

Reliability of systems, equipment and components

Part 17. Reliability growth: statistical test and estimation methods

Committees responsible for this British Standard

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 Association of Consulting Engineers
 Association of Project Managers
 British Gas plc
 British Railways Board
 British Telecommunications plc
 Chartered Institution of Building Services Engineers
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National foreword

This Part of BS 5760 has been prepared by Technical Committee DS/1. It is identical with IEC 1164 : 1995, *Reliability growth — Statistical test and estimation methods*, published by the International Electrotechnical Commission (IEC).

This Part describes the power law reliability growth model and related projection model. It provides procedures to estimate some or all of the quantities listed in clause 9 of BS 5760 : Part 6 : 1991.

BS 5760 provides comprehensive guidance on many aspects of reliability management. Sixteen Parts of this standard have been published and these may be summarized as follows:

Part 0. Introductory guide to reliability. This Part provides guidance for senior management and non-specialists on the importance of reliability and explains how measures to achieve reliability should be integrated with other aspects of project management.

Part 1. Guide to reliability and maintainability programme management. This Part discusses the essential features of a comprehensive reliability and maintainability programme to produce reliable and maintainable products.

Part 2. Guide to the assessment of reliability. This Part explains the purpose and problems of assessing reliability. A range of available assessment techniques is reviewed and the principle advantages and limitations are outlined. Sections are included on the assessment of software reliability, human reliability and one-shot devices. The extension of these techniques to the assessment of availability and to the assessment of reliability of services is considered. This use of a range of statistical distributions to analyse reliability data is described.

Part 3. Guide to reliability practices : examples. This Part contains authentic practical examples illustrating the principles established in BS 5760 : Parts 1 and 2.

Part 4. Guide to specification clauses relating to the achievement and development of reliability in new and existing items. This Part provides detailed guidance on the specification of reliability.

Part 5. Guide to failure modes, effects, and criticality analysis (FMEA and FMECA). This Part gives guidance on the application of these techniques.

Part 6. Guide to programmes for reliability growth. This Part describes procedures for exposing and removing weaknesses in hardware and software items in order to achieve acceptable reliability in a product. It explains basic concepts, management and test procedures and describes techniques for analysis and correction of failures.

Part 7. Guide to fault tree analysis. This Part gives guidance on the application of fault tree analysis. This technique may be used to identify factors affecting the reliability, performance and safety characteristics of a system.

Part 9. Guide to the block diagram technique. This Part describes the use of the block diagram technique for modelling and evaluating the reliability of both elementary and more complex systems. The extension of the method to calculate availability is also outlined.

Part 10. Guide to reliability of testing

Section 10.1 General requirements. This Section provides guidance on general principles, procedures and considerations for reliability testing in the laboratory and in the field.

Section 10.2 Design of test cycles. This Section provides a general procedure for design of test cycles, where no applicable preferred test cycles can be found.

Section 10.3 Compliance test procedures for standby-state availability. This Section provides guidance on techniques for testing the availability performance of frequently maintained items under defined conditions.

Section 10.5 Compliance test plans for success ratio. This Section gives procedures for preparing and applying compliance test plans for success ratio or failure ratio. Plans are provided for three main types of test.

NOTE 1. Further Sections of Part 10 based on other Parts of IEC 605 and related IEC standards are planned.

Part 11. Guide to the collection of reliability, maintainability, availability and maintenance support data from the field. This Part provides guidance for the collection of data relating to reliability, maintainability, availability and maintenance support performance of items operating in the field. It deals in general terms with the practical aspects of data collection and presentation and briefly explores the related topics of data analysis and presentation of results.

Part 12. Guide to the presentation of reliability, maintainability and availability predictions. This Part gives guidance on the presentation of quantitative predictions of reliability, maintainability and availability. The items to be included when presenting prediction information are listed and explained with the object of facilitating comparisons between projects and reports.

Part 13. Guide to preferred test conditions for equipment reliability testing.

Section 13.1 Indoor portable equipment : low degree of simulation

Section 13.2 Equipment for stationary use in weather protected locations : high degree of simulation

Section 13.3 Equipment for stationary use in partially weather protected locations : low degree of simulation

Section 13.4 Equipment for portable and non-stationary use : low degree of simulation

Part 14. Guide to formal design review. This Part provides guidelines for planning and conducting design reviews. Much of the guidance given is widely applicable and relevant to the general conduct of design reviews but it also includes details of the specific contributions to be made by various specialists.

Part 15. Guide to the application of Markov techniques. This Part provides guidance on the application of Markov techniques to dependability analysis. Further information, including advice on the application, benefits and limitations of Markov techniques, is given in BS 5760 : Part 2 : Section 11.

Further Parts of BS 5760 are envisaged in order to provide guidance on other techniques of reliability management (see note 4).

Whilst mainly addressing system and equipment level reliability, many of the techniques described in the Parts of BS 5760 may also be applied at the component level. Further guidance on component reliability is given in CECC 00804.

Guidance on specific aspects of maintainability is provided in the various Parts of BS 6548 *Maintainability of equipment*.

NOTE 2. This Part of BS 5760 makes reference to BS 4778 *Quality vocabulary*, and in particular to this Part of BS 4778 : Section 3.1, which contains definitions relating to concepts applicable to this Part of BS 5760. It is essential that these definitions and concepts should be fully understood if this guide is to be interpreted correctly. For this reason it is recommended that BS 4778 should be read in conjunction with BS 5760.

NOTE 3. BS 4778 : Section 3.2 deprecates the use of the terms 'failure modes and effects analysis' and 'failure modes, effects and criticality analysis'. It favours the use of 'fault modes and effects analysis' and 'fault modes, effects and criticality analysis', and these terms are used throughout BS 5760 : Part 2. However, the older terms have been retained in BS 5760 : Part 5 in order to align it with the current versions of IEC 812.

NOTE 4. *Part 8. Guide to the assessment of reliability of systems containing software* is currently in preparation and is available as DD 198 : 1991.

Cross-references

Publications referred to	Corresponding British Standard
IEC 50(191) : 1990	BS 4778 <i>Quality vocabulary</i> Part 3 <i>Availability, reliability and maintainability terms</i> Section 3.2 : 1991 <i>Glossary of international terms</i> (Identical)
IEC 605-1 : 1978	BS 5760 <i>Reliability of systems, equipment and components</i> Part 10 <i>Guide to reliability testing</i> Section 10.1 : 1993 <i>General requirements</i> (Identical)
IEC 1014 : 1989	BS 5760 <i>Reliability of systems, equipment and components</i> Part 6 : 1991 <i>Guide to programmes for reliability growth</i> (Identical)

Compliance with a British Standard does not of itself confer immunity from legal obligations.

INTRODUCTION

This International Standard describes the power law reliability growth model and related projection model and gives step-by-step directions for their use. There are several reliability growth models available, the power law model being one of the most widely used. This standard provides procedures to estimate some or all of the quantities listed in clause 9 of IEC 1014. An input is required consisting of a data set of accumulated test times at which relevant failures occurred, or were observed, for a single system, and the time of termination of the test, if different from the time of the final failure. It is assumed that the collection of data as input for the model begins after the completion of any preliminary tests, such as environmental stress screening, intended to stabilize the system's initial failure intensity.

Model parameters estimated from previous results may be used to plan and predict the course of future reliability growth programmes, if the conditions are similar.

Some of the procedures may require computer programs, but these are not unduly complex. This standard presents algorithms for which computer programs should be easy to construct.

RELIABILITY GROWTH – STATISTICAL TEST AND ESTIMATION METHODS

1 Scope

This International Standard gives models and numerical methods for reliability growth assessments based on failure data from a single system which were generated in a reliability improvement programme. These procedures deal with growth, estimation, confidence intervals for system reliability and goodness-of-fit tests.

2 Normative references

The following normative documents contain provisions which, through reference in this text, constitute provisions of this International Standard. At the time of publication, the editions indicated were valid. All normative documents are subject to revision, and parties to agreements based on this International Standard are encouraged to investigate the possibility of applying the most recent editions of the normative documents listed below. Members of IEC and ISO maintain registers of currently valid International Standards.

IEC 50(191): 1990, *International Electrotechnical Vocabulary (IEV) – Chapter 191: Dependability and quality of service*

IEC 605-1: 1978, *Equipment reliability testing – Part 1: General requirements*

IEC 605-4: 1986, *Equipment reliability testing – Part 4: Procedures for determining point estimates and confidence limits from equipment reliability determination tests*

IEC 605-6: 1986, *Equipment reliability testing – Part 6: Tests for the validity of a constant failure rate assumption*

IEC 1014: 1989, *Programmes for reliability growth*

3 Definitions

For the purposes of this standard the terms and definitions of IEC 50(191) and IEC 1014 apply, together with the following additional terms and definitions:

3.1 delayed modification: A corrective modification which is incorporated into the system at the end of a test.

NOTE – A delayed modification is not incorporated during the test.

3.2 improvement effectiveness factor: The fraction by which the intensity of a systematic failure is reduced by means of corrective modification.

3.3 type I test: A test which is terminated at a predetermined time or test with data available through a time which does not correspond to a failure.

NOTE – Type I test is sometimes called time terminated test.

3.4 type II test: A reliability growth test which is terminated upon the accumulation of a specified number of failures, or test with data available through a time which corresponds to a failure.

NOTE – Type II test is sometimes called failure terminated test.

4 Symbols

For the purpose of this international standard, the following symbols apply:

λ, β	scale and shape parameters for the power law model
CV	critical value for hypothesis test
d	number of intervals for grouped data analysis
$\bar{E}, E_i, E_j$	mean and individual improvement effectiveness factors
I	number of distinct types of category B failures observed
i, j	general purpose indices
K_A	number of category A failures
K_B	number of category B failures
K_i	number of i -th type category B failures observed; $K_B = \sum_{i=1}^k K_i$
M	parameter of the Cramér-von Mises test (statistical)
N	number of relevant failures
N_i	number of relevant failures in i -th interval
$N(T)$	accumulated number of failures up to test time T
$E[N(T)]$	expected accumulated number of failures up to test time T
$t(i-1); t(i)$	endpoints of i -th interval of test time for grouped data analysis
T	current accumulated relevant test time
T_i	accumulated relevant test time at the i -th failure
T_N	total accumulated relevant test times for type II test
T^*	total accumulated relevant test times for type I test
$\chi^2_{\gamma}(v)$	γ fractile of the χ^2 distribution with v degrees of freedom
z	general symbol for failure intensity
u_{γ}	γ fractile of the standard normal distribution
z_p	projected failure intensity
$z(T)$	current failure intensity at time T
$\theta(T)$	current instantaneous mean time between failures
θ_p	projected mean time between failures

5 The power law model

The statistical procedures for the power law reliability growth model use the original relevant failure and time data from the test. Except in the projection technique (see 7.6), the model is applied to the complete set of relevant failures (as in IEC 1014, figure 2, characteristic (3)) without subdivision into categories.

The basic equations for the power law model are given in this clause. Background information on the model is given in annex B.

The expected accumulated number of failures up to test time T is given by:

$$E[N(T)] = \lambda T^\beta, \text{ with } \lambda > 0, \beta > 0, T > 0$$

where

λ is the scale parameter

β is the shape parameter (a function of the general effectiveness of the improvements; $0 < \beta < 1$, corresponds to reliability growth; $\beta = 1$ corresponds to no reliability growth; $\beta > 1$ corresponds to negative reliability growth).

The current failure intensity after T h of testing is given by:

$$z(T) = \frac{d}{dT} E[N(T)] = \lambda \beta T^{\beta-1}, \text{ with } T > 0$$

Thus, parameters λ and β both affect the failure intensity achieved in a given time. The equation represents in effect the slope of a tangent to the $N(T)$ vs. T characteristic at time T as shown in IEC 1014, figure 6.

The current mean time between failures after T h of testing is given by:

$$\theta(T) = \frac{1}{z(T)}$$

Methods are given in 7.1 and 7.2 for maximum likelihood estimation of the parameters λ and β . Subclause 7.3 gives goodness-of-fit tests for the model, and 7.4 and 7.5 discuss confidence interval procedures. An extension of the model for reliability growth projections is given in 7.6.

The model has the following characteristic features:

- it is simple to evaluate;
- when the parameters have been estimated from past programmes it is a convenient tool for planning future programmes employing similar conditions of testing and equal improvement effectiveness (see clause 5, and IEC 1014, clause 6);
- it gives the unrealistic indications that $z(T) = \infty$ at $T = 0$ and that growth can be unending, that is $z(T)$ tends to zero as T tends to infinity; however, these limitations do not generally affect its practical use;
- it is relatively slow and insensitive in indicating growth immediately after a corrective modification, and so may give a low (that is, pessimistic) estimate of the final $\theta(T)$, unless projection is used (see 7.6);