

Guide to data analysis, quality control and improvement using cusum techniques —

Part 3: Cusum methods for process/quality control using measured data

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- Clay Pipe Development Association (CPDA)
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Contents

	Page
Committees responsible	Inside front cover
Foreword	ii
Introduction	1
1 Scope	1
2 Normative references	1
3 Terms and definitions	1
4 Symbols (and abbreviated terms)	1
5 Cusum methods for process/quality control	2
6 Cusum schemes for monitoring mean values	5
7 Cusum schemes for monitoring variation	16
8 Special situations	25
Annex A (informative) Examples	28
Bibliography	35
Figure 1 — Flowchart for setting up a cusum scheme to monitor the mean value	5
Figure 2 — Example cusum graph paper	10
Figure 3 — Cusum mask	11
Figure 4 — An example of an out-of-control cusum plot	12
Figure 5 — Schematic example of a look-up graph for the shift from target	14
Figure 6 — Intercept chart for mask parameters (assuming a normal variable)	15
Figure 7 — Flowchart for setting up a cusum scheme to monitor the range	17
Figure A.1 — Cusum plot and mask for mean	29
Figure A.2 — Cusum plot and mask for range	30
Figure A.3 — Cusum plot and mask for mean	32
Figure A.4 — Cusum plot and mask for range	33
Table 1 — Standard cusum schemes for subgroup means	6
Table 2 — Comparison of performance of standard cusum schemes for subgroup means	7
Table 3 — d_2 factor for estimating the within-subgroup standard deviation from within-subgroup range	8
Table 4 — Table for cusum calculations	9
Table 5 — Standard cusum schemes for subgroup ranges	18
Table 6 — Comparison of performance (ARL) of standard cusum schemes for subgroup ranges	19
Table 7 — Cusum graph paper for range scale coefficients	20
Table 8 — Standard cusum schemes for subgroup standard deviations	22
Table 9 — Comparison of performance (ARL) of standard cusum schemes for subgroup standard deviations	23
Table 10 — c_4 factor for estimating the within-subgroup standard deviation	24
Table 11 — Cusum graph paper scale coefficients for within-subgroup standard deviation cusum	25
Table 12 — Critical intervals for the correlation coefficient	26
Table A.1 — Table for cusum calculations	28
Table A.2 — Table for cusum calculations (for subgroup)	31

Foreword

This part of BS 5703 was prepared by Technical Committee SS/4. It supersedes BS 5703-3:1981, which is withdrawn.

It was deemed necessary to revise BS 5703 for the following reasons:

- to present the text from the ordinary users view point rather than that of the statistician;
- to incorporate modern cusum computer software routines;
- to reduce the quantity of text and eliminate repetition;
- to improve the diagrams;
- to bring statistical symbols into line with standard practice.

BS 5703 is published in four parts:

- *Part 1: Uses and value of cusum charts in business, industry, commerce and public service;*
- *Part 2: Introduction to decision-making using cusum techniques;*
- *Part 3: Cusum methods for process/quality control using measured data;*
- *Part 4: Cusum methods for discrete (count/attribute) data.*

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Summary of pages

This document comprises a front cover, an inside front cover, pages i and ii, pages 1 to 35 and a back cover.

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Introduction

The basis of cusum methods was established in BS 5703-1 and BS 5703-2. These described the method of calculation, the type of plot made of cusum and how to detect any significant change. This, part 3, of the standard describes how the cusum technique can be applied to monitoring a production process or a product characteristic.

Outlined are the necessary steps to set up a cusum scheme for measured data: how to select or set the target value, how to set up the cusum stationery, and what shape and size to make the cusum mask to achieve the desired level of performance. Cusum schemes for mean value, range and standard deviation are described as well as the types of data to be encountered.

The potential for the application of cusum methods is enormous, e.g. monitoring monthly sales figures and the value of shares in companies, controlling budgets where the data are often of a one-at-a-time nature and controlling industrial processes, and the products they produce, where samples can be taken at frequent intervals. The clauses given in this part of BS 5703 cover all of these circumstances.

1 Scope

BS 5703-3 provides procedures for setting up cusum schemes for process and quality control using measured data.

2 Normative references

The following referenced documents are indispensable for the application of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

BS ISO 3534-1, *Statistics — Vocabulary and symbols — Part 1: Probability and general statistical terms*.

BS ISO 3534-2, *Statistics — Vocabulary and symbols — Part 2: Statistical quality control*.

BS EN ISO 9000:2000, *Quality management systems — Fundamentals and vocabulary*.

3 Terms and definitions

For the purposes of this part of BS 5703, the terms and definitions given in BS ISO 3534-1, BS ISO 3534-2 and BS EN ISO 9000:2000, Clause 3 apply.

4 Symbols (and abbreviated terms)

a	scale factor;
C	cusum value;
c_4	factor for estimating the within-subgroup standard deviation;
d_2	factor for estimating the within-subgroup standard deviation from within-subgroup range;
F	calculated value of the slope of the mask decision line;
f	slope of the mask decision line;
H	calculated value of the decision interval;
h	decision interval or half mask height;
k	number of subgroups;
L_0	average run length when the process is at target level;
L_δ	average run length when the process average has shifted from the target by δ ;
n	the number of individual items comprising a subgroup (sample);
$\bar{R}$	mean range (of a subgroup);
s	sample standard deviation;
$\bar{s}$	mean sample standard deviation;
$s_{\bar{x}}$	standard deviation of subgroup means;

T	target value;
X	random variable;
x	realized value of a random variable, an observation;
$\bar{x}$	arithmetic mean value (of a subgroup);
$\bar{\bar{x}}$	mean of subgroup means;
Δ	standardized shift in the process average from target;
δ	shift in the process average from target;
σ	population parameter representing the standard deviation of a population;
σ_e	standard error, or standard deviation, of the variable of interest.

5 Cusum methods for process/quality control

5.1 Definition of a process

A process is a set of interrelated or interacting activities that transforms inputs into outputs. Processes include everything we do and can be large or small, simple or complex. Processes have customers and suppliers, internal and external, and extend across organizational and functional boundaries. They make up a network that is the “Company”.

5.2 Process parameters

A process parameter is an element of the process that is required to perform the transformation of process inputs into outputs. An example of this is the heat applied to a mould used in a plastic injection moulding process. If the temperature of the mould is wrong the moulded components could be dimensionally incorrect or they could have undesirable visual defects. Another example is the pressure applied when forming a ceramic to give a certain shape before firing in a kiln. For non-manufacturing processes such as invoicing, the process parameters might be the time allowed for each task or the standard of the data that are to be used in invoicing.

The correct identification of the key process parameter(s) that have the greatest influence over the process outputs is an essential step in performing process control. Cusums can be used to assist in managing the process parameter(s). If a target value for the parameter is specified, periodic observations can be compared to the target and in this way a cusum can be calculated, then plotted and then interpreted for control. If any departure from target takes place, the cusum will detect and identify it and so allow the operational personnel to take the appropriate corrective actions.

5.3 Product characteristics

A product characteristic is a feature that will be found, or is associated with, the product. It is often a dimension found on the product such as a length or a diameter. The product characteristics are the results of the process through which the product has come and when charting is applied to these measures it is usually grouped under the heading of Statistical Quality Control (SQC) as the data is always “after-the-event”.

Products will possess numerous such characteristics and although some might wish to chart and control all of them this will usually be far too onerous a task. Therefore only those characteristics that are of greatest importance should be selected. The criteria often used to make such a selection are whether the characteristic is linked to some safety aspect of the product or whether the characteristic influences the function that the product should perform. The cusum technique is well suited for the ongoing monitoring of such characteristics.

5.4 Nature of changes to be detected

5.4.1 *Size of change to be detected*

BS 5703-2 contains a discussion on how the cusum method works and how the decision methods operate. When designing a cusum system to monitor either a process parameter or a product characteristic, consideration should be given to the size of shift or change within the parameter or characteristic that it is important to detect. This decision will influence the shape of any “V-mask” that might be used to observe any out-of-control signals. When controlling a parameter or a characteristic, many practitioners take this as the smallest shift for which the process can be corrected. There is little point in seeking a shift smaller than this for the effect on the cusum plot is likely to create the phenomenon of “hunting” (see 5.4.5).

Changes that occur can be classified as “step”, “drift” or “cyclic”.

5.4.2 *Step changes*

A step change is one where the data from measurements made on a process parameter or product characteristic suddenly jump or “step” to a new level. An example of this is where a new batch of a raw material is used that differs in some way from that previously used, or where an inexperienced clerk takes over an administrative task and, until the person properly learns the tasks required, makes more errors than an experienced person. A cusum chart will identify this change by showing a significant gradient.

5.4.3 *Drifting*

This type of change is often associated with wear patterns of equipment or tooling but can happen where, in the human case, standards alter over time, e.g. inspection standards. The pattern would be detected by the cusum plot and depicted as an increasing (or decreasing) gradient.

5.4.4 *Cyclic behaviour*

A pattern that changes over time and repeats itself as a pattern is named a cyclic change. For example, it could happen in a factory where there are three work shifts and all three workers perform differently the same tasks. Since there is a given sequence of the shifts, e.g. Shift B always follows Shift A, a cyclic pattern will emerge. The cusum plots show this pattern as periods where the gradient goes in one direction followed by another where it changes back again, etc.

5.4.5 *Hunting*

Hunting will occur when the parameter or characteristic cannot be exactly adjusted to the desired target value and following an out-of-control signal the adjustment made takes the location of the parameter or characteristic to the other side of the target. The cusum plot then develops a gradient in the opposite direction and eventually the signal is received to reverse the adjustment which had previously been done. In this way a “zigzag” pattern will be detected on the cusum plot. Clearly this would be a most unsatisfactory situation and should be avoided by careful selection of the original “target value” and the subsequent minimum adjustment to be sought.

5.5 Selecting target values

5.5.1 *General*

The correct selection of the target value is of prime importance in the setting up of a cusum scheme. A target value that is in between two possible values will create “hunting” as described in 5.4.5.

5.5.2 *Standard (given) value*

The simplest target value to assign is a “given” or “pre-assigned” value. When this option is selected, the target value is often set equal to some specification value such as a nominal or mid-tolerance value. These are found on specification documents or drawings when the application is based around engineering. If the cusum application is non-manufacturing, the given target might be some performance level such as the expected time taken to process an invoice or the budgeted monthly expenditure for a department within a company.

It is possible for the target itself to vary. For example, if the sales of ice-cream were to be monitored by a cusum chart on a monthly basis, the target value to be used each month is likely to be different according to the time of year. It may be anticipated that more ice-cream will be sold during the summer months than the winter months and so a different target may be used for each month. Failure to recognize the sales pattern and instead to use a constant value per month would lead to a misleading plot on the cusum graph paper. The cusum value is likely to rise during one period of the year and then fall in another. If the target were varied the cusum would be better equipped to indicate whether there was any significant change in the level of sales of ice-cream with the “seasonality” removed.

Inappropriate target values can result in the phenomenon “hunting” described in 5.4.5 and careful consideration should therefore be exercised when choosing a target of this sort.

The norm is for a given value approach to be used when monitoring the mean or average level (location) for the parameter or characteristic in question. Although the same approach could be used to set the target value for monitoring the variability (dispersion) such as a subgroup standard deviation or range, this practice is not recommended within this standard. It is preferred to proceed using the guidance contained within 5.5.3 and later clauses.

5.5.3 *Performance-based value*

The target value can be set from current performance levels. This approach is compatible with performance-based control charting where the controls are set according to the recent historical performance of a process parameter or product characteristic.

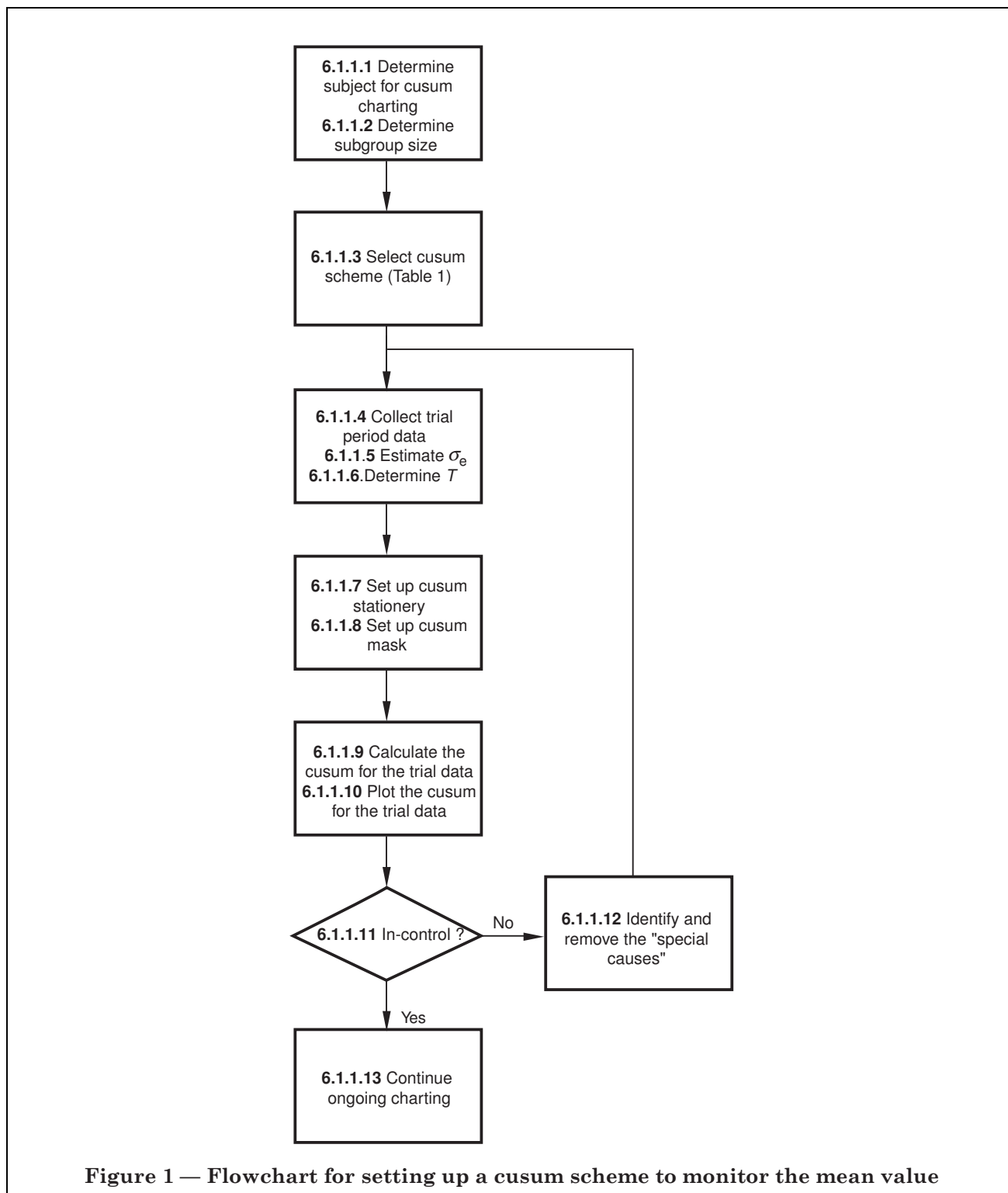
For monitoring the location or dispersion it is essential to capture data in a “trial” or “data collection” phase. Such a period should be long enough for the inherent variability to be fully observed and this will be a matter of judgement. Typically the trial should be long enough to provide for 25 points on the cusum plot. From this data estimates should be made of the mean value and the standard deviation.

Once determined, these target values should be used for the calculation of the cusum(s) but might require alteration at some later time if the cusum indicates a change in level. If it is not possible to make any process adjustment following such a change, or if the new level is acceptable, the only action that may be taken is to modify the target value. This is usually done after evaluating what the new level is from the most recent data and making this the new target. Thereafter the cusum will monitor the parameter or characteristic with reference to its new target value.

6 Cusum schemes for monitoring mean values

6.1 Standard schemes

6.1.1 Protocol for setting up a scheme: the following steps should be completed



6.1.1.1 Determine the subject for cusum charting

Determine the process parameter or product characteristic to be monitored.

NOTE This might be an instruction from a customer or a key process parameter or a significant product characteristic. The subject might also be identified during a problem solving exercise.

6.1.1.2 Determine the subgroup size

Subgroups of samples should be taken to provide the data for the cusum. A decision is required about how many samples will be in each subgroup.

If a process parameter is the chosen cusum subject, the most appropriate subgroup size is usually one. This is because the parameters, e.g. the temperature of a solution or the pressure in a vessel, are not likely to vary over a short time. Taking several consecutive repeat measures one after each other is unlikely to show any difference in the measurements. This would lead to technical problems when determining the standard deviation and the correct set up of the cusum mask.

If the data are genuinely one-at-a-time, such as the sales value for a particular month, then the rational subgroup size will be one.

When a product characteristic has been chosen, the rational subgroup size is often greater than one and typically five. Common sense should be exercised here, e.g. a four-cavity mould for an injection moulding operation would more logically have four samples (one from each cavity) as the basis for the subgroup size. The subgroup size is so selected to represent the random variation in the process.

6.1.1.3 Select cusum scheme

Table 1 indicates a set of standard schemes that provide for a range of typical requirements for a cusum scheme. The table provides two basic schemes, one that gives rather long average run lengths (ARL) at zero shift, i.e. a CS1 scheme, and another which has shorter ARL, i.e. a CS2 scheme. In other words, a CS2 scheme will give a few more “false signals” than a CS1 scheme. However, the CS2 scheme will detect the important shift a little quicker than the corresponding CS1 scheme. Whoever is responsible for the selection of the scheme must determine which scenario is the more important and then select the appropriate scheme. Table 2 illustrates the differences in performance of these standard schemes.

Table 1 — Standard cusum schemes for subgroup means

Important shift in the mean to be detected ^a	CS1 schemes		CS2 schemes	
	<i>h</i>	<i>f</i>	<i>h</i>	<i>f</i>
i) $< 0.75\sigma_e$	8.0	0.25	5.0	0.25
ii) $0.75 \text{ to } 1.50\sigma_e$	5.0	0.50	3.5	0.50
iii) $> 1.50\sigma_e$	2.5	1.00	1.8	1.00
NOTE 1 CS1 schemes give average run lengths, L_0 , in the region of 700 to 1 000 when the actual shift is zero.				
NOTE 2 CS2 schemes give average run lengths, L_0 , in the region of 140 to 200 when the actual shift is zero.				
^a For individual results (subgroup size = 1) σ_e represents the standard deviation. When the subgroup size is more than one σ_e represents the standard error of the mean.				

Once the decision has been made concerning which of CS1 or CS2 to select, the next decision is about the size of the important shift. Three typical levels of shift are provided for in the table. Depending on this selection the values for *h* and *f* can be read from the table.

If it is unclear which scheme should be selected, custom and practice indicates that a good starting scheme is to select scheme CS1 ii), i.e. *h* = 5.0 and *f* = 0.50.

BS 5703-2 describes what *h* and *f* represent. In summary, they are the parameters of a cusum mask where *h* represents the “half-mask height” or “decision interval” and *f* represents the slope of the “mask decision line”.

Table 2 — Comparison of performance of standard cusum schemes for subgroup means

Values are average run lengths (ARL)

Shift in the mean from target value (in units of σ_e) ^a	CS1 schemes			CS2 schemes		
	(i)	(ii)	(iii)	(i)	(ii)	(iii)
0	730	930	715	140	200	170
0.75	16.4	17.0	27.0	10.5	11.5	15.0
1.00	11.4	10.5	13.4	7.4	7.4	8.8
1.50	7.1	5.8	5.4	4.7	4.3	4.0
NOTE The values given are ARL. The reader should be aware that the actual run length taken to detect an actual shift will vary and might be shorter than or longer than the ARL. When it is of particular interest, the reader should examine the distribution of run lengths for particular shifts from target to know the expected range of run lengths that might be experienced.						
^a For individual results (subgroup size = 1) σ_e represents the standard deviation. When the subgroup size is more than one σ_e represents the standard error of the mean.						

Whatever scheme is selected, the values for these parameters should be multiplied by the estimated variability (or σ_e) to determine the actual size and shape of the mask. This is described in **6.1.1.8**.

6.1.1.4 *Collect trial period data*

As indicated in **5.5.3** data should be captured that will describe the nature of the variability in the process, so that the cusum scheme can be properly “tuned”, and to assist in establishing the target value if necessary.

Determine a trial period during which all the sources of process variation will be observed. This should be long enough or the sampling frequency high enough to produce at least 25 subgroups of data.

Take care not to introduce extra sources of variation, e.g. process adjustments, during this period as this will distort the variation pattern. If there is an interruption of data collection, a decision should be made as to whether the trial period requires to be done again or whether sufficient data was generated during the shortened trial period. In general, if the number of subgroups gathered was 20 or more and if it is judged that all potential sources of variation were observed within the 20 subgroups, then this number of subgroups will be satisfactory and the trial period closed. The data from the trial should then be used to establish the levels of variability the cusum scheme will operate under. This is described in **6.1.1.5** and **6.1.1.6**.

6.1.1.5 *Estimate σ_e from the trial period data*

6.1.1.5.1 *General*

The following subclauses outline the method for estimating σ_e . Special circumstances might occur where a different approach might be required. This different approach might require σ_e to be evaluated by looking at the standard deviation between the subgroup means.

6.1.1.5.2 *Subgroup sizes more than one ($n > 1$)*

- Calculate the range (largest minus smallest observation) of each subgroup.
- Calculate the average range ($\bar{R}$) of all of the subgroup ranges.
- Estimate the within-subgroup standard deviation (σ_0) by dividing the average range by the appropriate d_2 value taken from Table 3.
- Estimate σ_e by dividing σ_0 by the square root of the subgroup size, i.e. $\sigma_e = \sigma_0 / \sqrt{n}$.

Table 3 — d_2 factor for estimating the within-subgroup standard deviation from within-subgroup range

Subgroup size, n^a	d_2
2	1.128
3	1.693
4	2.059
5	2.326
6	2.534
7	2.704
8	2.847
9	2.970
10	3.078

NOTE For subgroups greater than 10 other methods may be more efficient at estimating the within-subgroup standard deviation.

^a Values of d_2 exist for $n > 10$. See BS 5702-1 or other textbooks or standards.

6.1.1.5.3 Subgroup size is one ($n = 1$)

The approach taken to estimate σ_e is to use the method of successive difference.

The data gathered during the trial period should be put in the sequence it was collected. The range (difference) between the first and second results should be calculated, then the range between second and third, etc. If there are k subgroups there will be $k - 1$ ranges. Calculate the average of these ranges ($\bar{R}$).

The estimate of σ_e can then be found by dividing the average range by 1.128.

6.1.1.6 Determine the target value, T

6.1.1.6.1 General

As described in 5.5 the target value may be a given value or a performance-based value determined from data.

6.1.1.6.2 Given value

The value of the target is a specified value. It may come from a specification document or a drawing and may be a nominal size, in the case of a product characteristic, or some expected level of performance given by management in the case of a non-manufacturing process.

6.1.1.6.3 Performance-based value

Here the target value should be determined from the data obtained during the trial period.

- Calculate the mean value ($\bar{x}$) for each subgroup.
- Calculate the average ($\bar{\bar{x}}$) of these means.
- Assign $\bar{\bar{x}}$ as the target, T .

6.1.1.7 Set up the cusum stationery

6.1.1.7.1 General

BS 5703-1 and BS 5703-2 provide guidance on setting up cusum stationery.