703 is essentially at appreciation level, it is intended to provide adequate information for a gainful first application by a typical user in most everyday situations. BS 5703-2, BS 5703-3 and BS 5703-4 provide a more rigorous, statistically based, approach to cusum.

1 Scope

BS 5703-1 describes a simple pictorial cumulative sum method of enhancing the presentation and understanding of data arranged in a meaningful sequence.

2 Normative references

The following referenced documents are indispensable for the application of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

BS ISO 3534-1, *Statistics — Vocabulary and symbols — Part 1: Probability and general statistical terms*.

BS ISO 3534-2, *Statistics — Vocabulary and symbols — Part 2: Statistical quality control*.

BS EN ISO 9000:2000, *Quality management systems — Fundamentals and vocabulary*.

3 Terms and definitions

For the purposes of this part of BS 5703, the terms and definitions given in BS ISO 3534-1, BS ISO 3534-2 and BS EN ISO 9000:2000, Clause 3 apply.

4 Principal features of cumulative sum (cusum) charts

A cusum chart is essentially a running total of deviations from some pre-selected reference value. The mean of any group of consecutive values is represented visually by the current slope of the graph. The principal features of a cusum chart are the following.

- a) It is sensitive in detecting changes in the mean.
- b) Any change in the mean, and the extent of the change, is indicated visually by a change in the slope of the graph:
 - 1) horizontal graph indicates an on target or reference value;
 - 2) downwards slope indicates a mean less than the reference or target value: the steeper the slope the bigger the difference;
 - 3) upwards slope indicates a mean more than the reference or target value: the steeper the slope the bigger the difference.
- c) It can be used retrospectively for investigative purposes, on a running basis for control, and for prediction of performance in the immediate future.

5 Basic steps in the construction of cusum charts

The steps in setting up a cusum chart for individual values are:

- Step 1:** Choose a *reference*, target, control or preferred *value*, RV. The average of past results will generally provide good discrimination.
- Step 2:** Tabulate the results in a meaningful (e.g. chronological) sequence. Subtract the reference value from each result.
- Step 3:** Progressively sum the values obtained in Step 2. These sums are then plotted as a cusum chart.
- Step 4:** To obtain the best visual effect set up a horizontal scale no wider than about 2.5 mm between plotting points.
- Step 5:** For reasonable discrimination, without undue sensitivity, the following options are recommended.
- Choose a vertical scale relative to the horizontal scale = [short-term spread of results (see Note)]/3. Round off as appropriate.
 - Alternatively, where it is required to detect a known change, say C , choose a vertical scale such that:

$$\frac{\text{length of one unit on horizontal scale}}{\text{length of one unit on vertical scale}} \text{ is between } C \text{ and } 2C. \text{ Round off as appropriate.}$$

NOTE The short-term spread of results can be estimated from the average of successive differences of results over a stable period

6 Example 1: measured data

6.1 The process

An example of the application of the cusum method and value of the results achieved is now illustrated.

Suppose a set of 40 values in chronological sequence is obtained of a particular characteristic. These happen to be voltages, taken in order of production, on fractional horsepower motors at an early stage of production. But they could be any individual values taken in a meaningful sequence and expressed on a continuous scale. These are now shown:

9, 16, 11, 12, 16, 7, 13, 12, 13, 11, 12, 8, 8, 11, 14, 8, 6, 14, 4, 13, 3, 9, 7, 14, 2, 6, 4, 12, 8, 8, 12, 6, 14, 13, 12, 14, 13, 10, 13, 13.

The reference or target voltage value is 10 V.

6.2 Simple plot of results

In order to gain a better understanding of the underlying behaviour of the process, by determining patterns and trends, a standard approach would be simply to plot these values in their natural order as shown in Figure 1a).

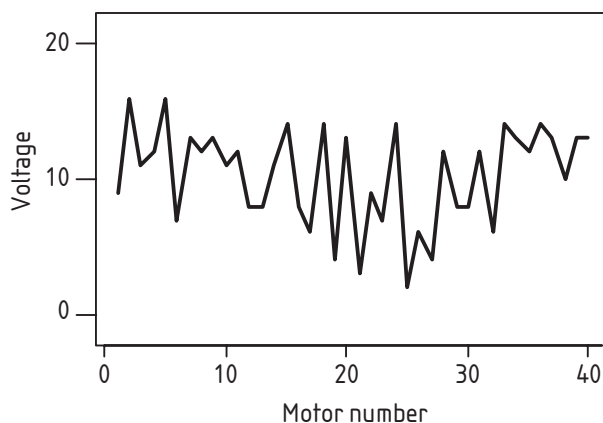


Figure 1a) — Simple plot of motor voltages

Apart from indicating a general drop away in the middle portion from a high start and with an equally high finish, Figure 1a) is not very revealing because of the extremely noisy, or spiky, data throughout.

6.3 Standard control chart for individuals

The next level of sophistication would probably be to establish a standard control chart for individuals as in Figure 1b).

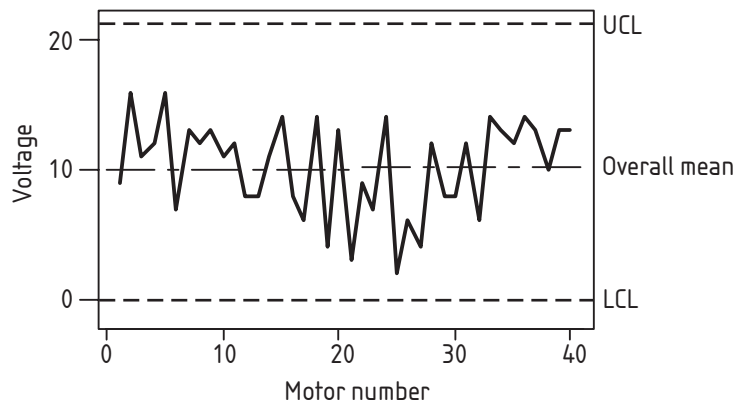


Figure 1b) — Standard control chart for individuals

Figure 1b) is even less revealing than the previous figure. It is, in fact, quite misleading. The standard statistical process control criteria, to test for process stability and control are:

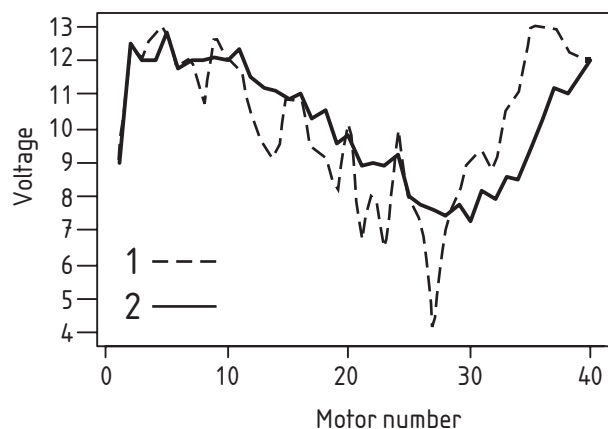
- a) no points lying above the upper control limit (UCL) or below the lower control limit (LCL);
- b) no runs of seven or more points upwards or downwards;
- c) no runs of seven points above or below the centreline.

The answer to all these criteria is no. Hence one would be led to the conclusion that this is a stable process, one that is “in control” around its overall average value of about 10 V, which is the target value. Further standard analysis would reveal that although the process is stable, it is not capable of meeting specification requirements. However, this analysis would not in itself provide any further clues as to why it is incapable of meeting the requirements.

The reason for the inability of the standard control chart for individuals to be of value here is that the control limits are based on actual process performance and not on desired or specified requirements. Consequently if the process naturally exhibits a large variation the control limits are correspondingly wide. What is required is a method that is better at indicating patterns and trends, or even pinpointing points of change, in order to help determine and remove primary sources of variation.

6.4 Standard moving average graphs

Recognizing the noisiness of the data, one could be led into constructing a moving average plot to smooth out the spikes. This is the usual reaction to this type of situation. A difficulty with this course of action is in choosing an appropriate length of moving average. This is always a problem with using fixed interval moving average plots. Moving averages of 3 and 10, for instance, are superimposed on the original plot in Figure 1c).



Key

- 1 --- Moving average = 3.
- 2 — Moving average = 10.

Figure 1c) — Plot of moving averages of 3 and 10

A comparison of the moving average plots in Figure 1c) indicates that, while both go some way to smooth the data, they differ somewhat in the fairly limited information on patterns and trends that they convey. This is always a problem with using fixed interval moving average plots.

6.5 Cusum chart: overall perspective

Another option here, the one recommended, would be to plot a cusum chart. A decided advantage over the “predetermined fixed interval moving average” approach is that “flexible length moving averages” can be tailored to the actual data after preliminary analysis of the cusum plot. How this is achieved is shown in 6.7.

Figure 1d) illustrates the cusum plot of the same data.

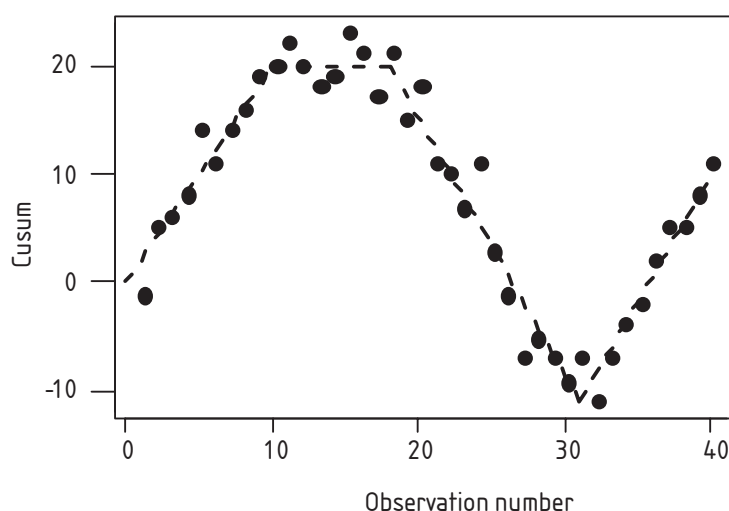


Figure 1d) — Cusum chart

It was not immediately apparent from the previous charts where, or whether, any significant changes in process level occurred; whereas the cusum chart indicates a well defined pattern. The best fitting (by eye) dashed lines drawn through the plotted points indicate three changes in process level, at their intersections, corresponding to the 10th, 18th and 31st motor.