

Guide to data analysis, quality control and improvement using cusum techniques —

Part 2: Introduction to decision-making using cusum techniques

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- Clay Pipe Development Association (CPDA)
- Federation of Small Businesses (FSB)
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Foreword

This part of BS 5703 was prepared by Technical Committee SS/4. It supersedes BS 5703-2:1980, which is withdrawn.

It was deemed necessary to revise BS 5703 for the following reasons:

- to present the text from the ordinary users view point rather than that of the statistician;
- to incorporate modern cusum computer software routines;
- to reduce the quantity of text and eliminate repetition;
- to improve the diagrams;
- to bring statistical symbols into line with standard practice.

BS 5703 is published in four parts:

- *Part 1: Uses and value of cusum charts in business, industry, commerce and public service;*
- *Part 2: Introduction to decision-making using cusum techniques;*
- *Part 3: Cusum methods for process/quality control using measured data;*
- *Part 4: Cusum methods for discrete (count/attribute) data.*

This publication does not purport to include all the necessary provisions of a contract. Users are responsible for its correct application.

Compliance with a British Standard does not of itself confer immunity from legal obligations.

Summary of pages

This document comprises a front cover, an inside front cover, pages i to iv, pages 1 to 33 and a back cover.

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Introduction

BS 5703-1 demonstrated the versatility and usefulness of a very simple, yet powerful, pictorial method of interpreting data arranged in any meaningful sequence.

These data can range from overall business figures such as turnover, profit or overheads to detailed operational data such as stock-outs and absenteeism to the control of individual process parameters and product characteristics. The data can either be expressed sequentially as individual values on a continuous scale (e.g. 24.60, 31.21, 18.97...); on a binomial scale in yes/no, good/bad, success/failure format; or as summary measures (e.g. mean, range, counts of events).

The method has a rather unusual name, cumulative sum, or, in short, “cusum”. This name relates to the process of subtracting a predetermined value, e.g. a target, preferred or reference value, from each figure in a sequence and progressively cumulating (i.e. adding) the differences. The graph of the series of cumulative differences is known as a cusum chart. Such a simple arithmetical process has a remarkable effect on the visual interpretation of the data.

The cusum method is already used unwittingly by golfers throughout the world. By scoring a round as plus 4, or perhaps, even, minus 2, golfers are using the cusum method in a numerical sense. They subtract the par value from their actual score for each hole and add (cumulate) the resulting differences. This is the cusum method in action. However it remains largely unknown and hence is a grossly underused tool throughout business, industry, commerce and public service. This is probably due to cusum methods generally being presented in statistical language rather than in the language of the workplace.

BS 5703-2 extends the cusum technique beyond that of BS 5703-1, which was concerned principally with using cusum charting as an effective means of visual presentation of data set out in any logical sequence. BS 5703-2 deals with quantifying and minimizing the risks involved in decision-making arising from the use of the cusum method. This leads to better and more consistent decision-making under conditions of uncertainty.

Of necessity some basic statistical concepts, terminology and symbols are introduced. However, these are kept to a bare minimum. Throughout, the language chosen, as with BS 5703-1, is that of the workplace rather than that of the statistician. The aim is to make the standard readily comprehensible to the very extensive range of prospective users and so facilitate widespread communication and understanding of the method.

It is advisable that those who are not familiar with the cusum technique absorb BS 5703-1 before reading BS 5703-2.

1 Scope

BS 5703-2 describes general-purpose methods of decision-making using cumulative sum (cusum) techniques for monitoring, control and retrospective analysis.

2 Normative references

The following referenced documents are indispensable for the application of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

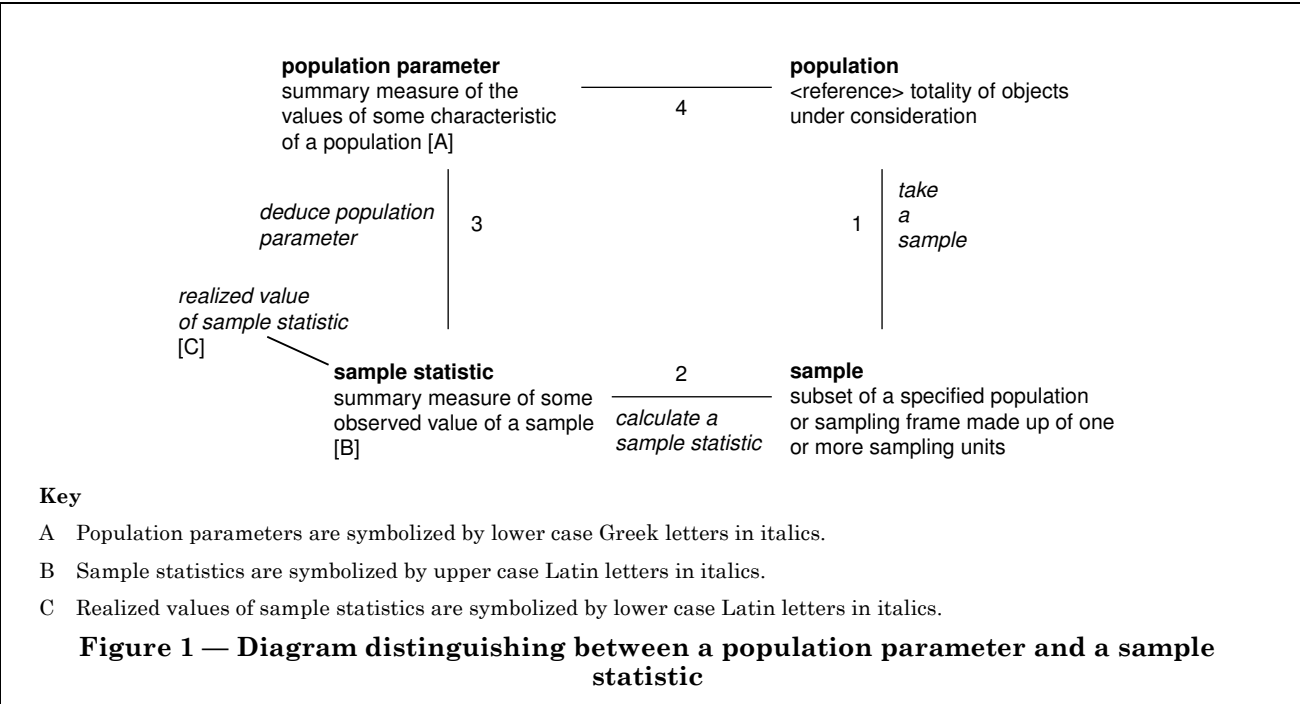
- BS ISO 3534-1, *Statistics — Vocabulary and symbols — Part 1: Probability and general statistical terms.*
- BS ISO 3534-2, *Statistics — Vocabulary and symbols — Part 2: Statistical quality control.*
- BS EN ISO 9000:2000, *Quality management systems — Fundamentals and vocabulary.*

3 Terms, definitions, abbreviations and symbols

3.1 General

For the purposes of this part of BS 5703, the terms, definitions, abbreviations and symbols given in BS ISO 3534-1, BS ISO 3534-2, BS EN ISO 9000:2000, Clause 3 and those in 3.2 apply.

NOTE While the rigour of the statistical treatment in this standard is kept to a minimum it is essential that there is a clear understanding of the distinction between what is a “population parameter” and what is a “sample statistic”, together with the associated notation. This is brought out in Figure 1.



- As an illustration of the application of this diagram and the distinguishing between “A”, “B” and “C”:
- a) the lower case Greek letter sigma, σ , a population parameter, represents the standard deviation of a population;
 - b) S , a sample statistic, represents the sample estimate of the standard deviation;
 - c) s represents the realized, or measured value, of the sample statistic, S .

Similarly:

- 1) the lower case Greek letter mu, μ , a population parameter, represents the arithmetic mean of the population;
- 2) $\bar{X}$, a sample statistic, represents the sample estimate of the arithmetic mean;
- 3) $\bar{x}$ represents the realized, or measured, value of the sample statistic $\bar{X}$.

Another standard deviation that is used to set up controls on cusum charts is the standard deviation of the variable plotted. It is frequently termed the standard error and is denoted by the symbol, σ_e . The standard error of the mean is the most familiar example.

A distinction is made between cusum design parameters relating to an observed variable as opposed to a standardized variable, e.g., one having a standard normal distribution, $\mu = 0$ and $\sigma = 1$. The upper case forms, H and F (see 3.2.3 and 3.2.4), relate to an observed variable and the lower case forms, h and f (see 3.2.3 and 3.2.4), to a standardized variable. For example, $H = h\sigma_e$.

3.2 Additional terms, definitions, abbreviations and symbols

3.2.1

target value

T

value for which departure in average level requires to be detected

NOTE 1 With a charted cusum deviations from the reference value are cumulated.

NOTE 2 The target value is often referred to as the reference value or the nominal control value. If so it should be acknowledged that it is not necessarily the most desirable or preferred value. It is simply a convenient target value for constructing a cusum chart.

3.2.2

datum value

<tabulated cusum> value from which differences are calculated

NOTE Upper datum value = $T + f\sigma_e$, for monitoring upward shift. Lower datum value = $T - f\sigma_e$, for monitoring downward shift.

3.2.3

reference shift

F, f

<tabulated cusum> difference between the target value (3.2.1) and datum value (3.2.2)

NOTE It is necessary to distinguish between f that relates to a standardized variable and F to an observed variable. $F = f\sigma_e$.

3.2.4

reference shift

F, f

<truncated V-mask> slope of the arm of the mask (tangent of the mask angle)

NOTE It is necessary to distinguish between f that relates to a standardized variable and F to an observed variable. $F = f\sigma_e$.

3.2.5

decision interval

H, h

<tabulated cusum> cumulative sum of deviations from a datum value (3.2.2) required to yield a signal

NOTE It is necessary to distinguish between h that relates to a standardized variable and H to an observed variable. $H = h\sigma_e$.

3.2.6

decision interval

H, h

<truncated V-mask> half-height at the datum of the mask

NOTE It is necessary to distinguish between h that relates to a standardized variable and H to an observed variable. $H = h\sigma_e$.

3.2.7

average run length

ARL

average number of samples taken up to the point at which a signal occurs

NOTE ARL is normally related to a particular process level in which case it carries an appropriate subscript, as, for example, $ARL_{in-control}$ and $ARL_{out-of-control}$, $ARL_{0.1}$ and $ARL_{0.3}$, $ARL_{acceptable}$ and $ARL_{rejectable}$.

4 Review of fundamentals

4.1 Principal features of cumulative sum (cusum) charts

A cusum chart is essentially a running total of deviations from some pre-selected target value. The mean of any group of consecutive values is indicated visually by the current slope of the graph. The principal features of a cusum chart are the following.

- a) It is sensitive in detecting changes in the mean.
- b) Any change in the mean, and the extent of the change, is indicated visually by a change in slope of the graph:
 - 1) horizontal graph indicates an on the reference or target value;
 - 2) downwards slope indicates a mean less than the reference or target value: the steeper the slope the bigger the difference;
 - 3) upwards slope indicates a mean more than the reference or target value: the steeper the slope the bigger the difference.
- c) It can be used retrospectively for investigative purposes, on an ongoing basis for monitoring and control, and for prediction of performance in the immediate future.

4.2 Basic steps in the construction of cusum charts

The steps in setting up a cusum chart for individual values are:

Step 1: Choose a *reference*, target, control or preferred *value*, RV. The average of past results will give the best discrimination.

Step 2: Tabulate the results in a meaningful (e.g. chronological) sequence. Subtract the target value from each result.

Step 3: Progressively sum the values obtained in Step 2. These sums are then plotted as a cusum chart.

Step 4: To obtain the best visual effect set up a horizontal scale no wider than about 2.5 mm between plotting points.

Step 5: For reasonable discrimination, without undue sensitivity, the following options are recommended.

- a) Choose: a vertical scale relative to the horizontal scale = (short term spread of results)/3. Round off as appropriate.
- b) Alternatively, where it is required to detect a known change, say C , choose a vertical scale such that:

$\left(\frac{\text{length of one unit on horizontal scale}}{\text{length of one unit on vertical scale}} \right)$ is between C and $2C$. Round off as appropriate.

4.3 Visual interpretation of a cusum chart

4.3.1 Introduction

In BS 5703-1 of this standard the focus was on visual interpretation of data using the cusum technique. Quite often this approach, in itself, is quite adequate. However, in all applications requiring decisions to be made in the presence of variation in the observed data, risks of erroneous decisions occur. Two types of error can occur:

- a) incorrect decision that a real change has occurred when it has not. This reflects “over-control” in a process-monitoring situation when action is taken when it should not be. In statistics it is termed a “type I error”. In acceptance sampling it is associated with the “producer’s risk¹⁾” and corresponds to the erroneous rejection of a satisfactory batch;
- b) incorrect decision that a change has not occurred when indeed it has. This reflects “under-control” in a process-monitoring situation when action is not taken when it should be. In statistics it is termed a “type II error”. In acceptance sampling it is associated with the “consumer’s risk²⁾” and corresponds to the erroneous acceptance of an unsatisfactory batch.

Before applying decision rules to cusums the decision has to be made as to whether they are necessary or not. The power of the purely visual approach is indicated by example.

4.3.2 Example

Figure 2a) shows a simple plot of motor voltages in production sequence. Due to the noisiness of the data it is not intuitively obvious, from the plot, whether or not any significant changes in mean voltage occurred.

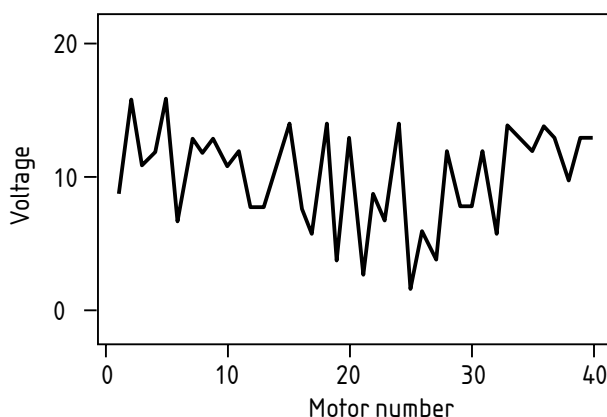


Figure 2a) — Simple plot of individual motor voltages

A minimum number of straight lines drawn through the cusum plot of the same data shown in Figure 2b) clearly indicates three changes in mean, at motors 11, 19 and 32. The reference, or target, voltage is 10.

¹⁾ Producer’s risk is the probability of a type I error.

²⁾ Consumer’s risk is the probability of a type II error.

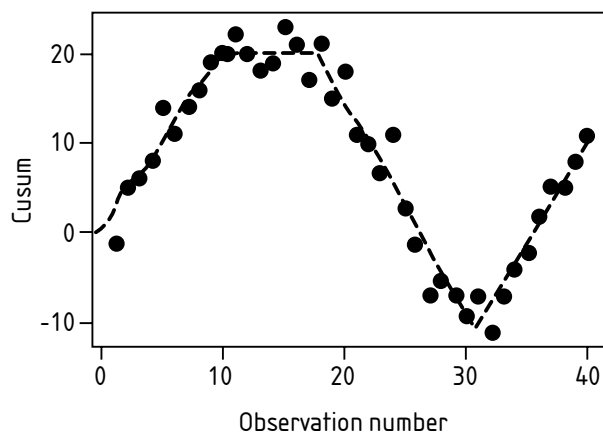


Figure 2b) — Cusum plot of motor voltages

This cusum plot is further enhanced by the Manhattan diagram of Figure 2c). This shows a noiseless plot of estimated mean voltages over the 40 motors.

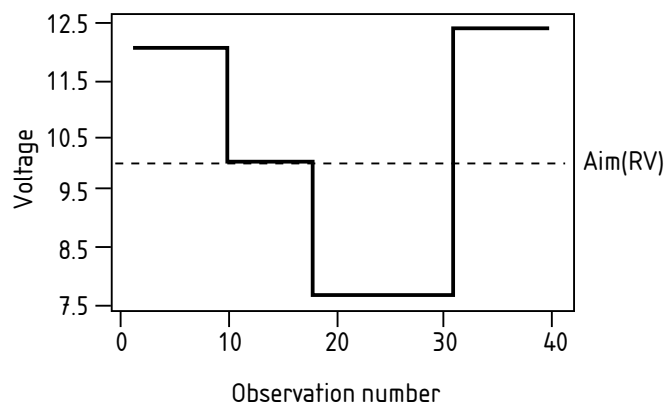


Figure 2c) — Manhattan diagram of motor voltages

A more detailed treatment of this and similar examples are given in BS 5703-1.

5 Fundamentals of cusum-based decision-making

5.1 Need for decision rules

Decision rules might be needed to rationalize the interpretation of a cusum chart. When an appropriate decision rule so indicates, some action is taken, depending on the nature of the process. Typical actions are:

- a) in-process control: adjustment of process conditions;
- b) in an improvement context: investigation of the underlying cause of the change;
- c) in a forecasting mode: analysis of and, if necessary, adjustment to the forecasting model or its parameters.