

Specification for

Quantities, units and symbols —

Part 11: Mathematical signs and symbols for use in the physical sciences and technology

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- Department of Trade and Industry (National Physical Laboratory)
- Department of Trade and Industry (National Weights and Measures Laboratory)
- EEA (the Association of Electronics, Telecommunications and Business Equipment Industries)
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- Institute of Trading Standards Administration
- Institution of Chemical Engineers
- Institution of Electrical Engineers
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- Schools Mathematics Project
- Society of Chemical Industry

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National foreword

This Part of BS 5775 has been prepared under the direction of the Systems Department Steering Committee and is identical with ISO 31-11:1992 *Quantities and units — Part 11: Mathematical signs and symbols for use in the physical sciences and technology*, published by the International Organization for Standardization (ISO). ISO 31-11 was prepared by Technical Committee 12, Quantities, units, symbols, conversion factors, with the active participation and approval of the UK.

This edition of BS 5775-11 supersedes BS 5775-11:1979 which is withdrawn. The principal changes made in this revised version are as follows:

- a) a new clause on coordinate systems has been added;
- b) some new items have been added in the old clauses.

BS 5775 comprises the following Parts, each of which is identical with the corresponding Part of ISO 31.

- *Part 0: General principles;*
- *Part 1: Space and time;*
- *Part 2: Periodic and related phenomena;*
- *Part 3: Mechanics;*
- *Part 4: Heat;*
- *Part 5: Electricity and magnetism;*
- *Part 6: Light and related electromagnetic radiations;*
- *Part 7: Acoustics;*
- *Part 8: Physical chemistry and molecular physics;*
- *Part 9: Atomic and nuclear physics;*
- *Part 10: Nuclear reactions and ionizing radiations;*
- *Part 11: Mathematical signs and symbols for use in the physical sciences and technology;*
- *Part 12: Characteristic numbers;*
- *Part 13: Solid state physics.*

Cross-reference

International standard	Corresponding British Standard
ISO 31-0:1992	BS 5775 <i>Specification for quantities, units and symbols</i> Part 0:1993 <i>General principles</i> (Identical)

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Summary of pages

This document comprises a front cover, an inside front cover, pages i and ii, pages 1 to 26, an inside back cover and a back cover.

This standard has been updated (see copyright date) and may have had amendments incorporated. This will be indicated in the amendment table on the inside front cover.

Introduction

0.1 General

If more than one sign, symbol or expression is given for the same item, they are on an equal footing. Signs, symbols and expressions in the “Remarks” column are given for information.

Where the numbering of an item has been changed in the revision of a part of ISO 31, the number in the preceding edition is shown in parentheses below the new number for the item; a dash is used to indicate that the item in question did not appear in the preceding edition.

0.2 Variables, functions and operators

Variables, such as x , y , etc., and running numbers, such as i in $\Sigma_i x_i$ are printed in italic (sloping) type. Also parameters, such as a , b , etc., which may be considered as constant in a particular context, are printed in italic (sloping) type. The same applies to functions in general, e.g. f , g .

An explicitly defined function is, however, printed in Roman (upright) type, e.g. $\sin$, $\exp$, $\ln$, Γ .

Mathematical constants, the values of which never change, are printed in Roman (upright) type, e.g. $e = 2,718\ 281\ 8\dots$; $\pi = 3,141\ 592\ 6\dots$; $i^2 = -1$. Well defined operators are also printed in upright style, e.g. $\div$, δ in δx and each d in df/dx .

Numbers expressed in the form of digits are always printed upright, e.g. 351 204; 1,32; 7/8.

The argument of a function is written in parentheses after the symbol for the function, without a space between the symbol for the function and the first parenthesis, e.g. $f(x)$, $\cos(\omega t + \varphi)$. If the symbol for the function consists of two or more letters and the argument contains no operation sign, such as $+$; $-$; $\times$; $:$; or $/$, the parentheses around the argument may be omitted. In these cases, there should be a thin space between the symbol for the function and the argument, e.g. $\text{ent } 2,4$; $\sin n\pi$; $\text{arcosh } 2A$; $\text{Ei } x$.

If there is any risk of confusion, parentheses should always be inserted. For example, write $\cos(x) + y$ or $(\cos x) + y$; do not write $\cos x + y$, which could be mistaken for $\cos(x + y)$.

If an expression or equation must be split into two or more lines, the line-breaks should preferably be immediately after one of the signs $=$; $+$; $-$; $\pm$; or $"$; or, if necessary, immediately after one of the signs x ; $:$; or $/$. In this case, the sign works like a hyphen at the end of the first line, informing the reader that the rest will follow on the next line or even on the next page. The sign should not be repeated at the beginning of the following line; two minus signs could for example give rise to sign errors.

0.3 Scalars, vectors and tensors

Scalars, vectors and tensors are used to denote certain physical quantities. They are as such independent of the particular choice of coordinate system, whereas each component of a vector or a tensor depends on that choice.

It is important to distinguish between the “components of a vector” $\mathbf{a}$, i.e. a_x , a_y and a_z , and the “component vectors”, i.e. $a_x \mathbf{e}_x$, $a_y \mathbf{e}_y$ and $a_z \mathbf{e}_z$.

The cartesian components of the position vector are equal to the cartesian coordinates of the point given by the position vector.

Instead of treating each component as a physical quantity (i.e. numerical value $\times$ unit), the vector could be written as a numerical-value vector multiplied by the unit. All units are scalars.

EXAMPLE

$$\begin{array}{ccc}
 \text{component } F_x & & \text{numerical-value vector} \\
 \downarrow & & \downarrow \\
 \mathbf{F} = (3\ \text{N}, -2\ \text{N}, 5\ \text{N}) & = & (3, -2, 5)\ \text{N} \\
 \begin{array}{cc} \downarrow & \downarrow \\ \text{numerical value} & \text{unit} \end{array} & & \downarrow \\
 & & \text{unit}
 \end{array}$$

The same considerations apply to tensors of second and higher orders.

1 Scope

This part of ISO 31 gives general information about mathematical signs and symbols, their meanings, verbal equivalents and applications.

The recommendations in this part of ISO 31 are intended mainly for use in the physical sciences and technology.

2 Normative reference

The following standard contains provisions which, through reference in this text, constitute provisions of this part of ISO 31. At the time of publication, the edition indicated was valid. All standards are subject to revision, and parties to agreements based on this part of ISO 31 are encouraged to investigate the possibility of applying the most recent edition of the standard indicated below. Members of IEC and ISO maintain registers of currently valid International Standards.

ISO 31-0:1992, *Quantities and units — Part 0: General principles*.

3 MATHEMATICAL LOGIC				
Item No.	Symbol, sign	Application	Name of symbol	Meaning, verbal equivalent and remarks
11-3.1 (11-2.1)	$\wedge$	$p \wedge q$	conjunction sign	p and q
11-3.2 (11-2.2)	$\vee$	$p \vee q$	disjunction sign	p or q (or both)
11-3.3 (11-2.3)	$\neg$	$\neg p$	negation sign	negation of p ; not p ; non p
11-3.4 (11-2.4)	$\Rightarrow$	$p \Rightarrow q$	implication sign	if p then q ; p implies q Can also be written $q \Leftarrow p$. Sometimes $\rightarrow$ is used.
11-3.5 (11-2.5)	$\Leftrightarrow$	$p \Leftrightarrow q$	equivalence sign	$p \Rightarrow q$ and $q \Rightarrow p$; p is equivalent to q Sometimes $\leftrightarrow$ is used.
11-3.6 (11-2-6)	$\forall$	$\forall x \in A p(x)$ ($\forall x \in A$) $p(x)$	universal quantifier	for every x belonging to A , the proposition $p(x)$ is true If it is clear from the context which set A is being considered, the notation $\forall x p(x)$ can be used.
11-3.7 (11-2.7)	$\exists$	$\exists x \in A p(x)$ ($\exists x \in A$) $p(x)$	existential quantifier	For $x \in A$, see 11-4.1 there exists an x belonging to A for which $p(x)$ is true If it is clear from the context which set A is being considered, the notation $\exists x p(x)$ can be used. For $x \in A$, see 11-4.1. $\exists!$ or $\exists_1$ is used to indicate the existence of one only one and element for which $p(x)$ is true.

4 SETS				
Item No.	Symbol, sign	Application	Meaning, verbal equivalent	Remarks and examples
11-4.1 (11-1.1)	$\in$	$x \in A$	x belongs to A ; x is an element of the set A	The symbol $\notin$ is also used.
11-4.2 (11-1.2)	$\notin$	$y \notin A$	y does not belong to A ; y is not an element of the set A	
11-4.3 (11-1.3)	$\ni$	$A \ni x$	the set A contains x (as element)	$A \ni x$ has the same meaning as $x \in A$.
11-4.4 (11-1.4)	$\nexists$	$A \nexists y$	the set A does not contain y (as element)	$A \nexists y$ has the same meaning as $y \notin A$. The symbol $\nexists$ is also used.
11-4.5 (11-1.5)	$\{\}$	$\{x_1, x_2, \dots, x_n\}$	set with elements $x_1, x_2, \dots, x_n$	Also $\{x_i; i \in I\}$, where I denotes a set of indices.
11-4.6 (11-1.6)	$\{ \}$	$\{x \in A \mid p(x)\}$	set of those elements of A for which the proposition $p(x)$ is true	EXAMPLE $\{x \in \mathbb{R} \mid x \leq 5\}$ If it is clear from the context which set A is being considered, the notation $\{x \mid p(x)\}$ can be used. EXAMPLE $\{x \mid x \leq 5\}$
11-4.7 (—)	card	card (A)	number of elements in A ; cardinal of A	$\mathbb{N} = \{0, 1, 2, 3, \dots\}$ Exclusion of zero from the sets 11-4.9 to 11-4.13 is denoted by an asterisk, e.g. $\mathbb{N}^*$. $\mathbb{N}_k = \{0, 1, \dots, k-1\}$ $\mathbb{Z} = \{\dots, -2, -1, 0, 1, 2, \dots\}$ See remark to 11-4.9. See remark to 11-4.9. See remark to 11-4.9.
11-4.8 (11-1.7)	$\emptyset$		the empty set	
11-4.9 (11-1.8)	$\mathbb{N}$ N		the set of natural numbers; the set of positive integers and zero	
11-4.10 (11-1.9)	$\mathbb{Z}$ Z		the set of integers	
11-4.11 (11-1.10)	$\mathbb{Q}$ Q		the set of rational numbers	
11-4.12 (11-1.11)	$\mathbb{R}$ R		the set of the real numbers	See remark to 11-4.9.
11-4.13 (11-1.12)	$\mathbb{C}$ C		the set of complex numbers	See remark to 11-4.9.

4 SETS				
Item No.	Symbol, sign	Application	Meaning, verbal equivalent	Remarks and examples
11-4.14 (—)	[,]	$[a, b]$	close interval in $\mathbb{R}$ from a (included) to b (included)	$[a, b] = \{x \in \mathbb{R} \mid a \leq x \leq b\}$
11-4.15 (—)	] ,] (,]	$]a, b]$ $(a, b]$	left half-open interval, in $\mathbb{R}$ from a (excluded) to b (included)	$]a, b] = \{x \in \mathbb{R} \mid a < x \leq b\}$
11-4.16 (—)	[, [(,)	$[a, b[$ $[a, b)$	right half-open interval in $\mathbb{R}$ from a (included) to b (excluded)	$[a, b[= \{x \in \mathbb{R} \mid a \leq x < b\}$
11-4.17 (—)	] , [(,)	$]a, b[$ (a, b)	open interval in $\mathbb{R}$ from a (excluded) to b (excluded)	$]a, b[= \{x \in \mathbb{R} \mid a < x < b\}$
11-4.18 (11-1.13)	$\subseteq$	$B \subseteq A$	B is included in A ; B is a subset of A	Every element of B belongs to A . $\subset$ is also used, but see remark to 11-4.19.
11-4.19 (11-1.14)	$\subset$	$B \subset A$	B is properly included in A ; B is a proper subset of A	Every element of B belongs to A , but B is not equal to A . If $\subset$ is used for 11-4.18, then $\subsetneq$ shall be used for 11-4.19.
11-4.20 (11-1.15)	$\nsubseteq$	$C \nsubseteq A$	C is not included in A ; C is not a subset of A	$\not\subset$ is also used. The symbols $\nsubseteq$ and $\not\subset$ are also used.
11-4.21 (11-1.16)	$\supseteq$	$A \supseteq B$	A includes B (as subset)	A contains every element of B . $\supset$ is also used, but see remark to 11-4.22. $A \supseteq B$ has the same meaning as $B \subseteq A$.
11-4.22 (11-1.17)	$\supset$	$A \supset B$	A includes B properly	A contains every element of B , but A is not equal to B . If $\supset$ is used for 11-4.21, then $\supsetneq$ shall be used for 11-4.22. $A \supset B$ has the same meaning as $B \subset A$.
11-4.23 (11-1.18)	$\nsubseteq$	$A \nsubseteq C$	A does not include C (as subset)	$\not\supset$ is also used. The symbols $\nsubseteq$ and $\not\supset$ are also used. $A \nsubseteq C$ has the same meaning as $C \nsubseteq A$.
11-4.24 (11-1.19)	$\cup$	$A \cup B$	union of A and B	The set of elements which belong to A or to B or to both A and B . $A \cup B = \{x \mid x \in A \vee x \in B\}$

4 SETS				
Item No.	Symbol, sign	Application	Meaning, verbal equivalent	Remarks and examples
11-4.25 (11-1.20)	$\cup$	$\bigcup_{i=1}^n A_i$	Union of a collection of sets $A_1, \dots, A_n$	$\bigcup_{i=1}^n A_i = A_1 \cup A_2 \cup \dots \cup A_n$, the set of elements belonging to at least one of the sets $A_1, \dots, A_n$. $\bigcup_{i=1}^n$ and $\bigcup_{i \in I}$, $\bigcup_{i \in I}$ are also used, where I denotes a set of indices.
11-4.26 (11-1.21)	$\cap$	$A \cap B$	intersection of A and B , read as A inter B	The set of elements which belong to both A and B . $A \cap B = \{x \mid x \in A \wedge x \in B\}$
11-4.27 (11-1.22)	$\cap$	$\bigcap_{i=1}^n A_i$	intersection of a collection of sets $A_1, \dots, A_n$	$\bigcap_{i=1}^n A_i = A_1 \cap A_2 \cap \dots \cap A_n$, the set of elements belonging to all sets $A_1, A_2, \dots$ and A_n . $\bigcap_{i=1}^n$ and $\bigcap_{i \in I}$, $\bigcap_{i \in I}$ are also used, where I denotes a set or indices.
11-4.28 (11-1.23)	$\setminus$	$A \setminus B$	difference between A and B ; A minus B	The set of elements which belong to A , but not to B . $A \setminus B = \{x \mid x \in A \wedge x \notin B\}$ $A - B$ should not be used.
11-4.29 (11-1.24)	$\complement$	$\complement_A B$	complement of subset B of A	The set of those elements of A which do not belong to the subset B . If it is clear from the context which set A is being considered, the symbol A is often omitted. Also $\complement_A B = A \setminus B$
11-4.30 (11-1.25)	$(,)$	(a, b)	ordered pair a, b ; couple a, b	$(a, b) = (c, d)$ if and only if $a = c$ and $b = d$. $\langle a, b \rangle$ is also used.
11-4.31 (11-1.26)	$(, \dots,)$	$(a_1, a_2, \dots, a_n)$	ordered n -tuple	$\langle a_1, a_2, \dots, a_n \rangle$ is also used.
11-4.32 (11-1.27)	$\times$	$A \times B$	cartesian product of A and B	The set of ordered pairs (a, b) such that $a \in A$ and $b \in B$. $A \times B = \{(a, b) \mid a \in A \wedge b \in B\}$ $A \times A \times \dots \times A$ is denoted by A^n , where n is the number of factors in the product.
11-4.33 (—)	Δ	Δ_A	set of pairs (x, x) of $A \times A$, where $x \in A$; diagonal of the set $A \times A$	$\Delta_A = \{(x, x) \mid x \in A\}$ id_A is also used.

5 MISCELLANEOUS SIGNS AND SYMBOLS				
Item No.	Symbol, sign	Application	Meaning, verbal equivalent	Remarks and examples
11-5.1 (11-3.1)	=	$a = b$	a is equal to b	$\equiv$ may be used to emphasize that a particular equality is an identity.
11-5.2 (11-3.2)	$\neq$	$a \neq b$	a is not equal to b	The symbol $\neq$ is also used.
11-5.3 (11-3.3)	$\stackrel{\text{def}}{=}$	$a \stackrel{\text{def}}{=} b$	a is by definition equal to b	EXAMPLE $p \stackrel{\text{def}}{=} mv$, where p is momentum, m is mass and v is velocity. $\stackrel{\text{d}}{=}$ and $\stackrel{:=}{=}$ are also used.
11-5.4 (11-3.4)	$\triangleq$	$a \triangleq b$	a corresponds to b	EXAMPLES When $E = kT$, $1 \text{ eV} \triangleq 11\,604,5 \text{ K}$. When 1 cm on a map corresponds to a length of 10 km, one may write $1 \text{ cm} \triangleq 10 \text{ km}$.
11-5.5 (11-3.5)	$\approx$	$a \approx b$	a is approximately equal to b	The symbol $\simeq$ is reserved for “is asymptotically equal to”. See 11-7.7.
11-5.6 (11-3.6)	$\sim$ $\propto$	$a \sim b$ $a \propto b$	a is proportional to b	
11-5.7 (11-3.7)	$<$	$a < b$	a is less than b	
11-5.8 (11-3.8)	$>$	$b > a$	b is greater than a	
11-5.9 (11-3.9)	$\leq$	$a \leq b$	a is less than or equal to b	The symbols $\leq$ and $\leqslant$ are also used.
11-5.10 (11-3.10)	$\geq$	$b \geq a$	b is greater than or equal to a	The symbols $\geq$ and $\geqslant$ are also used.
11-5.11 (11-3.11)	$\ll$	$a \ll b$	a is much less than b	
11-5.12 (11-3.12)	$\gg$	$b \gg a$	b is much greater than a	
11-5.13 (11-3.13)	∞		infinity	
11-5.14 (—)	() [] { } < >	$(a + b) c$ $[a + b]c$ $\{a + b\}c$ $\langle a + b \rangle c$	$ac + bc$, parentheses $ac + bc$, square brackets $ac + bc$, braces $ac + bc$, angle brackets	In ordinary algebra the sequence of (), [], { } and < > in order of nesting is not standardized. Special uses are made of (), [], { } and < > in particular field.
11-5.15 (—)	//	$AB \parallel CD$	the line AB is parallel to the line CD	
11-5.16 (—)	$\perp$	$AB \perp CD$	the line AB is perpendicular to the line CD	