

Machine tool ball screws —

Part 4: Method of calculation of static axial rigidity

UDC 621.9:621.83.05-219.51

Committees responsible for this British Standard

The preparation of this British Standard was entrusted by the Machine, Engineers and Hand Tools Standards Committee (MTE/-) to Technical Committee MTE/1 upon which the following bodies were represented:

Advanced Manufacturing Technology Research Institute
Department of Trade and Industry (Mechanical and Electrical Engineering Division)
Federation of British Engineers' Tool Manufacturers
GAMBICA (BEAMA Ltd.)
Health and Safety Executive
Institution of Production Engineers
Ministry of Defence
Coopted members

This British Standard, having been prepared under the direction of the Machine, Engineers and Hand Tools Standards Committee, was published under the authority of the Board of BSI and comes into effect on 31 July 1987

© BSI 12-1999

The following BSI references relate to the work on this standard:
Committee reference MTE/1
Draft for comment 85/72760 DC

ISBN 0 580 15441 6

Amendments issued since publication

Amd. No.	Date of issue	Comments
6264	November 1989	Indicated by a sideline in the margin

Contents

	Page
Committees responsible	Inside front cover
Foreword	ii
0 Introduction	1
1 Scope	1
2 Definitions	1
3 Notation	1
4 Method of determining rigidity	2
4.1 General	2
4.2 Overall rigidity	3
4.3 Rigidity of the ball screw shaft	3
4.4 Rigidity of nut body and screw shaft under radial component due to axial load	4
4.5 Rigidity in the ball/ball track area	4
4.6 Rigidity of a symmetrically preloaded nut system	7
4.7 Rigidity of the ball screw within the loaded ball nut area	7
4.8 Accuracy factor	7
4.9 Overall rigidity	7
Appendix A A graphical solution to 4.5. Rigidity in the ball/ball track area	8
Appendix B Worked example of calculation of rigidity	16
Figure 1 — Notation of the ball screw	1
Figure 2 — Ball screw shaft rigidly mounted at one end	3
Figure 3 — Ball screw shaft rigidly mounted at both ends	3
Figure 4 — Relative displacement between ball nut and ball screw shaft due to axial backlash	5
Figure 5 — Preloaded ball nuts	5
Figure 6 — Graphs to illustrate calculation of rigidity	9
Table 1 — Accuracy factor	7
Publications referred to	Inside back cover

Foreword

This Part of BS 6101 has been prepared under the direction of the Machine, Engineers and Hand Tools Standards Committee. It has been produced in advance of related work being prepared in the International Organization for Standardization (ISO) in order to facilitate discussion between manufacturers and purchasers of machine tool ball screws.

A British Standard does not purport to include all the necessary provisions of a contract. Users of British Standards are responsible for their correct application.

Compliance with a British Standard does not of itself confer immunity from legal obligations.

Summary of pages

This document comprises a front cover, an inside front cover, pages i and ii, pages 1 to 18, an inside back cover and a back cover.

This standard has been updated (see copyright date) and may have had amendments incorporated. This will be indicated in the amendment table on the inside front cover.

Symbol	Term	Symbol	Term
c_E	Material constant (for steel $c_E = 0.4643$)	l	Length (in mm)
c_k	Geometry factor	P	Lead (in mm)
d_c	Diameter of load application on the ball screw shaft (in mm)	r	Radius (in mm)
D_c	Diameter of load application on the ball nut (in mm)	R	Rigidity (in N/μm)
dl	Elastic deflection (in μm)	R_{tot}	Overall rigidity (in N/μm)
D_w	Ball diameter (in mm)	Y	Auxiliary value according to Hertz for the description of the elliptic integrals of the 1st and 2nd kind
d_0	Nominal diameter (in mm) of ball screw	Z_1	Number of loaded balls per turn
d_1	Ball screw shaft outer diameter (in mm)	ϕ	Lead angle (in degrees)
D_1	Ball nut outer diameter (in mm)	α	Contact angle (in degrees)
D_3	Ball nut internal diameter (in mm)	ρ	Reciprocal curvature radius (in 1/mm)
E	Young's modulus of elasticity (in N/mm ²) (for steel = 2.1×10^5)	τ	Ratio of the semimajor to the semiminor axes of the contact ellipse
F_a	Axial force (in N)	Subscript	
F_{am}	Limit load of anti-backlash double-nut system (in N)	a	Axial direction
F_N	Normal force (in N)	b/t	Ball and ball track area
F_P	Preload force (in N)	m	Mean
F_1	Actual axial force applied to ball nut 1 (in N)	n	Ball nut
F_2	Actual axial force applied to ball nut 2 (in N)	n/s	Nut body and screw shaft area
f_{ar}	Accuracy factor (rigidity)	p	Preload
f_r	Conformity factor (ratio of ball track radius divided by ball diameter)	s	Screw shaft
i	Number of loaded turns	u	Ball screw within the loaded nut area
k	Rigidity characteristic (in N/μm ^{2/3})	u'	Ball screw within the loaded nut area modified by accuracy factor (f_{ar})

4 Method of determining rigidity

4.1 General

The static axial rigidity R (in N/μm) is the resistance to deformation and denotes the force F (in N) which is required to effect a component deflection dl_a by 1 μm in the direction of load applications. The value of R shall be calculated from the equation:

$$R = \frac{F_a}{dl_a} \quad (1)$$

4.2 Overall rigidity

The inverse of the overall rigidity R_{tot} (in N/μm) is the sum of the inverse rigidity values of the components and shall be calculated from the equation:

$$\frac{1}{R_{\text{tot}}} = \frac{1}{R_s} + \frac{1}{R_u'} \quad (2)$$

R_u is the sum of $R_{n/s}$ and $R_{b/t}$. The methods of calculating the rigidity values of the components R_s , $R_{n/s}$, $R_{b/t}$ and R_u' are given in 4.3, 4.4, 4.5 and 4.7 respectively.

4.3 Rigidity of the ball screw shaft

4.3.1 General. The rigidity of the ball screw shaft R_s (in N/μm) follows from the elastic deflection of the shaft dl s caused by the axial force F_a and is dependent on the bearing arrangement. The value of R_s shall be calculated as described in 4.3.1 and 4.3.2.

4.3.2 Rigid mounting of the ball screw shaft at one end. (See Figure 2.)

The rigidity R_{s1} (in N/μm) of the ball screw shaft rigidly mounted at one end shall be calculated from the following equation:

$$R_{s1} = \left(\frac{\pi d_c^2 E}{4 l_s} \right) 10^{-3} \quad (3)$$

where

$$d_c = d_0 - D_w \cos \alpha \quad (4)$$

4.3.3 Rigid mounting of the ball screw shaft at both ends. (See Figure 3.)

The rigidity of the ball screw shaft rigidly mounted at both ends R_{s2} (in N/μm) shall be calculated from the following equation:

$$R_{s2} = \frac{\pi d_c^2 E}{4 l_{s2}} \left(\frac{l_s}{l_s - l_{s2}} \right) \times 10^{-3} \quad (5)$$

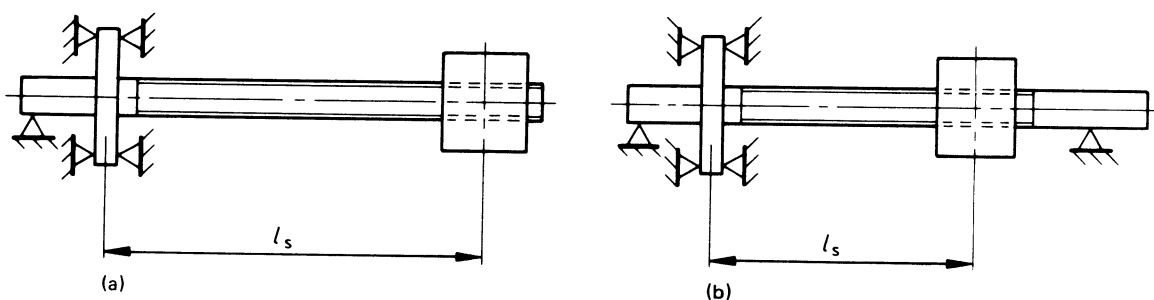


Figure 2 — Ball screw shaft rigidly mounted at one end

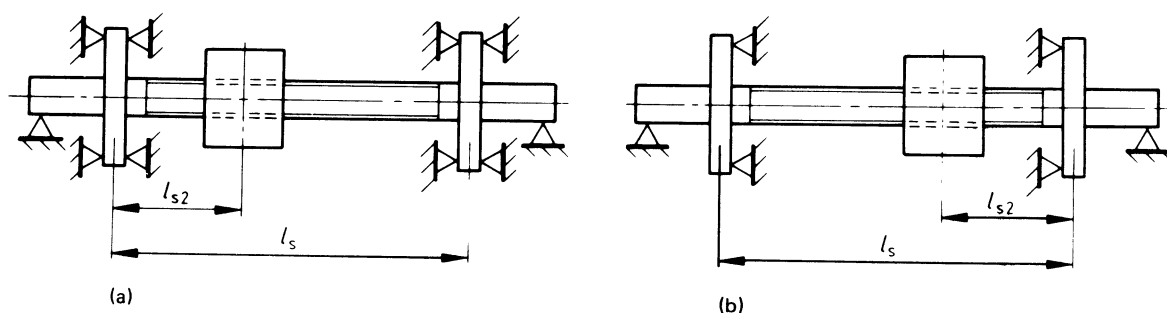


Figure 3 — Ball screw shaft rigidly mounted at both ends

The minimum of rigidity shall be calculated from the following equation:

$$l_{s2} = \frac{l_s}{2}$$

and thus

$$R_{s2 \min} = \left(\frac{\pi d_c^2 E}{l_s} \right) \times 10^{-3} \quad (6)$$

4.4 Rigidity of nut body and screw shaft under radial component due to axial load

The rigidity of the nut body and screw shaft under radial component due to axial load, $R_{n/s}$ (in N/μm), shall be calculated from the following equation:

$$R_{n/s} = \frac{F_a}{dl_{n/s}} \quad (7)$$

$$dl_{n/s} = \frac{1}{R_{n/s}} F_a \quad (8)$$

The axial rigidity of nut body and screw $R_{n/s}$ (in N/μm) under this type of load shall be calculated from the following equation:

$$R_{n/s} = \frac{2 \pi i P E \tan^2 \alpha}{\left(\frac{D_1^2 + D_c^2}{D_1^2 - D_c^2} + 1 \right)} \times 10^{-3} \quad (9)$$

where

$$D_c = d_0 + D_w \cos \alpha \quad (10)$$

NOTE It has been assumed that the ball screw shaft is solid and that the ball screw shaft and ball nut have the same Young's modulus and Poisson's ratio.

As both nut bodies act like preloaded rings, the rigidity $R_{n/sp}$ of a double nut is twice as high as that of a single nut, and shall be calculated from:

$$R_{n/sp} = 2 \times R_{n/s} \quad (11)$$

4.5 Rigidity in the ball/ball track area

NOTE 1 An alternative graphical method, suitable for approximation only, is given in Appendix A.

NOTE 2 The following have been disregarded in the calculation:

- the nut expansion factor (common ball screws are assumed to have a sufficiently large thread cross section);
- uneven distribution of load on the balls and threads;
- machining inaccuracies;
- change of contact angle;
- relative displacement between ball nut and ball screw shaft due to the axial backlash [see Figure 4(a) and Figure 4(b)].

4.5.1 General

In order to obtain high rigidity in the ball/ball track area, nut systems are preloaded (see Figure 5). Thus the backlash in the individual nut and the relatively large ball/ball track deflection at low load are eliminated.

The preload to be applied has to be determined carefully as excessive preload will reduce life.

The preload F_p for symmetrical double nuts shall be calculated from the following equation:

$$F_p = \frac{1}{2.83} F_{am} \quad (12)$$

NOTE This equation gives an approximate value for F_p .

For loads beyond F_{am} , the point at which the preload nut is unloaded, the double-nut system then exhibits single nut behaviour.

The extent of deflection in the ball/ball track area is a function of the following:

- load applied;
- number of loaded balls;
- conformity;
- contact angle;
- ball size;
- nominal diameter.

The axial deflection in the ball/ball track area can be derived from Hertzian formula but shall be approximated by the following equation:

$$dl_{b/t} = \frac{dl_{sb/t} + dl_{nb/t}}{\cos \phi \sin \alpha} \quad (13)$$

Following Hertz, the approach of the components shall be calculated from:

$$dl_{s,n \ b/t} = Y_{s,n} \times c_E^2 \times \sqrt[3]{(F_N^2 \Sigma \rho_{s,n})} \quad (14)$$

where

$$F_N = \frac{F_a}{i Z_1 \cos \phi \sin \alpha} \quad (15)$$