

BRITISH STANDARD

Sampling procedures for inspection by variables –

Part 1: Guide to single sampling plans indexed by acceptance quality limit (AQL) for lot-by-lot inspection for a single quality characteristic and a single AQL

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Foreword

Publishing information

This part of BS 6002 was published by BSI and came into effect on 28 February 2007. It was prepared by Technical Committee SS/5, *Applications of statistical methods, Acceptance sampling*. A list of organizations represented on this committee can be obtained on request to its secretary.

Relationship with other publications

BS 6002-1 and BS 6002-2 together supersede BS 6002-1:1993, which is withdrawn.

Information about this document

The most significant differences between BS 6002-1:2007 and BS 6002-1:1993 are as follows:

- The acronym AQL now stands for *acceptance quality limit* rather than *acceptable quality level*, in order to reflect more accurately its function.
- The coverage of this part of BS 6002 is constrained to a single, normally distributed variable with a single class of nonconformity. Part 1 includes the case of combined control of double specification limits, but procedures for separate or complex control of double specification limits are deferred to Part 2. (More general procedures that can be used for multiple characteristics and/or multiple AQLs are also given in Part 2.)
- The plans have been modified so that their operating characteristic curves match those of the plans in BS 6001-1 more closely. A consequence is that the sample sizes for the “s” method and for the “ σ ” method are no longer constant along rows of the master tables.
- All acceptability constants (see Annexes B, C, G and I) have been revised and tabulated to three decimal places for an extended range of AQLs corresponding to BS 6001-1:1999.
- All tabulated values of operating characteristics have been recalculated and related directly to reduced inspection as well as to normal and tightened inspection.
- The annex containing the general statistical theory has been removed. It is planned ultimately to reintroduce this within a guidance document to sampling procedures for inspection by variables.
- Tables that are required for implementing the procedures have been relocated into annexes.
- The annex dealing with the “R” method has been eliminated, now that calculators with a standard deviation function key are so cheap and so widely available. Data for acceptance sampling by variables is often substantially more expensive to acquire than data for sampling by attributes, and the “s” method makes more efficient use of this data.

BS 6002 currently consists of the following parts, under the general title *Sampling procedures for inspection by variables*:

- *Part 1: Guide to single sampling plans indexed by acceptance quality limit (AQL) for lot-by-lot inspection – Single quality characteristic and single AQL;*
- *Part 2: General guide to single sampling plans indexed by acceptance quality limit (AQL) for lot-by-lot inspection;*
- *Part 3: Guide to double sampling schemes indexed by acceptance quality limit (AQL) for lot-by-lot inspection.*

Fourteen annexes are provided. Annexes A to I provide the tables needed to support the procedures. Annex J indicates how the sample standard deviation, s , and the presumed known value of the process standard deviation, σ , should be determined. Annex K provides the statistical theory underlying the calculation of the consumer's risk qualities, together with tables showing these quality levels for normal, tightened and reduced inspection as well as for the "s" and " σ " methods. Annex L provides similar information for the producer's risks. Annex M gives the general formula for the operating characteristic of the " σ " method. Annex N provides the statistical theory underlying the estimation of the process fraction nonconforming under the "s" method for sample sizes 3 and 4, which for technical reasons are treated differently from the other sample sizes in this British Standard.

Presentational conventions

The provisions in this standard are presented in roman (i.e. upright) type. Its recommendations are expressed in sentences in which the principal auxiliary verb is "should".

Commentary, explanation and general informative material is presented in smaller italic type, and does not constitute a normative element.

Contractual and legal considerations

This publication does not purport to include all the necessary provisions of a contract. Users are responsible for its correct application.

Compliance with a British Standard cannot confer immunity from legal obligations.

Introduction

This British Standard describes an acceptance sampling system of single sampling plans for inspection by variables. It is indexed in terms of the acceptance quality limit (AQL), and is designed for users who have simple requirements. (A more comprehensive and technical treatment is given in BS 6002-2.) BS 6002-1 is complementary to BS 6001-1.

The objectives of the methods laid down in this British Standard are to ensure that lots of an acceptable quality have a high probability of acceptance and that the probability of not accepting inferior lots is as high as practicable. This is achieved by means of the switching rules, which provide:

- a) automatic protection to the consumer (by means of a switch to tightened inspection or discontinuation of sampling inspection) should a deterioration in quality be detected;
- b) an incentive (at the discretion of the responsible authority) to reduce inspection costs (by means of a switch to a smaller sample size) should consistently good quality be achieved.

In this British Standard, the acceptability of a lot is implicitly determined from an estimate of the percentage of nonconforming items in the process, based on a random sample of items from the lot.

BS 6002-1 is intended for application to a continuing series of lots of discrete products all supplied by one producer using one production process. If there are different producers or production processes, BS 6002-1 is applied to each one separately.

BS 6002-1 is intended for application to a single quality characteristic that is measurable on a continuous scale. For two or more such quality characteristics, see BS 6002-2.

It is assumed in this British Standard that measurement error is negligible (see ISO 10576-1:2003). For information on allowing for measurement error, see reference [1] in the Bibliography.

For double specification limits, this British Standard treats combined control. For other types of control, refer to BS 6002-2.

Caution The procedures in this British Standard are not suitable for application to lots that have been screened previously for nonconforming items.

Inspection by variables for percent nonconforming items, as described in the present document, includes several possible modes, the combination of which leads to a presentation that may appear quite complex to the user:

- unknown standard deviation, or originally unknown then estimated with fair precision, or known since the start of inspection;
- a single specification limit, or combined control of double specification limits;
- normal inspection, tightened inspection or reduced inspection.

The summary table is intended to facilitate the use of the standard by directing the user to the paragraphs and tables concerning any situation with which he may be confronted. The table only deals with Clauses 15, 16, 20, 21 and 22; in every case, it is necessary first of all to have read the other clauses.

Summary table

	Single specification limit						Double specification limits with combined control					
	“s” method			“σ” method			“s” method			“σ” method		
	Clauses or sub-clauses	Tables/ annexes	Charts	Clauses or sub-clauses	Tables/ annexes	Charts	Clauses or sub-clauses	Tables/ annexes	Charts	Clauses or sub-clauses	Tables/ annexes	Charts
Normal inspection	15.1, 15.2, 15.3 and 20.1	A.1, A.2, B.1, B to R	B to R	16.1, 16.2 and 20.1	A.1, A.2, C.1, B to R ^{A)}	B to R ^{A)}	15.1, 15.4 and 20.1	A.1, A.2, D.1, F.1 (for $n = 3$), G.1 (for $n = 3$ or 4), B to R ^{A)}	s-D to s-R, B to R ^{A)}	16.1, 16.3 and 20.1	A.1, A.2, C.1, E.1, B to R ^{A)}	B to R ^{A)}
Switching between normal and tightened inspection	20.2, 20.3	B.1, B.2	B to R	20.2, 20.3	C.1, C.2	B to R ^{A)}	20.2, 20.3	D.1, D.2	s-D to s-R, B to R ^{A)}	20.2, 20.3	C.1, C.2, E.1	B to R ^{A)}
Switching between normal and reduced inspection	20.4, 20.5	B.1, B.3	B to R	20.4, 20.5	C.1, C.3	B to R ^{A)}	20.4, 20.5	D.1, D.3 G.1 (for $n = 3$ or 4)	s-D to s-R, B to R ^{A)}	20.4, 20.5	E.1	B to R ^{A)}
Switching between tightened and discontinued inspection	21	B.2	B to R	21	C.2	B to R ^{A)}	21	D.2	s-D to s-R, B to R ^{A)}	21	E.1	B to R ^{A)}
Switching between the “s” and “σ” methods	22	Annex J		22	Annex J		22	Annex J		22	Annex E Annex J	

A) But see 8.4.

1 Scope

This British Standard describes an acceptance sampling system primarily designed for use under the following conditions:

- a) where the inspection procedure is to be applied to a **continuing series of lots** of discrete products all supplied by one producer using one production process;
- b) where only a **single quality characteristic x** of these products is taken into consideration, which must be **measurable on a continuous scale**;
- c) where the measurement error is negligible, i.e. with a standard deviation no more than 10% of the process standard deviation;
- d) where production is stable (under statistical control) and the quality characteristic x is distributed according to a **normal distribution** or a close approximation to the normal distribution;
- e) where a contract or standard defines a **lower specification limit L** , an **upper specification limit U** , or **both**; an item is qualified as conforming if and only if its measured quality characteristic x satisfies the appropriate one of the following inequalities:
 - 1) $x \geq L$ (i.e. the lower specification limit is not violated);
 - 2) $x \leq U$ (i.e. the upper specification limit is not violated);
 - 3) $x \geq L$ and $x \leq U$ (i.e. neither the lower nor the upper specification limit is violated).

Inequalities 1) and 2) are called cases with a single specification limit, and 3) a case with double specification limits.

Where double specification limits apply, it is assumed in this British Standard that conformance to both specification limits is equally important to the integrity of the product; in such cases, it is appropriate to apply a single AQL to the combined percentage of product outside the two specification limits. This is referred to as **combined control**.

2 Normative references

The following referenced documents are indispensable for the application of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

BS 2846-7 [ISO 5479], *Guide to statistical interpretation of data – Tests for departure from normality*

BS 6001-1 [ISO 2859-1], *Sampling procedures for inspection by attributes – Part 1: Sampling schemes indexed by acceptance quality limit (AQL) for lot-by-lot inspection*

BS 6001-2 [ISO 2859-2], *Sampling procedures for inspection by attributes – Part 2: Sampling plans indexed by limiting quality (LQ) for lot-by-lot inspection*

BS 6002-2, *Sampling procedures for inspection by variables – Part 2: General guide to single sampling plans indexed by acceptance quality limit (AQL) for lot-by-lot inspection of independent quality characteristics*

BS ISO 3534-1, *Statistics – Vocabulary and symbols – Part 1: Probability and statistical terms*

BS ISO 3534-2, *Statistics – Vocabulary and symbols – Part 2: Statistical quality control*

BS ISO 5725-2, *Accuracy (trueness and precision) of measurement methods and results – Part 2: Basic methods for the determination of repeatability and reproducibility of a standard measurement method*

3 Terms and definitions

For the purposes of this British Standard, the terms and definitions given in BS 6001-1, BS ISO 3534-1, and BS ISO 3534-2 apply, except where redefined as follows. References are given in square brackets for unchanged definitions that have been repeated here for convenience.

3.1 inspection by variables

inspection by measuring the magnitude of a characteristic of an item [BS ISO 3534-2]

3.2 sampling inspection

inspection of selected items in the group under consideration [BS ISO 3534-2]

3.3 acceptance sampling inspection acceptance sampling

sampling inspection (3.2) to determine whether, or not, to accept a lot or other amount of product, material or service [BS ISO 3534-2]

3.4 acceptance sampling inspection by variables

acceptance sampling inspection (3.3) in which the acceptability of the process is determined statistically from measurements on specified quality characteristics of each item in a sample from a lot

3.5 process fraction nonconforming

rate at which nonconforming items are generated by a process, expressed as a proportion

3.6 acceptance quality limit AQL

NOTE See Clause 5.

worst tolerable **process fraction nonconforming (3.5)** when a continuing series of lots is submitted for **acceptance sampling (3.3)**

3.7 quality level

quality expressed as a rate of occurrence of nonconforming units

3.8 limiting quality LQ

NOTE See 13.1.

quality level (3.7), when a lot is considered in isolation, which, for the purposes of **acceptance sampling inspection (3.3)**, is limited to a low probability of acceptance (in this British Standard: 10%) [BS ISO 3534-2]

3.9 nonconformity

non-fulfilment of a requirement

3.10 nonconforming unit

unit with one or more nonconformities [BS ISO 3534-2]

NOTE See Clause 15.

3.11 “s” method acceptance sampling plan

acceptance sampling (3.3) plan by variables using the sample standard deviation [BS ISO 3534-2]

NOTE See Clause 16.

3.12 “σ” method acceptance sampling plan

acceptance sampling (3.3) plan by variables using the presumed value of the process standard deviation [BS ISO 3534-2]

3.13 specification limit

conformance boundary specified for a characteristic [BS ISO 3534-2]

3.14 lower specification limit

specification limit (3.13) that defines the lower conformance boundary [BS ISO 3534-2]

3.15 upper specification limit

U

specification limit (3.13) that defines the upper conformance boundary [BS ISO 3534-2]

NOTE See 5.3.

3.16 combined control

requirement when both upper and lower limits are specified for the quality characteristic and an **AQL (3.6)** is given that applies to the combined percent nonconforming beyond the two limits

*NOTE The use of combined control implies that nonconformity beyond either **specification limit (3.13)** is believed to be of equal, or at least roughly equal, importance to the lack of integrity of the product.*

3.17 acceptability constant

k

NOTE See 15.2 and 16.2.

constant depending on the specified value of the **acceptance quality limit (3.6)** and the sample size, used in the criteria for accepting the lot in an **acceptance sampling (3.3)** plan by variables [BS ISO 3534-2]

3.18 quality statistic

Q

NOTE See 15.2 and 16.2.

function of the **specification limit (3.13)**, the sample mean, and the sample or process standard deviation, used in assessing the acceptability of a lot [BS ISO 3534-2]

*NOTE For the case of a single **specification limit (3.13)**, the lot may be sentenced on the result of comparing *Q* with the **acceptability constant (3.17)** *k*.*

3.19 lower quality statistic

Q_L

NOTE See Clause 4, 15.2 and 16.2.

function of the **lower specification limit (3.14)**, the sample mean, and the sample or process standard deviation

*NOTE For a single, lower, **specification limit (3.14)**, the lot is sentenced on the result of comparing *Q_L* with the **acceptability constant (3.17)** *k*. [BS ISO 3534-2]*

NOTE See Clause 4, 15.2 and 16.2.

3.20 upper quality statistic

Q_U

function of the **upper specification limit (3.15)**, the sample mean, and the sample or process standard deviation

*NOTE For a single, **upper, specification limit (3.15)**, the lot is sentenced on the result of comparing Q_U with the **acceptability constant (3.17)** k [BS ISO 3534-2]*

3.21 maximum sample standard deviation MSSD

$s_{\max}$

largest sample standard deviation for a given sample size code letter and **acceptance quality limit (3.6)** for which it is possible to satisfy the acceptance criterion for the combined control of double **specification limits (3.13)** when the process variability is unknown

NOTE See 15.4.

3.22 maximum process standard deviation MPSD

$\sigma_{\max}$

largest process standard deviation for a given sample size code letter and **acceptance quality limit (3.6)** for which it is possible to satisfy the acceptance criterion for the combined control of double **specification limits (3.13)** under tightened inspection when the process variability is known

NOTE See 16.3.

3.23 switching rule

instruction within an **acceptance sampling (3.3)** scheme for changing from one **acceptance sampling (3.3)** plan to another of greater or lesser severity based on demonstrated quality history [BS ISO 3534-2]

NOTE See Clause 20.

NOTE Normal, tightened or reduced inspection, or discontinuation of inspection, are examples of “severity”.

3.24 measurement

set of operations to determine the value of some quantity [BS ISO 3534-2]

4 Symbols

The symbols used are as follows:

c_u	Factor for determining the upper control limit for the sample standard deviation (see Annex H)
f_s	Factor that relates the maximum sample standard deviation to the difference between U and L (see Annex D)
f_σ	Factor that relates the maximum process standard deviation under tightened inspection to the difference between U and L (see Annex E)
k	Form k acceptability constant for use with a single quality characteristic and a single specification limit (see Annex B for the “s” method or Annex C for the “ σ ” method)
L	Lower specification limit. (As a suffix to a variable, denotes its value at L .)
μ	Process mean
N	Lot size (number of items in a lot)
n	Sample size (number of items in a sample)

$\hat{p}$	Estimate of the process fraction nonconforming
$\hat{p}_L$	Estimate of the process fraction nonconforming below the lower specification limit
$\hat{p}_U$	Estimate of the process fraction nonconforming above the upper specification limit
p^*	Maximum acceptable value for the estimate of the process fraction nonconforming
P_a	Probability of acceptance
Q	Quality statistic
Q_L	Lower quality statistic <i>NOTE</i> Q_L is defined as $(\bar{x} - L)/s$ when the process standard deviation is unknown, and as $(\bar{x} - L)/\sigma$ when it is presumed to be known.
Q_U	Upper quality statistic. <i>NOTE</i> Q_U is defined as $(U - \bar{x})/s$ when the process standard deviation is unknown, and as $(U - \bar{x})/\sigma$ when it is presumed to be known.
s	Sample standard deviation of the measured values of the quality characteristic (also an estimate of the standard deviation of the process), i.e.

$$s = \sqrt{\frac{\sum_{j=1}^n (x_j - \bar{x})^2}{n-1}}$$

(See Annex J.)

$s_{\max}$	Maximum sample standard deviation (MSSD)
σ	Standard deviation of a process that is under statistical control <i>NOTE</i> σ^2 , the square of the process standard deviation, is known as the process variance.
$\sigma_{\max}$	Maximum process standard deviation (MPSD)
U	Upper specification limit. (As a suffix to a variable, denotes its value at U .)
x_j	Measured value of the quality characteristic for the j th item of the sample
$\bar{x}$	The arithmetic mean of the measured values of the quality characteristic in the sample, i.e.

$$\bar{x} = \frac{\sum_{j=1}^n x_j}{n}$$

5 Acceptance quality limit (AQL)

5.1 Concept

The AQL is the quality level that is the worst tolerable process fraction nonconforming when a continuing series of lots is submitted for acceptance sampling. Although individual lots with quality as bad as the acceptance quality limit may be accepted with fairly high probability, the designation of an acceptance quality limit does not suggest that this is a desirable quality level.

The sampling schemes found in this standard, with their rules for switching and for discontinuation of sampling inspection, are designed to encourage suppliers to keep the process fractions nonconforming consistently better than the respective AQLs. Otherwise, there is a high risk that the inspection severity will be switched to tightened inspection, under which the criteria for lot acceptance become more demanding. Once on tightened inspection, unless action is taken to improve the process, it is very likely that the rule requiring discontinuation of sampling inspection will be invoked pending such improvement.

5.2 Use

The AQL, together with the sample size code letter, is used to index the sampling plans in this British Standard.

5.3 Specifying AQLs

The AQL to be used will be designated in the product specification, contract or by the responsible authority. Where both upper and lower specification limits are given, this part of BS 6002 addresses only the case of an overall AQL applying to the combined percent nonconforming beyond the two limits; this is known as “combined control”. (See Part 2 of BS 6002 for “separate” and “complex” control of double specification limits.)

5.4 Preferred AQLs

The sixteen AQLs given in this British Standard, ranging in value from 0.01% to 10% nonconforming, are described as preferred AQLs. If, for any product or service, an AQL is designated other than a preferred AQL, then this British Standard is not applicable. (See **13.2**.)

5.5 Caution

From this definition of the AQL, it follows that the desired protection can only be assured when a continuing series of lots is provided for inspection.

5.6 Limitation

The designation of an AQL should not imply that the supplier has the right to supply knowingly any nonconforming product.

6 Switching rules for normal, tightened and reduced inspection

Switching rules discourage the producer from operating at a quality level that is worse than the AQL. This British Standard prescribes a switch to tightened inspection when inspection results indicate that the AQL is being exceeded. It further prescribes a discontinuation of sampling inspection altogether if tightened inspection fails to stimulate the producer into rapidly improving his production process.

Tightened inspection and the discontinuation rule are integral, and therefore obligatory, procedures of this British Standard if the protection implied by the AQL is to be maintained.

This British Standard also provides the possibility of switching to reduced inspection when inspection results indicate that the quality level is stable and reliable at a level better than the AQL. This practice is, however, optional (at the discretion of the responsible authority).

When there is sufficient evidence from the control charts (see **19.1**) that the variability is in statistical control, consideration should be given to switching to the “ σ ” method. If this appears advantageous, the consistent value of s (the sample standard deviation) should be taken as σ (see Clause **22**).

When it has been necessary to discontinue acceptance sampling inspection, inspection under this British Standard should not be resumed until action has been taken by the producer to improve the quality of the submitted product.

Details of the operation of the switching rules are given in Clauses **20**, **21** and **22**.

7 Relation to BS 6001-1

7.1 Similarities

The similarities to BS 6001-1 are as follows.

- a) This British Standard is complementary to BS 6001-1; the two standards share a common philosophy and, as far as possible, their procedures and vocabulary are the same.
- b) Both use the AQL to index the sampling plans and the preferred values used in this document are identical with those given for percent nonconforming in BS 6001-1 (i.e. from 0.01% to 10%).
- c) In both British Standards, lot size and inspection level (inspection level II in default of other instructions) determine a sample size code letter. Then general tables give the sample size to be taken and the acceptability criterion, indexed by the sample size code letter and the AQL. Separate tables are given for the “ s ” and “ σ ” methods, and for normal, tightened and reduced inspection.
- d) The switching rules are essentially equivalent.

7.2 Differences

The differences from BS 6001-1 are as follows.

- a) **Determination of acceptability.** Acceptability for a BS 6001-1 attributes sampling plan for percent nonconforming is determined by the number of nonconforming items found in the sample. Acceptability for a plan for inspection by variables is based on the distance of the estimated value of the process mean from the specification limit(s) in terms of the estimated or presumed value of the process standard deviation. In this British Standard, two methods are considered: the “s” method for use when the process standard deviation σ is unknown and the “ σ ” method for use when σ is presumed to be known. In the case of a single specification limit, the acceptability may be calculated from a formula (see **15.2** and **16.2**), but for the “s” method it is also easily established by a graphical method (see **15.3**). In the case of combined control of double specification limits under the “s” method, this British Standard provides only for a graphical method of determining acceptability (see **15.4**); for combined control of double specification limits under the “ σ ” method, a numerical method is given.
- b) **Normality.** In BS 6001-1, there is no requirement relating to the distribution of the characteristics. However, in this British Standard it is necessary for the efficient operation of the plans that the measurements be distributed according to a normal distribution or a close approximation to a normal distribution.
- c) **Operating characteristic curves (OC curves).** The OC curves of the variables plans in this standard are not identical to those of the corresponding attributes plans in BS 6001-1. The curves have been matched by minimizing the area between the curves representing the squares of the OC values, a method that gives greater emphasis to the match at the top of the OC curves.
- d) **Producer’s risk.** For process quality precisely at the AQL, the producer’s risk that a lot will not be accepted tends to decrease with one-step increases in sample size coupled with one-step decreases in AQL, i.e. down diagonals of the master tables running from top right to bottom left. The progressions of probabilities are similar, but not identical, to those in BS 6001-1. (The producer’s risks of the plans are given in Annex L.)
- e) **Sample sizes.** The variables sample sizes for given combinations of sample size code letter and AQL are usually smaller than the corresponding attributes sample sizes. This is particularly true for the “ σ ” method. Moreover, due to the method by which the variables plans were derived, their sample sizes vary over AQL for a given sample size code letter.
- f) **Double sampling plans.** Double sampling plans by variables are presented separately, in BS 6002-3.
- g) **Multiple sampling plans.** No multiple sampling plans by variables are given in this British Standard.

- h) **Average outgoing quality limit (AOQL).** The AOQL concept is mainly of value when 100% inspection and rectification is feasible for non-accepted lots. It follows that the AOQL concept cannot be applied under destructive or expensive testing. As variables plans will generally be used under these circumstances, no tables of AOQL have been included in this British Standard.

8 Limiting quality protection

8.1 Use of individual plans

This British Standard is intended to be used as a system employing tightened, normal and reduced inspection on a continuing series of lots to provide consumer protection while assuring the producer that acceptance will be very likely to occur if quality is better than the AQL.

Some users may select specific individual plans from this British Standard and use them without the switching rules. For example, a purchaser may be using the plans for verification purposes only. This is not the intended application of the system given in this British Standard and its use in this way should not be referred to as “inspection in compliance with BS 6002-1”. When used in such a way, BS 6002-1 simply represents a collection of individual plans indexed by the AQL. The operating characteristic curves and other measures of a plan so chosen should be assessed individually from the tables provided.

8.2 Consumer’s risk quality tables

If the series of lots is not long enough to allow the switching rules to be applied, it may be desirable to limit the selection of sampling plans to those, associated with a designated AQL value, that give a consumer’s risk quality not worse than the specified limiting quality protection. Sampling plans for this purpose can be selected by choosing a consumer’s risk quality (CRQ) and a consumer’s risk to be associated with it. Annex K gives values of consumer’s risk quality for the “s” method and “ σ ” methods corresponding to a consumer’s risk of 10%.

However, application of this British Standard to isolated lots is deprecated, as the theory of sampling by variables applies to a *process*. For isolated lots, it is appropriate and more efficient to use plans for sampling by attributes, such as from BS 6001-2. (See also reference [2] in the Bibliography.)

8.3 Producer’s risk tables

Annex L gives the probability of non-acceptance under the “s” and “ σ ” methods for lots produced when the process fraction nonconforming equals the AQL. This probability is called the producer’s risk.

8.4 Operating characteristic curves

The tables for consumer's risk quality and producer's risk provide information about only two points on the operating characteristic curves. The degree of consumer protection provided by an individual sampling plan at any process quality level may, however, be judged from its operating characteristic (OC) curve. OC curves for the normal inspection "s" method sampling plans of this British Standard are given in Charts B to R, which should be consulted when choosing a sampling plan. Also given are tables of process qualities at nine standard probabilities of acceptance for all of the "s" method sampling plans in this British Standard.

These OC curves and tables apply to a single specification limit under the "s" method. Most of them also provide a good approximation to the " σ " method, and to the case of combined control of double specification limits, particularly for the larger sample sizes. If more accurate OC values are required for the " σ " method, refer to Annex M.

9 Planning

The choice of the most suitable variables plan, if one exists, requires experience, judgement and some knowledge both of statistics and the product to be inspected. Clauses **10** to **13** of this British Standard are intended to help those responsible for specifying sampling plans in making this choice. They suggest the considerations that should be borne in mind when deciding whether a variables plan would be suitable, and the choices to be made when selecting an appropriate standard plan.

10 Choice between variables and attributes

The first question to consider is whether it is desirable to inspect by variables rather than by attributes. The following points should be taken into account.

- a) In terms of economics, it is necessary to compare the total cost of the relatively simple inspection of a larger number of items by means of an attributes scheme with the generally more elaborate procedure required by a variables scheme, which is usually more time-consuming and costly per item.
- b) In terms of the knowledge gained, the advantage lies with inspection by variables, as the information obtained indicates more precisely how good the product is. Earlier warning may therefore be given if the quality is slipping.
- c) An attributes scheme can be more readily understood and accepted; for example, it may at first be difficult to accept that, when inspecting by variables, a lot can be rejected on measurements taken of a sample that does not contain any nonconforming items. (See the examples in **15.4.2** and **15.4.4**.)

- d) From a comparison of the size of the samples required for the same AQL from standard plans for inspection by attributes, such as from BS 6001-1, and the standard plans in this British Standard, it will be seen that the smallest samples are generally required by the “ σ ” method (used when the process standard deviation is presumed to be known). The sample sizes for the “ s ” method (used when the process standard deviation is unknown) are also in general substantially smaller than for sampling by attributes.
- e) Inspection by variables is particularly appropriate in conjunction with the use of control charts for variables.
- f) Variables sampling has a substantial advantage when the inspection process is expensive, for example in the case of destructive testing.
- g) A variables scheme becomes relatively more complicated to operate as the number of measurements to be taken on each item increases. (For two or more quality characteristics, this part of BS 6002 does not apply. See Part 2 for details.)
- h) The use of this British Standard is only applicable when there is reason to believe that the distribution of measurements of the quality characteristic is normal. In case of doubt, the responsible authority should be consulted.

NOTE 1 BS 2846-7 gives detailed procedures for tests for departure from normality.

NOTE 2 Departure from normality is also dealt with in section two of BS ISO 5725-2, which provides examples of graphical methods that can be used to verify that the distribution of the data is sufficiently normal to justify the use of sampling by variables.

11 Choice between the “ s ” and “ σ ” methods

If it is desired to apply inspection by variables, the next question is whether to use the “ s ” method or the “ σ ” method. The “ σ ” method is the most economical in terms of sample size but, before this method may be employed, the value of σ has to be established.

Initially, it will be necessary to begin with the “ s ” method but, subject to the agreement of the responsible authority and provided the quality remains satisfactory, the standard switching rules will permit a switch to reduced inspection and the use of a smaller sample size.

The question then is, if the variability is under control and lots continue to be accepted, whether it will be economical to change to the “ σ ” method. The size of the sample will generally be smaller and the acceptability criterion simpler under the “ σ ” method. (See 16.2.) On the other hand, it will still be necessary to calculate the sample standard deviation, s , for record purposes and to keep the control charts up to date. (See Clause 19.) The calculation of s can appear daunting at first sight, but the difficulty is more apparent than real; this is especially true if a calculator or computer is available. Methods of determining s and σ are given in Annex J.

12 Choice of inspection level and AQL

For a standard sampling plan, the inspection level in conjunction with the size of the lots and the AQL determines the size of the sample to be taken, and governs the severity of the inspection. The appropriate OC curve from Charts B to R (see Clause 24) or appropriate table from Tables B to R (see Clause 25) show the extent of the risk that is involved in such a plan.

The choice of inspection level and AQL is governed by a number of factors, but is mainly a balance between the total cost of inspection and the consequences of nonconforming items passing into service.

The normal practice is to use inspection level II, unless special circumstances indicate that another level is more appropriate.

13 Choice of sampling plan

13.1 Standard plans

The standard procedure can be used only when the production of lots is continuing.

This standard procedure, with its semi-automatic steps from lot size to sample size, using inspection level II and beginning with the “s” method, has been found in practice to produce workable sampling plans; but it rests on the assumption that the order of priority is first the AQL, second the sample size and last, the limiting quality.

The acceptability of this system is due to the fact that the consumer is protected by the switching rules (see Clauses 20, 21 and 22), which quickly increase the severity of inspection and finally terminate inspection altogether if the quality of the process remains worse than the AQL.

NOTE It should also be remembered that the limiting quality is the quality which, if offered for inspection, would have a 10% probability of acceptance. The actual risk taken by the consumer varies according to the probability that goods of such a low quality are offered for inspection.

However, if, in certain circumstances, the limiting quality has a higher priority than the sample size, a suitable plan in this British Standard may be selected by using Chart A. Construct a vertical line through the acceptable value for the limiting quality and a horizontal line through the desired quality with a 95% probability of acceptance (i.e. approximately equal to the AQL). The point of intersection of these two lines will lie on, or under, a line indexed with the sample size code letter of a standard normal inspection plan that meets the specified requirements. This should be verified by inspecting the OC curve from among Charts B to R relating to this code letter and AQL.

However, use of this method is deprecated for isolated lots or short series of lots. (See 8.2.)

EXAMPLE

Suppose that an acceptable value for the limiting quality is 6.0% nonconforming and that the desired quality with a 95% probability of acceptance is 2.0% nonconforming. A vertical line on Chart A at 6.0% nonconforming and a horizontal line at 2.0% nonconforming intersect just below the sloping line indexed by the letter L. Examining Chart L, it is seen that a plan with sample size code letter L and AQL 1.5% meets the requirements.

If the lines intersect at a point above the line marked R in Chart A, this implies that the specification cannot be met by any of the plans in this British Standard.

13.2 Special plans

If standard plans are not acceptable, it will be necessary to devise a special plan. It then has to be decided which combination of AQL, limiting quality, and sample size is most suitable, remembering that these are not independent for, when any two have been chosen, the third follows.

NOTE This choice is not completely unfettered; the fact that the size of the sample is necessarily a whole number imposes some limitations. If a special scheme is necessary, it should be devised only with the assistance of a statistician experienced in quality control.

14 Preliminary operations

Before starting inspection by variables,

- a) check that production is considered to be continuing and that the distribution of the quality characteristic can be considered to be normal;

NOTE 1 For tests for departure from normality, see BS 2846-7.

NOTE 2 If lots have been screened for nonconforming items prior to acceptance sampling, then the distribution will have been truncated and this British Standard will not be applicable.

- b) check whether the “s” method is to be used initially or whether the standard deviation is stable and known, in which case the “ σ ” method should be used;
- c) check that the inspection level to be used has been designated. If none has been given, inspection level II should be used;
- d) check, for a quality characteristic with double specification limits, that nonconformities beyond each limit are of equal importance. If this is not the case, refer to BS 6002-2;
- e) check that an AQL has been designated, and that it is one of the preferred AQLs for use with this British Standard. If it is not, then the tables are not applicable.

15 Standard procedure for the “s” method

15.1 Obtaining a plan, sampling and preliminary calculations

The procedure for obtaining and implementing a plan is as follows.

- a) With the inspection level given (normally this will be II) and with the lot size, obtain the sample size code letter using Table A.1.
- b) For a single specification limit, enter Table B.1, B.2 or B.3 as appropriate with this code letter and the AQL, and obtain the sample size n and the acceptability constant k . For combined control of double specification limits when the sample size is 5 or more, find the appropriate acceptance curve from among Charts s-D to s-R (see Clause 25).
- c) Take a random sample of size n , measure the characteristic x in each item and then calculate $\bar{x}$ the sample mean, and s , the sample standard deviation (see Annex J). If $\bar{x}$ is outside the specification limit(s), the lot can be judged unacceptable without even calculating s . It is, however, necessary to calculate s for record purposes.

15.2 Acceptability criteria for single specification limits

If single specification limits are given, calculate the quality statistic

$$Q_U = \frac{U - \bar{x}}{s}$$

or

$$Q_L = \frac{\bar{x} - L}{s}$$

as appropriate, then compare the quality statistic (Q_U or Q_L) with the acceptability constant k obtained from either Table B.1, B.2 or B.3 for normal, tightened or reduced inspection respectively. If the quality statistic is greater than or equal to the acceptability constant, the lot is acceptable; if less, then it is not acceptable.

Thus, if only the upper specification limit U is given, the lot is:

- acceptable if $Q_U \geq k$,
- not acceptable if $Q_U < k$,

or, if only the lower specification limit L is given, the lot is:

- acceptable if $Q_L \geq k$,
- not acceptable if $Q_L < k$.

EXAMPLE 1 Single, upper specification limit

The maximum temperature of operation for a certain device is specified as 60 °C. Production is inspected in lots of 100 items and the process standard deviation is unknown. Inspection level II, normal inspection with AQL = 2.5% is to be used. From Table A.1, the sample size code letter is found to be F; from Table B.1 it is seen that a sample size of 13 is required and that the acceptability constant k is 1.427. Suppose that the measurements are as follows:

53 °C; 57 °C; 49 °C; 58 °C; 59 °C; 54 °C; 58 °C; 56 °C; 50 °C; 50 °C; 55 °C; 54 °C; 57 °C.

Conformance with the acceptability criterion is to be determined.

Information needed	Values obtained
Sample size: n	13
Sample mean: $\bar{x} = \sum x/n$	54.615 °C
Sample standard deviation: $s = \sqrt{\sum_j (x_j - \bar{x})^2 / (n - 1)}$	3.330 °C
(See K.2 , Annex K.)	
Specification limit (upper): U	60 °C
Upper quality statistic: $Q_U = (U - \bar{x})/s$	1.617
Form k acceptability constant: k (from Table B.1)	1.427
Acceptability criterion: Is $Q_U \geq k$?	Yes (1.617 > 1.427)

The lot meets the acceptability criterion, and is therefore acceptable.

EXAMPLE 2 Single, lower specification limit, requiring the use of an arrow in the master table.

A certain pyrotechnic delay mechanism has a specified minimum delay time of 4.0 s. The process standard deviation is unknown. Production is inspected in lots of 1 000 items and inspection level II, normal inspection, is to be used with an AQL of 0.1% applied to the lower limit. From Table A.1 it is seen that the sample size code letter is J. However, on entering Table B.1 with sample size code letter J and AQL 0.1%, it is found that there is an arrow pointing to the cell below. This means that an entirely suitable plan is unavailable, and the next best plan is given by sample size code letter K, i.e. sample size 29 and acceptability constant $k = 2.579$. A random sample of size 29 is drawn. Suppose the sample delay times, in seconds, are as follows:

6.95 6.04 6.68 6.63 6.65 6.52 6.59 6.40 6.44 6.34 6.04
6.15 6.29 6.63 6.50 6.44 7.15 6.70 6.59 6.51 6.80 5.94
6.35 7.17 6.83 6.25 6.96 7.00 6.38

Conformance with the acceptability criterion is to be determined.

Information needed	Values obtained
Sample size : n	29
Sample mean: $\bar{x} = \sum x/n$	6.549 seconds
Sample standard deviation: $s = \sqrt{\sum_j (x_j - \bar{x})^2 / (n - 1)}$ (See K.2 , Annex K.)	0.319 4 seconds
Lower specification limit: L	4.0 seconds
Lower quality statistic: $Q_L = (\bar{x} - L)/s$	7.981
Form k acceptability constant: k (from Table B.1)	2.579
Acceptability criterion: Is $Q_L \geq k$?	Yes ($7.981 > 2.579$)
The lot meets the acceptability criterion, so it is acceptable.	

15.3 Graphical method for a single specification limit

When a graphical criterion is desired, draw the line

$$\bar{x} = U - ks \quad (\text{for an upper limit})$$

or

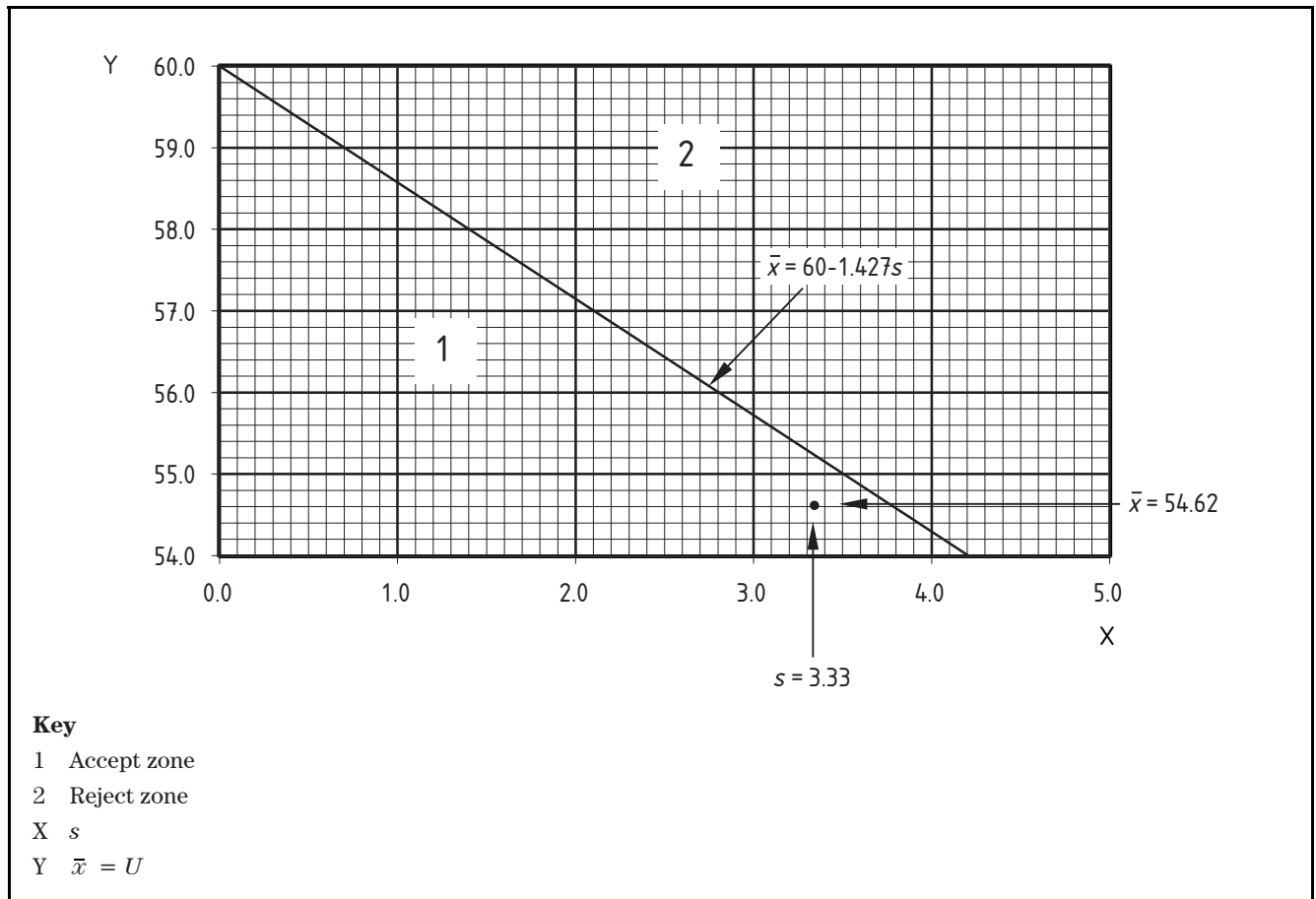
$$\bar{x} = L + ks \quad (\text{for a lower limit}),$$

as appropriate, on graph paper with $\bar{x}$ as the vertical axis and s as the horizontal axis. When the inspection concerns an upper specification limit, the accept zone is the zone below the line. When a lower specification limit is considered, the accept zone is the zone above the line. Plot the point $(s, \bar{x})$ on the graph. If this point lies in the accept zone, the lot is acceptable; if outside, it is not acceptable.

EXAMPLE Graphical method for upper specification limit

Using the data given in Example 1 of 15.2, mark the point $U = 60$ on the $\bar{x}$ (vertical) axis and draw a line through this point with a slope $-k$. [As $k = 1.427$, this means that the line passes through the points $(s = 1, \bar{x} = 58.573)$, $(s = 2, \bar{x} = 57.146)$, $(s = 3, \bar{x} = 55.719)$, etc.] Select a suitable point and draw a straight line through it and $(s = 0, \bar{x} = 60)$, i.e. $(0, U)$. The accept zone is then the area under this line. The calculated values of s and $\bar{x}$ are 3.330 and 54.615. The point $(s, \bar{x})$ is seen from Figure 1 to lie within the accept zone; the lot is therefore acceptable.

Figure 1 Example of the use of an acceptance chart for a single specification limit: “s” method



The graph can be prepared before beginning the inspection of a series of lots. Then, for each lot, the point (s , $\bar{x}$) may be plotted to determine whether or not the lot is acceptable.

15.4 Acceptability criterion for combined control of double specification limits

15.4.1 General

For the “s” method with combined control of the upper and lower specification limits, i.e. with an overall AQL for the percentage of items from the process outside the specification limits, this British Standard provides a graphical method for determining lot acceptability for all sample sizes except for sample sizes 3 and 4. (BS 6002-2 provides numerical methods only.) The larger the sample variability, the less likely it is that the requirement is satisfied. If the value of s exceeds the value of the maximum sample standard deviation (MSSD) obtained from Table D.1, D.2 or D.3, no further calculation or reference to graphs is required, for the lot should be immediately judged unacceptable.

Numerical methods are provided for the combined control of double specification limits for sample sizes 3 and 4.

15.4.2 Procedure for sample size 3

It may be seen from Annex B that there are four cases where the required sample size is 3 for the “s” method, i.e. for sample size code letter B under normal inspection for AQL 4%, for sample size code letter B under tightened inspection for AQL 6.5%, and for sample size code letters B to D under reduced inspection for AQLs 1.5% and 10%.

For sample size 3, proceed as follows. After calculating the sample mean $\bar{x}$ and the sample standard deviation s , find the applicable value of the coefficient f_s from Table D.1, D.2 or D.3. Determine the maximum sample standard deviation (i.e. the maximum allowable) from the formula

$$\text{MSSD}, s_{\max} = (U - L)f_s.$$

Then compare s with $s_{\max}$. If s is greater than $s_{\max}$, then the lot may be rejected without further calculation.

Otherwise, determine the values of $Q_U = (U - \bar{x})/s$ and $Q_L = (\bar{x} - L)/s$. Multiply Q_U and Q_L by $\sqrt{3}/2$ (i.e. approximately 0.866) and use Table F.1 to determine the estimates $\hat{p}_U$ and $\hat{p}_L$ of the fraction of items in the process that are nonconforming beyond the upper and lower limits respectively.

NOTE 1 Negative values of Q correspond to estimates of the process fraction nonconforming in excess of 0.5 at that specification limit and will consequently always result in lot non-acceptance under the provisions of this British Standard. However, in order to obtain a numerical value for record-keeping purposes, the estimate of the process fraction nonconforming may be obtained by entering Table F.1 with the absolute value of $\sqrt{3}Q/2$ and subtracting the result from 1.0. For example, if $Q_U = -0.156$ then $\sqrt{3}Q_U/2 = -0.135$; entering Table F.1 with 0.135 gives an estimate of 0.4569; subtracting this from 1.0 gives $\hat{p}_U = 0.5431$.

NOTE 2 The basis of Table F.1 is given in Annex K. Instead of using Table F.1, the estimate of the process fraction nonconforming beyond each specification limit when $n = 3$ may be calculated directly as

$$\hat{p} = \begin{cases} 0 & \text{if } Q > 2/\sqrt{3}, \\ \frac{2}{\pi} \arcsin \left\{ \sqrt{(1 - Q\sqrt{3}/2)/2} \right\} & \text{if } -2/\sqrt{3} \leq Q \leq 2/\sqrt{3}, \\ 1 & \text{if } Q < -2/\sqrt{3}. \end{cases}$$

These two estimates should be added to obtain the estimate $\hat{p} = \hat{p}_U + \hat{p}_L$ of the overall process fraction nonconforming. If $\hat{p}$ does not exceed the applicable maximum allowable value, p^* , given in Table G.1, the lot is considered to be acceptable; otherwise, the lot is considered to be not acceptable.

EXAMPLE Determination of acceptability for combined control of double specification limits when the sample size is 3

Torpedoes supplied in batches of 100 are to be inspected for accuracy in the horizontal plane. Positive or negative angular errors are equally unacceptable, so combined control of the double specification limits is appropriate. The specification limits are set at 10 m either side of the point of aim at a distance of one kilometre, with an AQL of 4%. Because testing is destructive and very costly, it has been agreed between the producer and the responsible authority that special inspection level S-2 is to be used. From Table A.1, the sample size code letter is found to be B. From Table B.1, it is seen that a sample of size 3 is required. Three torpedoes are tested, yielding errors –5.0 m, 6.7 m and 8.8 m. Conformance with the acceptability criterion under normal inspection is to be determined.

Information needed	Value obtained
Sample size: n	3
Sample mean: $\bar{x} = \frac{1}{n} \sum_{j=1}^n x_j$	3.5 m
Sample standard deviation:	
$s = \sqrt{\sum_{j=1}^n (x_j - \bar{x})^2 / (n - 1)}$	7.436 m
(See J.1.2 , Annex J.)	
Value of f_s for MSSD (Table D.1)	2.957
MSSD, $s_{\max} = (U - L)f_s = (10 - (-10)) \times 2.957$	59.14
As $s = 7.436 < s_{\max} = 59.14$, the lot <i>might</i> be acceptable, so continue with the calculations.	
$Q_U = (U - \bar{x})/s = (10 - 3.5)/7.436$	0.874 1
$Q_L = (\bar{x} - L)/s = (3.5 + 10)/7.436$	1.815
$\sqrt{3}Q_U/2$	0.757
$\sqrt{3}Q_L/2$	1.572
$\hat{p}_U$ (from Table F.1)	0.226 7
$\hat{p}_L$ (from Table F.1)	0.000 0
$\hat{p} = \hat{p}_U + \hat{p}_L$	0.226 7
p^* (from Table G.1, normal inspection)	0.192 4

As $\hat{p} > p^*$, the lot is not acceptable.

NOTE This lot is not acceptable even though all inspected items **in the sample** are within the specification limits.

15.4.3 Procedure for sample size 4

For sample size 4 under the “s” method, proceed as follows. After calculating the sample mean $\bar{x}$ and the sample standard deviation s , find the applicable value of the coefficient f_s from Table D.1, D.2 or D.3. Determine the maximum sample standard deviation (i.e. the maximum allowable) from the formula

$$\text{MSSD} = s_{\max} = (U - L)f_s.$$

Then compare s with the MSSD. If s is greater than the MSSD then the lot may be rejected without further calculation.

Otherwise determine the values of $Q_U = (U - \bar{x})/s$ and $Q_L = (\bar{x} - L)/s$. Calculate

$$(1) \quad \hat{p}_U = \begin{cases} 1 & \text{if } Q_U \leq -1.5 \\ 0.5 - Q_U/3 & \text{if } -1.5 < Q_U < 1.5 \\ 0 & \text{if } Q_U \geq 1.5 \end{cases}$$

and

$$(2) \quad \hat{p}_L = \begin{cases} 1 & \text{if } Q_L \leq -1.5 \\ 0.5 - Q_L/3 & \text{if } -1.5 < Q_L < 1.5 \\ 0 & \text{if } Q_L \geq 1.5 \end{cases}$$

Add these two estimates to obtain the estimate $\hat{p} = \hat{p}_U + \hat{p}_L$ of the overall process fraction nonconforming. If $\hat{p}$ does not exceed the applicable maximum allowable value, p^* , given in Table G.1, the lot is considered to be acceptable; otherwise, the lot is considered to be not acceptable.

NOTE The basis of formulae (1) and (2) is given in Annex N.

EXAMPLE

Items are being manufactured in lots of size 25. The lower and upper specification limits on their diameters are 82 to 84 mm. Items with diameters that are too large are equally unsatisfactory as those with diameters that are too small, and it has been decided to control the total fraction nonconforming using an AQL of 2.5% at inspection level II. Normal inspection is to be instituted at the beginning of inspection operations. From Table A.1, the sample size code letter is found to be C. From Table B.1, it is seen that a sample of size 4 is required. The diameters of four items from the first lot are measured, yielding diameters 82.4 mm, 82.2 mm, 83.1 mm and 82.3 mm. Conformance with the acceptability criterion under normal inspection is to be determined.

Information needed	Value obtained
Sample size: n	4
Sample mean: $\bar{x} = \frac{1}{n} \sum_{j=1}^n x_j$	82.50 mm
Sample standard deviation:	
$s = \sqrt{\sum_{j=1}^n (x_j - \bar{x})^2 / (n - 1)}$	0.408 2 mm
(See J.1.2 , Annex J.)	
Upper specification limit: U	84.0 mm
Lower specification limit: L	82.0 mm
Value of f_s for MSSD (Table D.1)	0.365
$s_{\max} = (U - L)f_s = (84 - 82) \times 0.365$	0.730 mm
As $s = 0.4082 < s_{\max} = 0.730$, the lot <i>might</i> be acceptable, so continue with the calculations.	

Further information needed	Value obtained
$Q_U = (U - \bar{x})/s = (84 - 82.5)/0.4082$	3.674 7
$Q_L = (\bar{x} - L)/s = (82.5 - 82)/0.4082$	1.224 9
$\hat{p}_U$ [from (1) above]	0.000 0
$\hat{p}_L$ [from (2) above]	0.091 7
$\hat{p} = \hat{p}_U + \hat{p}_L$	0.091 7
p^* (from Table G.1, normal inspection)	0.086 0

As $\hat{p} > p^*$, the lot is not acceptable.

15.4.4 Procedure for sample sizes greater than 4

After calculating the sample mean $\bar{x}$ and the sample standard deviation s , find the applicable value of the coefficient f_s from Table D.1, D.2 or D.3. Determine the maximum sample standard deviation (i.e. the maximum allowable) from the formula

$$\text{MSSD} = s_{\max} = (U - L)f_s.$$

Then compare s with $s_{\max}$. If s is greater than $s_{\max}$, then the lot may be rejected without further ado.

Otherwise, from among Charts s -D to s -R, consult the chart labelled with the appropriate sample size code letter and select the acceptance curve with the AQL specified for the two limits.

Then calculate the values of $s/(U - L)$ and $(\bar{x} - L)/(U - L)$ and plot a point representing these values on a copy of the graph. If the point lies inside the curve, the lot is acceptable; if outside, the lot is not acceptable.

For greater convenience, it is recommended that, before the inspection operations begin, the required acceptance curves for normal, tightened and reduced inspection be copied. The scales should be adjusted so that s and $\bar{x}$ can be plotted directly (i.e. the upper specification limit is given instead of 1.0 and the lower specification limit instead of 0.0 on the vertical scale).

Then plot a point on the chart representing the values of s and $\bar{x}$ found from the sample. If the point lies inside the curve, the lot is acceptable; if outside, it is not acceptable.

EXAMPLE Determination of acceptability for combined control of double specification limits when the sample size is 5 or more.

The minimum temperature of operation for a certain device is specified as 60 °C and the maximum temperature as 70 °C. Production is in inspection lots of 80 items. Inspection level II, normal inspection, with AQL = 1.5%, is to be used. From Table A.1, the sample size code letter is found to be E; from Table B.1 it is seen that a sample of 13 is required, and from Table D.1 that the value of f_s for the MSSD under normal inspection is 0.274. Suppose the measurements obtained are as follows: 63.5 °C; 61.9 °C; 65.2 °C; 61.7 °C; 68.4 °C; 67.1 °C; 60.0 °C; 66.4 °C; 62.8 °C; 68.0 °C; 63.4 °C; 60.7 °C; 65.8 °C. Conformance with the acceptability criterion is to be determined.

Information needed	Value obtained
Sample size: n	13
Sample mean: $\bar{x} = \sum x/n$	64.223 °C
Sample standard deviation: $s = \sqrt{\sum_j (x_j - \bar{x})^2 / (n - 1)}$	2.789 9 °C
(See K.2 , Annex K)	
Upper specification limit: U	70.0 °C
Lower specification limit: L	60.0 °C
Value of f_s for MSSD (Table F.1 for normal inspection)	0.274
MSSD = $s_{\max} = (U - L)f_s = (70 - 60) \times 0.274$	2.74 °C

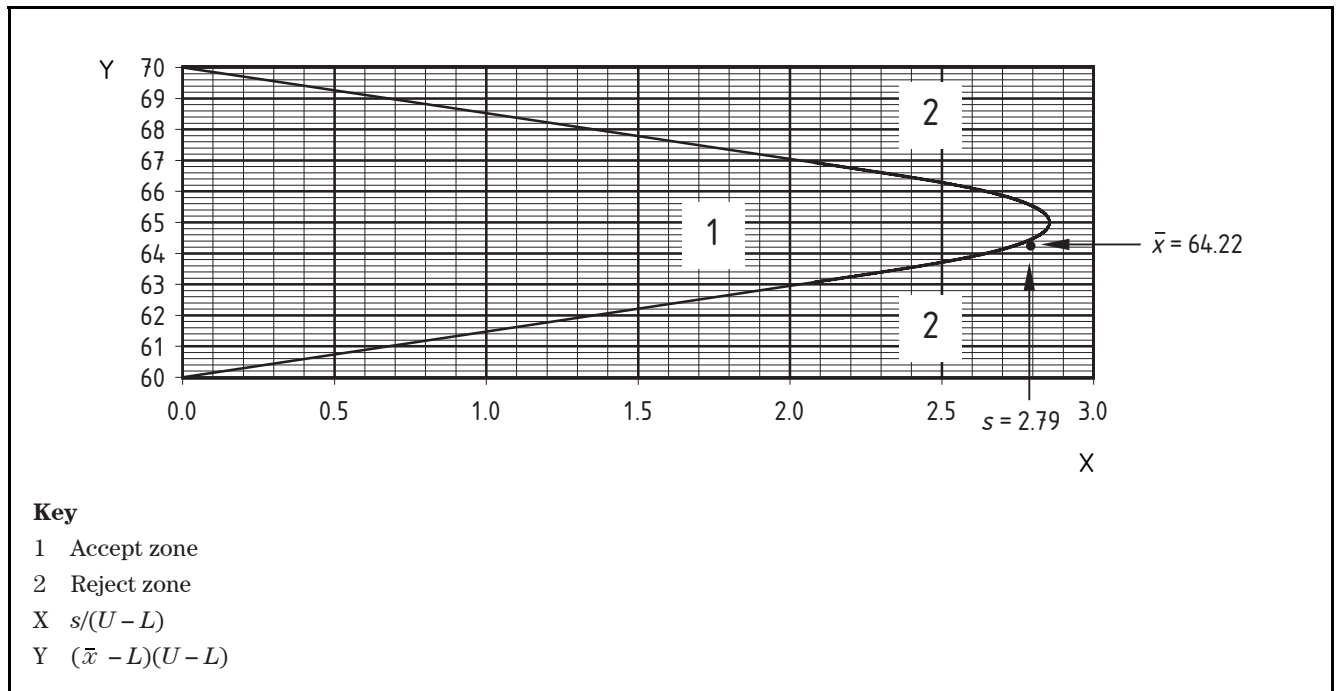
As the value of s exceeds $s_{\max}$, the lot may immediately be adjudged unacceptable.

NOTE This lot is not acceptable even though all inspected items **in the sample** are within the specification limits.

Suppose that the AQL had been 2.5% instead of 1.5%. In that case, the value of f_s would be 0.285, so $s_{\max}$ is equal to $(70 - 60) \times 0.285$ °C. As s is now less than $s_{\max}$, it is not possible to determine immediately whether or not the lot is acceptable.

The appropriate acceptance curve is taken from Chart s-E. If, as on Figure 2, the scales have been adjusted to the real measurements, plot the point ($s = 2.79$, $\bar{x} = 64.22$). The point lies just outside the acceptance curve for an AQL of 2.5%, so the lot is not acceptable.

Figure 2 Example of the use of an acceptance chart for combined control of double specification limits: "s" method with actual scales



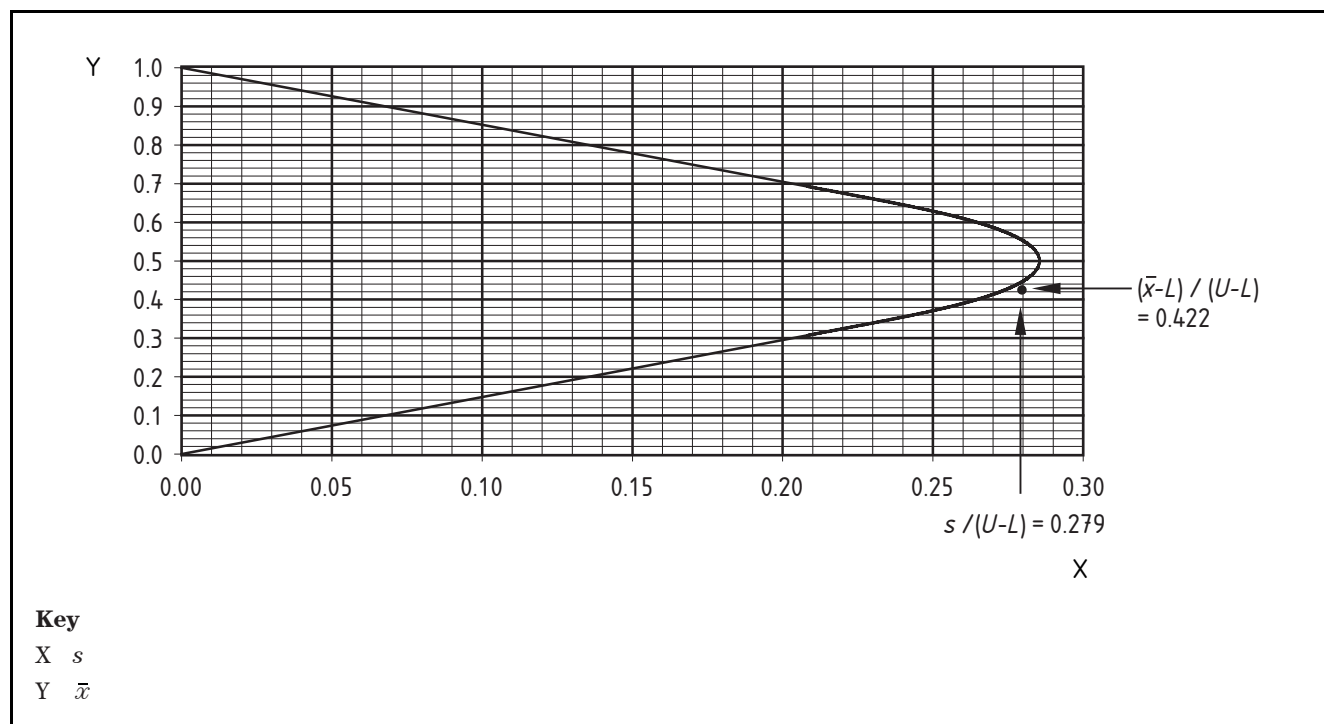
If the scale of the chart has not been adjusted to the values of s and $\bar{x}$, the following additional calculations are required:

$$\begin{aligned} \text{Standardized sample mean: } & (\bar{x} - L)/(U - L) \\ & = (64.22 - 60)/(70 - 60) = 0.422. \end{aligned}$$

$$\begin{aligned} \text{Standardized sample standard deviation: } & s/(U - L) \\ & = 2.79/(70 - 60) = 0.279. \end{aligned}$$

The point (0.279, 0.422) is plotted on Figure 3. As it lies outside the acceptance curve for AQL = 2.5%, the lot is not acceptable.

Figure 3 Example of the use of an acceptance chart for combined control of double specification limits: “s” method with normalized scales



16 Standard procedure for the “ σ ” method

16.1 Obtaining a plan, sampling and preliminary calculations

The “ σ ” method is only to be used when there is valid evidence that the standard deviation, σ , of the process can be considered constant with a known value.

From Table A.1 obtain the sample size code letter. Then, depending on the severity of inspection, enter Table C.1, C.2 or C.3 with the sample size code letter and the specified AQL to obtain the sample size n and acceptability constant k .

Take a random sample of this size, measure the characteristic under inspection, x , for all items of the sample and calculate the sample mean $\bar{x}$. (The sample standard deviation s should also be calculated, but only for the purpose of checking the continued stability of the process standard deviation. See Clause 19.)

16.2 Acceptability criteria for a single specification limit

The acceptability criterion can be found by following a similar procedure to that given for the “s” method. First replace the s derived from the individual samples by σ , the presumed known value of the standard deviation of the process, and then compare the calculated value of Q with the value of the acceptability constant k obtained from one of the Tables C.1, C.2 and C.3.

NOTE For example, the acceptability criterion $Q_U [= (U - \bar{x})/\sigma] \geq k$ for an upper specification may be written as $\bar{x} \leq U - k\sigma$. As U , k and σ are all known in advance, the acceptance value $\bar{x}_U [= U - k\sigma]$ should therefore be determined before inspection begins. For an upper specification limit, a lot will be

acceptable if $\bar{x} \leq \bar{x}_U [= U - k\sigma]$; not acceptable if $\bar{x} > \bar{x}_U [= U - k\sigma]$.

Similarly, for a lower specification limit, a lot will be

acceptable if $\bar{x} \geq \bar{x}_L [= L + k\sigma]$; not acceptable if $\bar{x} < \bar{x}_L [= L + k\sigma]$.

EXAMPLE Determination of acceptability for a single specification limit using the “ σ ” method.

The specified minimum yield point for certain steel castings is 400 N/mm². A lot of 500 items is submitted for inspection. Inspection level II, normal inspection, with AQL = 1.0%, is to be used. The value of σ is considered to be 21 N/mm². From Table A.1, it is seen that the sample size code letter is H. Then, from Table C.1, it is seen that for an AQL of 1.0%, the sample size n is 11 and the acceptability constant k is 1.830. Suppose the yield points of the sample specimens are 431; 417; 469; 407; 450; 452; 427; 411; 429; 420; 400. Conformance with the acceptability criterion is to be determined.

Information needed	Value obtained
Acceptability constant: k	1.830
Known σ	21 N/mm ²
Product: $k\sigma$	38.43 N/mm ²
Specification limit: L	400 N/mm ²
Acceptance value: $\bar{x}_L = L + k\sigma$	438.43 N/mm ²
Sum of measurement results: $\sum x$	4 713 N/mm ²
Sample size: n	11
Sample mean: $\bar{x}$	428.45 N/mm ²
Acceptability criterion: Is $\bar{x} \geq \bar{x}_L$?	No

The sample mean of the lot does not meet the acceptability criterion so the lot is not acceptable.

16.3 Acceptability criterion for combined control of double specification limits

For combined control of the upper and the lower specification limits, i.e. where there is an overall AQL for the percentage of the process outside the specification limits, the following approximative procedure may be used:

- Before sampling, enter Table E.1 with the AQL to determine the value of the factor f_σ .
- Calculate the maximum allowable value of the process standard deviation, using the formula $\sigma_{\max} = (U - L)f_\sigma$ for the MPSD.
- Compare the value of the process standard deviation σ with $\sigma_{\max}$. If σ exceeds $\sigma_{\max}$, the process is unacceptable and sampling inspection is pointless until it is demonstrated that the process variability has been adequately reduced.
- If $\sigma \leq \sigma_{\max}$, use the lot size and given inspection level to determine the sample size code letter from Table A.1.
- From the sample size code letter and inspection severity (i.e. whether inspection is normal, tightened or reduced) determine the sample size, n , and acceptability constant, k , from Table C.1, C.2 or C.3.
- Calculate the upper allowable bound, $\bar{x}_U$, to sample means from the formula $\bar{x}_U = U - k\sigma$, and the lower allowable bound $\bar{x}_L$ from the formula $\bar{x}_L = L + k\sigma$.
- Select a random sample of size n from the lot and calculate the sample mean $\bar{x}$. The acceptability criterion is: If $\bar{x}_L \leq \bar{x} \leq \bar{x}_U$, the lot is acceptable; if $\bar{x} < \bar{x}_L$ or $\bar{x} > \bar{x}_U$, the lot is not acceptable.

NOTE If $\bar{x}_L \leq \bar{x} \leq \bar{x}_U$ but $\sigma > 0.75 \sigma_{\max}$ and $\bar{x}$ is close to either $\bar{x}_L$ or $\bar{x}_U$, the exact method given in 17.3 of BS 6002-2 is preferred.

EXAMPLE Determination of acceptability for combined control under the “ σ ” method

The specification for electrical resistance of a certain electrical component is $(520 \pm 50) \Omega$. Production is at a rate of 1 000 items per inspection lot. Inspection level II, normal inspection, with a single AQL of 1.5%, is to be used for the two specification limits (470 Ω and 570 Ω). σ is known to be 18.5 Ω .

Information needed	Value obtained
Factor from Table E.1: f_σ	0.194
Upper specification limit: U	570 Ω
Lower specification limit: L	470 Ω
Maximum process standard deviation, $\sigma_{\max} = (U - L)f_\sigma$	19.4 Ω
Known σ	18.5 Ω

As σ is less than $\sigma_{\max}$, the sample is analysed further for lot acceptability.