

Guide to

Assessment of position, size and departure from nominal form of geometric features

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- Department of Trade and Industry (National Engineering Laboratory)
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Foreword

This British Standard has been prepared under the direction of the Advanced Manufacturing Technology Standards Policy Committee.

This standard is intended for use by manufacturers of coordinate measuring machines (CMMs) and software writers within the CMM industry.

It covers the assessment of geometric form of workpieces measured with a CMM. It gives information and guidance to promote the better use of CMMs for this purpose by the adoption of reliable software that provides well presented comprehensive information.

This British Standard contains mathematical concepts and notation; therefore it is assumed that the execution of its guidance is entrusted to appropriately qualified and experienced personnel.

A British Standard does not purport to include all the necessary provisions of a contract. Users of British Standards are responsible for their correct application.

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Summary of pages

This document comprises a front cover, an inside front cover, pages i and ii, pages 1 to 17 and a back cover.

This standard has been updated (see copyright date) and may have had amendments incorporated. This will be indicated in the amendment table on the inside front cover.

1 Scope

This British Standard provides information and guidance to manufacturers of coordinate measuring machines (CMMs), and particularly software writers within the CMM industry. It contains recommendations for determining the position, size and departure from nominal form of geometric features, given measurements of coordinates of points on a workpiece. The features covered correspond to the following geometric elements: lines, planes, circles, spheres, cylinders and cones.

This standard is concerned with software implementations of algorithms based on sound mathematical and computational principles, rather than automated versions of manual or graphical assessment procedures.

This standard does not provide detailed guidance on methods for treating data gathered in an unstable environment.

This standard does not cover secondary attributes, i.e. measures derived from departures from form of the above geometric elements, such as parallelism, concentricity or orthogonality.

This standard primarily relates to CMMs that operate with a right-handed Cartesian coordinate system. It also relates to other measurement systems that provide such coordinates. Some of the guidance provided also applies to other coordinate systems.

2 Definitions

For the purposes of this British Standard the following definitions apply.

2.1

algorithm

a step-by-step description in mathematical or other unambiguous terms of a process for solving a particular problem, e.g. the determination of the parameters describing a geometric feature

2.2

centroid

the point described by coordinates which are the arithmetic means of the corresponding coordinates of the data points

2.3

circularity

measure of departure from nominal form for a mathematical circle

2.4

cylindricity

measure of departure from nominal form for a mathematical cylinder

2.5

data point

either a raw data point or a raw data point that has been processed in some way

2.6

departure from nominal form

overall measure of the deviation of a workpiece from nominal form

NOTE The departure from nominal form is defined as the spread, or in terms of the spread.

2.7

deviation

the straight-line (Euclidean) distance of a data point from the reference, measured normal to the geometric element. The distance is regarded as positive or negative according to which side of the element the point lies. Where appropriate it is negative if the point lies in the material of the workpiece, and positive otherwise. In the case of a line in two dimensions or a plane all points on one side are taken to have a positive deviation and those on the other a negative deviation. The deviation for the i th data point is denoted by e_i

2.8

direction cosines (of a line)

the cosines of the angles between a line and the Cartesian axes

2.9

geometric element

line, plane, circle, cylinder, cone or sphere

2.10

geometric feature

(part of) an object nominally in the shape of one of the geometric elements

2.11

line

a straight line in two or three dimensions

2.12

measurement procedure

a strategy for obtaining a representative set of points on a workpiece

2.13

nominal form

the ideal geometric object of which the geometric feature under test is a machined or otherwise manufactured manifestation, e.g. sphere

2.14

normal

a line passing through a point on a curve or surface and perpendicular to the tangent line or plane at the point

NOTE A normal conventionally points out of the material.

2.15 outlier

a data point that is not regarded as a member of a set of data points representative of the geometric feature

NOTE An outlier may arise from malfunction of the CMM or an error on the part of its operator.

2.16 parameters

algebraic variables representing the size and position of a geometric element, e.g. the radius and centre coordinates of a circle

2.17 parameter values

numerical values of parameters

2.18 parametrization

a choice of algebraic variables to represent a geometric element

2.19 pre-processing

operations upon measured data intended to render it more suitable for purposes of assessment of form

2.20 range of the deviations

the difference between the largest and smallest signed deviations, i.e. $\max_i e_i - \min_i e_i$

2.21 raw data

measured coordinates of points on the boundary or surface of the geometric feature

2.22 reference

a computed geometric element to be used as a basis for assessment

2.23 representative set of points

a set of points that, for the purposes of the assessment, adequately represent the geometric feature

2.24 residual

a measure of the error of fit of a reference or trial reference at a point. At the i th point the residual is denoted by res_i

2.25 root mean square (r.m.s.) deviation

the square root of the quotient of the sum of the squares of the deviations and the number of degrees of freedom v , i.e. $\sqrt{(\sum_i e_i^2/v)}$

2.26 software, software implementation

a computer implementation of an algorithm

2.27 spread

a measure of the scatter of the deviations

NOTE Two useful definitions are the range of the deviations and the root mean square deviation. A further measure of spread is a suitable multiple of the root mean square deviation.

2.28 uniform pseudorandom number generator

an algorithm or software for producing a sequence of numbers that, according to statistical tests, appear to be samples from a rectangular distribution

2.29 workpiece

the object or component under test, containing the geometric feature being assessed

3 Symbols and abbreviations

For the purposes of this British Standard the following symbols and abbreviations apply. Several meanings are given to some of the symbols and the specific meaning is implied in each case by the context in which the symbols are used.

a	Direction cosine for x .
b	Direction cosine for y .
c	Direction cosine for z .
c	Constant in circle equation.
C	Circle.
C	Cylinder.
C	Cone.
d	Distance of a point from a geometric element.
e_i	Deviation of the i th data point from a reference.
E	Objective function used in computing a reference.
f	Parameter in circle equation.
f	Intermediate variable in distance formulae.
F	Measure of departure from nominal form.
g	Parameter in circle equation.
g	Intermediate variable in distance formula.

G	Point on L or P closest to the centroid of the data points.	y_0, y_1, y_2	y -coordinates of locating points for a straight line or axis of a cylinder or a cone.
h	Distance between two parallel planes.	y_p	y -coordinate of a general point on a geometric feature when computing distance from the point to a geometric element.
h	Height of frustum of cone.	z	Third Cartesian coordinate.
i	Subscript for data point.	$\bar{z}$	Arithmetic mean of values of z_i ($= \Sigma_i z_i / N$).
l	Length of generator of frustum of cone.	z_0, z_1, z_2	z -coordinate of locating points for a straight line or axis of a cylinder or a cone.
L	Straight line.	z_p	z -coordinate of a general point on a geometric feature when computing distance from the point to a geometric element.
n	Number of parameters necessary to describe a geometric element, which normally is also the same as the mathematical minimum number of points required to define the element.	θ	Bearing angle for cylindrical or spherical coordinates.
n_c	Number of approximately uniformly spaced parallel planes.	v	Number of degrees of freedom, given by $N - n$.
n_p	Number of measurements on or near a plane.	ϕ	Azimuth angle for spherical coordinates.
N	Number of measured points on a workpiece.	ψ	Apex angle of cone (equal to twice the angle between the cone's generator and axis).
P	Plane.	4 Outline of guide In order to obtain a reliable assessment of geometric form in any particular case, the corresponding geometric element should first be represented, i.e. parametrized, in a mathematically sound way. Recommended parametrizations are given in clause 5. It is recommended that the assessment process itself be carried out in four stages: - apply an appropriate measurement procedure, i.e. a strategy for obtaining a representative set of measurements on the workpiece (see clause 6); (optionally) pre-process the data, i.e. replace the measured data by modified values in order, for example, to smooth the data, to remove inappropriate points or to compensate for environmental effects (see clause 7); - compute the reference (e.g. an approximating circle in terms of its centre coordinates and radius), to give position and size (see clause 8); - assess, in terms of the reference, the departure from nominal form (see clause 9). Once the assessment has been carried out, it is recommended that the software provides the information detailed in clause 10. To avoid unnecessary numerical inaccuracies during the assessment, software writers should adopt the recommendations given in clause 11.	
q	Number of lobes on a nominally circular feature.		
r	Radius of circle, sphere or cylinder.		
r	Radial coordinate in a cylindrical or spherical coordinate system.		
r_1, r_2	Radii of ends of frustum of cone.		
r_i	Distance of i th data point from centre of reference circle.		
res_i	Residual evaluated at the i th data point.		
s	Difference between number of measurements on successive planes in a measurement procedure for a cone.		
s	Distance from the surface of a cone to a point on its axis.		
S	Sphere		
S	Set of data points.		
t	Parameter proportional to distance.		
u, v, w	Intermediate variables in distance formulae.		
x	First Cartesian coordinate.		
$\bar{x}$	Arithmetic mean of values of x_i ($= \Sigma_i x_i / N$).		
x_0, x_1, x_2	x -coordinates of locating points for a straight line or axis of a cylinder or a cone.		
x_p	x -coordinate of a general point on a geometric feature when computing distance from the point to a geometric element.		
y	Second Cartesian coordinate.		
$\bar{y}$	Arithmetic mean of values of y_i ($= \Sigma_i y_i / N$).		